الوحدة الخامسة

التكامل

التكامل غير المحدد

① اثبت ان $F(x) = 5 - \frac{1}{3}x^3$ هي مشتقة عكسية للدالة $f(x) = -x^2$ ثم اكتب مشتقة عكسية اخرى لها P.14

$$F'(x) = 0 - \left(\frac{1}{3}\right) 3x^2 = -x^2 = f(x)$$

$\because F'(x) = f(x)$

$\therefore F$ هي مشتقة عكسية للدالة f

مشتقة عكسية أخرى

$$F_1(x) = 10 - \frac{1}{3}x^3$$

P.14

② اثبت أن $F(x) = \frac{x^3+1}{x^2}$ هي مشتقة عكسية للدالة $f(x) = 1 - \frac{2}{x^3}$

$$F(x) = \frac{x^3+1}{x^2} = \frac{x^3}{x^2} + \frac{1}{x^2}$$

$$= x + \frac{1}{x^2} = x + x^{-2}$$

$$F'(x) = 1 - 2x^{-3} = 1 - \frac{2}{x^3} = f(x)$$

$\because F'(x) = f(x)$

$\therefore F$ هي مشتقة عكسية للدالة f

③ P. 15

ⓐ $\int 15\,dx = 15x + c$

ⓑ $\int 5x^4\,dx = \frac{5}{5}x^5 + c = x^5 + c$

④ P. 16

$$\int (3x^2 - 4x - 1)\,dx = \frac{3}{3}x^3 - \frac{4}{2}x^2 - x + c$$

$$= x^3 - 2x^2 - x + c$$

⑤ P. 17

ⓐ $$\int (2x-3)(x+4)\,dx = \int 2x^2 + 8x - 3x - 12\ dx$$
$$= \int 2x^2 + 5x - 12\ dx$$
$$= \frac{2}{3}x^3 + \frac{5}{2}x^2 + 12x + c$$

ⓑ $$\int \frac{x^2+5x+4}{x+1}\,dx = \int \frac{(\cancel{x+1})(x+4)}{(\cancel{x+1})}\,dx$$

$$= \int x - 4\ dx = \frac{1}{2}x^2 + 4x + c$$

ⓒ $$\int \left(\frac{3x^2 - x}{x}\right)^2 dx = \int \frac{(3x^2-x)^2}{x^2}\,dx = \int \frac{9x^4 - 6x^3 + x^2}{x^2}\,dx$$

$$= \int \frac{9x^4}{x^2} - \frac{6x^3}{x^2} + \frac{x^2}{x^2}\,dx$$

$$= \int (9x^2 - 6x + 1)\,dx$$

$$= \frac{9}{3}x^3 - \frac{6}{2}x^2 + x + c$$

$$= 3x^3 - 3x^2 + x + c$$

⑥ P. 17

ⓐ $\int x\sqrt{x}\,dx = \int x\,x^{\frac{1}{2}}\,dx = \int x^{\frac{3}{2}}\,dx$

$= \frac{2}{5}x^{\frac{5}{2}} + c = \frac{2}{5}\sqrt{x^5} + c$

ⓑ $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}} + c = 2\sqrt{x} + C$

ⓒ $\int \frac{x^2-3x}{\sqrt[3]{x}}\,dx = \int (x^2-3x)\,x^{-\frac{1}{3}}\,dx$

$= \int x^{\frac{5}{3}} - 3x^{\frac{2}{3}}\,dx = \frac{3}{8}x^{\frac{8}{3}} - 3\cdot\frac{3}{5}x^{\frac{5}{3}} + C$

$= \frac{3}{8}\sqrt[3]{x^8} - \frac{9}{5}\sqrt[3]{x^5} + c$

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⑦ P.18

$F(x) = \int (2x+5)\,dx$

$F(x) = x^2 + 5x + c$ $\qquad F(-1) = 0$

$F(-1) = 1 - 5 + c = 0$

$c = 4$

$\therefore F(x) = x^2 + 5x + 4$

$S(t)$ t $V(t)$ $a(t)$ (8) P.19

إزاحة الزمن السرعة العجلة

$a(t) = \frac{dv}{dt} = -9.8$ «سالب بسبب القذف لأعلى»

$V(t) = \int a(t)\,dt = \int -9.8\,dt$

$= -9.8t + c$

$V(0) = 12 \Rightarrow \quad 12 = 0 + c \Rightarrow c = 12$

$V(t) = -9.8t + 12$

(a) عند أقصى ارتفاع $V(t) = 0 \Leftarrow$

$0 = -9.8t + 12$

$t = \frac{12}{9.8} = 1.22$

(b) $S(t) = \int v(t)\,dt = \int (-9.8t + 12)\,dt$

$= \frac{-9.8}{2}t^2 + 12t + c$ لحظة القذف:

$80 = 0 + 0 + c \Rightarrow c = 80$ لحظة القذف:

$t = 0,\ S = 80$

$\therefore S(t) = -4.9t^2 + 12t + 80$

تصل الكرة الى سطح الأرض عندما $S(t) = 0$

$\therefore 4.9t^2 - 12t - 80 = 0$

$t = -3$ مرفوض أو $t = 5.45$

$\therefore$ تصل الكرة الى الأرض بعد 5.45 ثانية

التكامل بالتعويض

① P. 21

ⓐ $\int (x^3+4x^2+x)^7 (3x^2+8x+1)\, dx$

$u = x^3+4x^2+x$

$du = (3x^2+8x+1)\, dx$

$= \int u^7\, du = \frac{1}{8} u^8 + c$

$= \frac{1}{8} (x^3+4x^2+x)^8 + c$

ⓑ $\int \sqrt[3]{x^2-5x+2}\ (2x-5)\, dx$

$u = x^2-5x+2$

$du = (2x-5)\, dx$

$= \int u^{\frac{1}{3}}\, du = \frac{3}{4} u^{\frac{4}{3}} + c$

$= \frac{3}{4} \sqrt[3]{(x^2-5x+2)^4} + c$

ⓐ $\int \sqrt[5]{3x+7}\, dx =$ ② P.22

$u = 3x+7$

$du = 3\, dx$

$= \int \frac{1}{3} (3x+7)^{\frac{1}{5}}\, 3\, dx$

$= \int \frac{1}{3} u^{\frac{1}{5}}\, du = \frac{1}{3} \cdot \frac{5}{6} u^{\frac{6}{5}} + c = \frac{5}{18} u^{\frac{6}{5}} + c$

$= \frac{5}{18} \sqrt[5]{(3x+7)^6} + c$

② P. 22

(b) $\int \frac{3(\sqrt[3]{x}-5)}{\sqrt[3]{x^2}}\,dx =$

$= \int (3)\,3\,(x^{\frac{1}{3}}-5)\ \frac{1}{3}\,x^{-\frac{2}{3}}\,dx$

$u = x^{\frac{1}{3}} - 5$

$du = \frac{1}{3}\,x^{-\frac{2}{3}}\,dx$

$= \int 9\,u\,du$

$= \frac{9}{2}u^2 + c = \frac{9}{2}(\sqrt[3]{x}-5)^2 + c$

③ P. 23

$\int x(2x-1)^3\,dx =$

$u = 2x-1$

$2x = u+1 \Rightarrow 2dx = du$

$= \frac{1}{4}\int 2x\,(2x-1)^3\,2\,dx$

$= \frac{1}{4}\int (u+1)\,u^3\,du = \frac{1}{4}\left[\frac{u^5}{5} + \frac{u^4}{4}\right] + c$

$= \frac{1}{4}\left[\frac{(2x-1)^5}{5} + \frac{(2x-1)^4}{4}\right] + c$

④ P. 23

$\int x^5\sqrt{3+x^2}\,dx =$

$u = 3+x^2$

$du = 2x\,dx$

$x^2 = u-3$

$= \int \frac{1}{2}x^4(3+x^2)^{\frac{1}{2}}\,2x\,dx$

$= \int \frac{1}{2}(x^2)^2(3+x^2)^{1/2}\,2x\,dx$

$= \int \frac{1}{2}(u-3)^2(u)^{1/2}\,du$

$= \int \frac{1}{2}(u^2-6u+9)\,u^{\frac{1}{2}}\,du = \frac{1}{2}\int (u^{\frac{5}{2}} - 6u^{\frac{3}{2}} + 9u^{\frac{1}{2}})\,du$

$= \frac{1}{2}\left[\frac{2}{7}u^{\frac{7}{2}} - 6\times\frac{2}{5}u^{\frac{5}{2}} + 9\times\frac{2}{3}u^{\frac{3}{2}}\right] + c$

$= \frac{1}{7}(3+x^2)^{\frac{7}{2}} - \frac{6}{5}(3+x^2)^{\frac{3}{2}} + 3(3+x^2)^{\frac{3}{2}} + c$