مدونة المناهج السعودية

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الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

PHYS 101

Ch. 9

Rotation of Rigid Bodies

Chapter 9

Chapter Nine

Rotation of Rigid Bodies

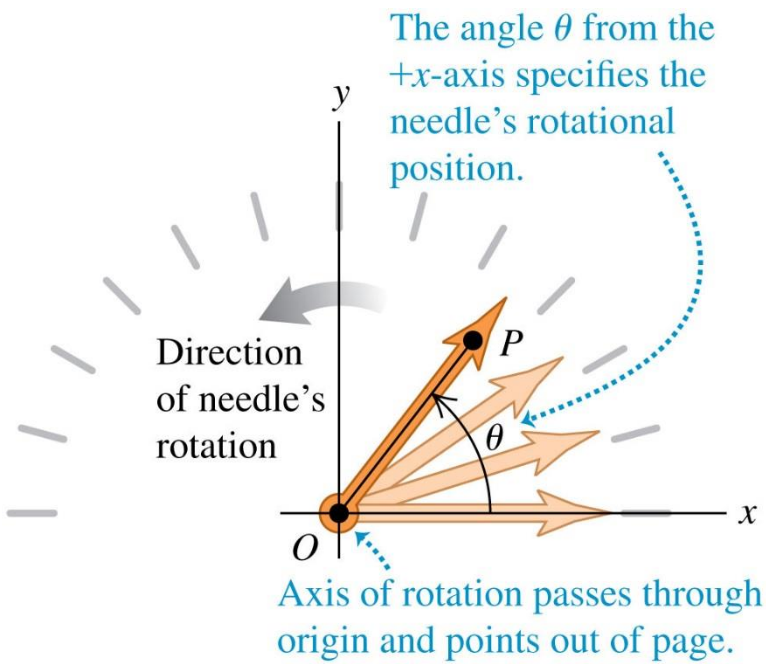
- ***Angular Velocity and Acceleration***
- ***Relating Linear and Angular Kinematics***

Angular Velocity and Acceleration

ANGULAR VELOCITY AND ACCELERATION

Angular coordinate

A car's speedometer needle rotates about a *fixed axis*.



Units of angles

One complete revolution is

360° = 2π radians.

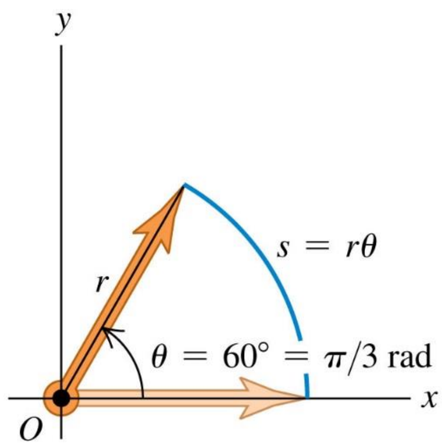
Angular Velocity and Acceleration

$$1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

$$\theta = \frac{s}{r}$$

$$s = r\theta$$

(θ in radians)

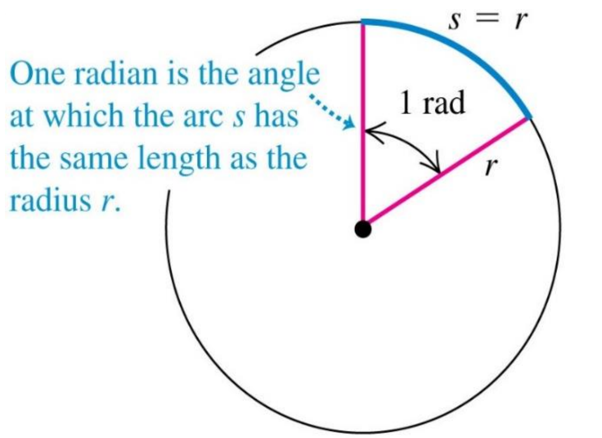


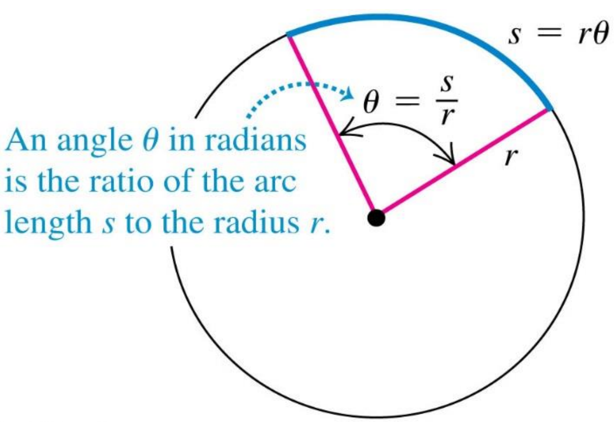
In any equation that relates linear quantities to angular quantities, the angles MUST be expressed in radians ...

RIGHT! ▶ $s = (\pi/3)r$

... never in degrees or revolutions.

WRONG ▶ $s = \cancel{60}r$





Angular Velocity and Acceleration

Example 1:

A turbine blade of radius 5 cm rotates an angle of 60°. Find the angle of rotation in radians:

Solution:

(A)

(A) π/3 rad
(B) π/2 rad
(C) π/4 rad
(D) 60 rad

Angular Velocity and Acceleration

Example 2:

A turbine blade of radius 5 cm rotates an angle of 60^o. Find the length of scanned arc:

Solution:

(B)

(A) 2.74 cm
(B) 5.23 cm
(C) 3.21 cm
(D) 10.48 cm

Angular Velocity and Acceleration

Example 3:

The angular position θ of a 0.18 m radius flywheel is given by $\theta = 2t^3$ rad. Find θ in radians at t = 2 s.

Solution:

(C)

(A) 4 rad
(B) 8 rad
(C) 16 rad
(D) 24 rad

Angular Velocity and Acceleration

Example 4:

The angular position θ of a 0.18 m radius flywheel is given by $\theta = 2\,t^3$ rad. Find θ in degrees at t = 2 s.

Solution:

(A)

(A) 920^o
(B) 180^o
(C) 360^o
(D) 720^o

Angular Velocity and Acceleration

Angular Velocity

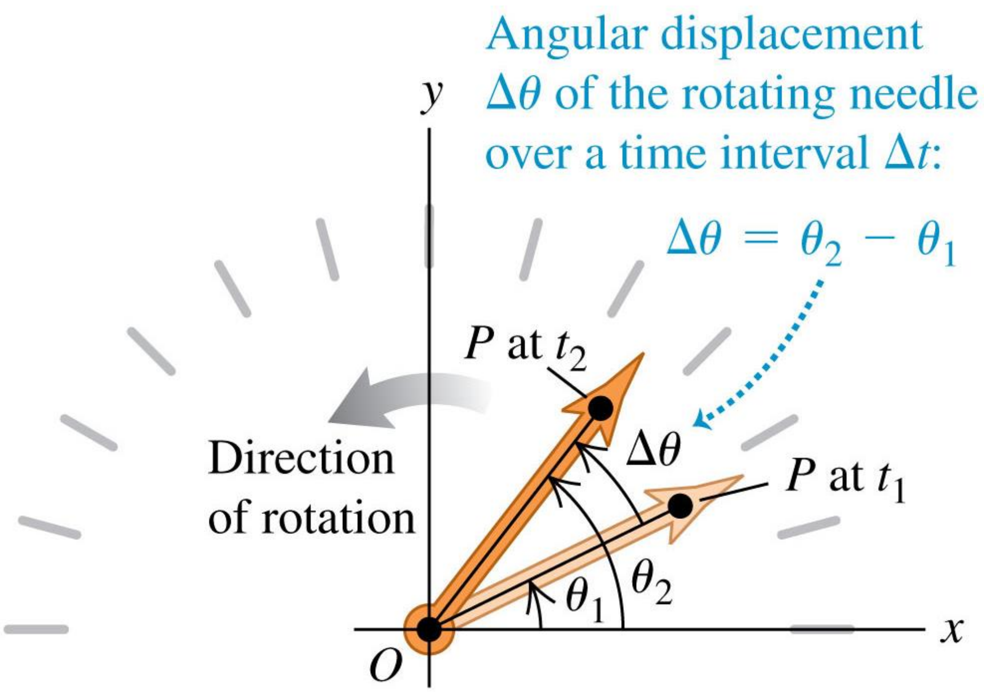
The average angular velocity of a body is given by

$$\omega_{\text{av-}z} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where z means that the rotation is about the z-axis.

The instantaneous angular velocity is

$$\omega_z = \frac{d\theta}{dt}$$



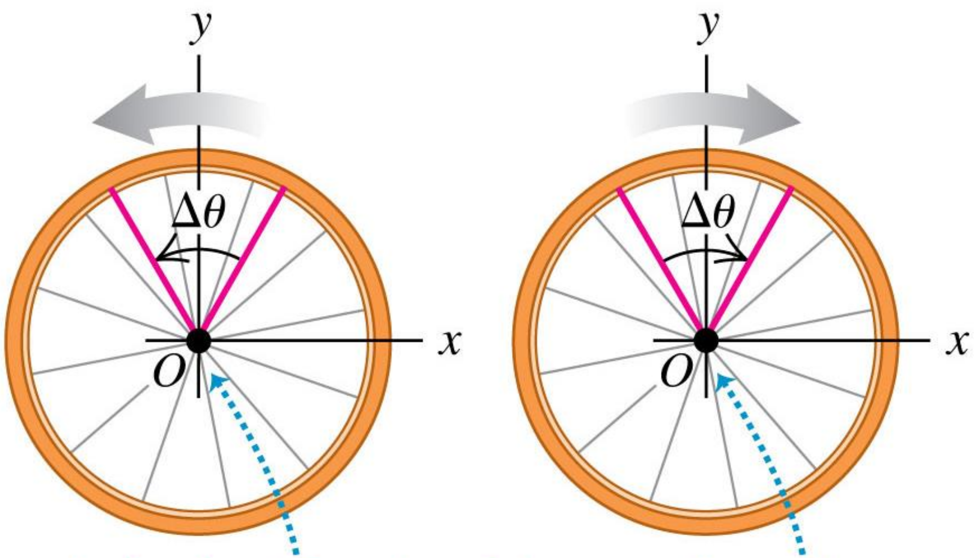
Angular Velocity and Acceleration

Counterclockwise rotation:

θ increases, so angular velocity is positive.
$\Delta\theta > 0$, so
$\omega_{av\text{-}z} = \Delta\theta/\Delta t > 0$

Clockwise rotation:

θ decreases, so angular velocity is negative.
$\Delta\theta < 0$, so
$\omega_{av\text{-}z} = \Delta\theta/\Delta t < 0$

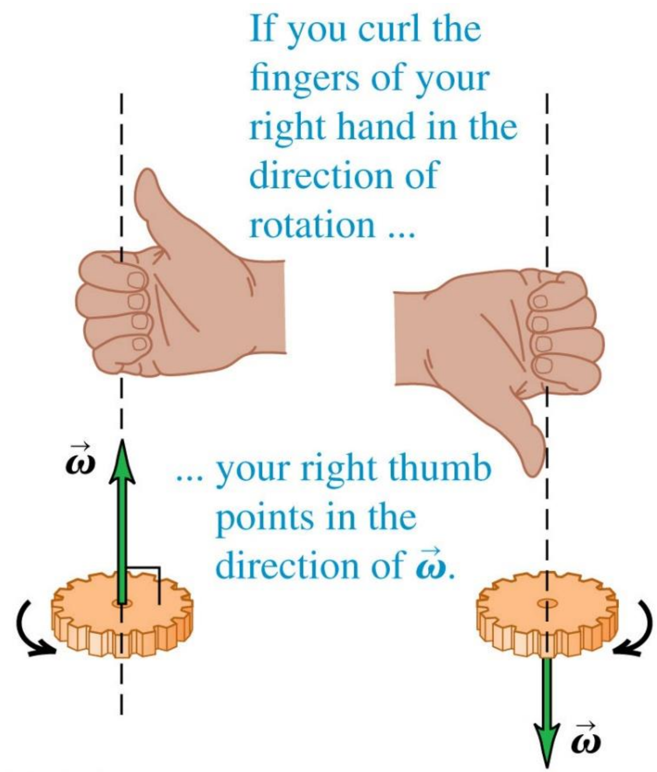


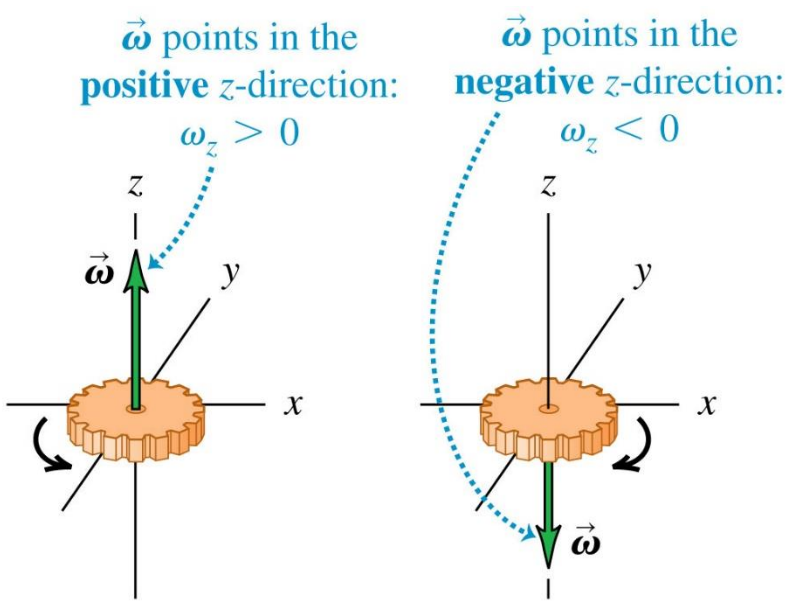
Axis of rotation (z-axis) passes through origin and points out of page.

Angular Velocity and Acceleration

Angular Velocity As a Vector

Angular velocity is defined as a vector whose direction is given by the right-hand rule.





The sign of ω_z for rotation along the z-axis

Angular Velocity and Acceleration

Example 5:

The angular position θ of a 0.18 m radius flywheel is given by $\theta = 2\ t^3$ rad. Find the distance that a particle on the flywheel rim moves from $t_1 = 2s$ to $t_2 = 5s$.

Solution:

(B)

(A) 48 m
(B) 42 m
(C) 24 m
(D) 28 m

Angular Velocity and Acceleration

Example 6:

The angular position θ of a 0.18 m radius flywheel is given by $\theta = 2\,t^3$ rad. Find the average angular velocity over that interval from $t_1 = 2s$ to $t_2 = 5s$.

Solution:

(C)

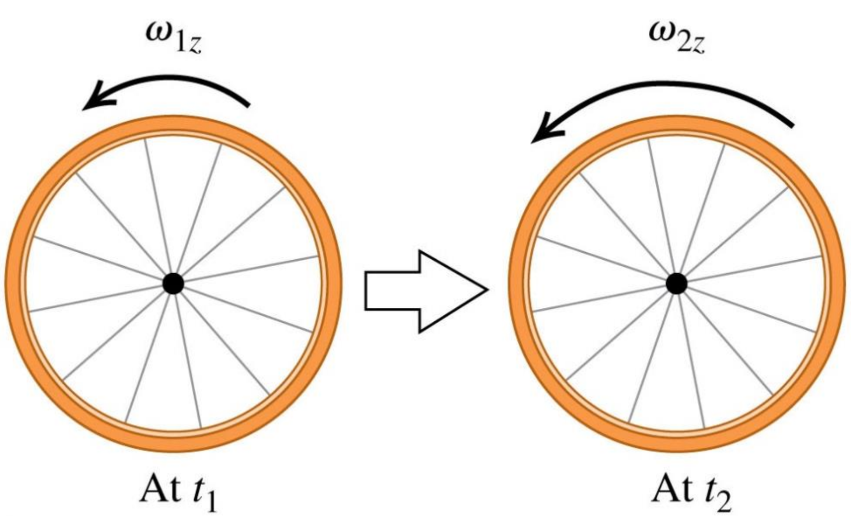
(A) 24 rad/s
(B) 150 rad/s
(C) 78 rad/s
(D) 12 rad/s

Angular Velocity and Acceleration

Angular Acceleration

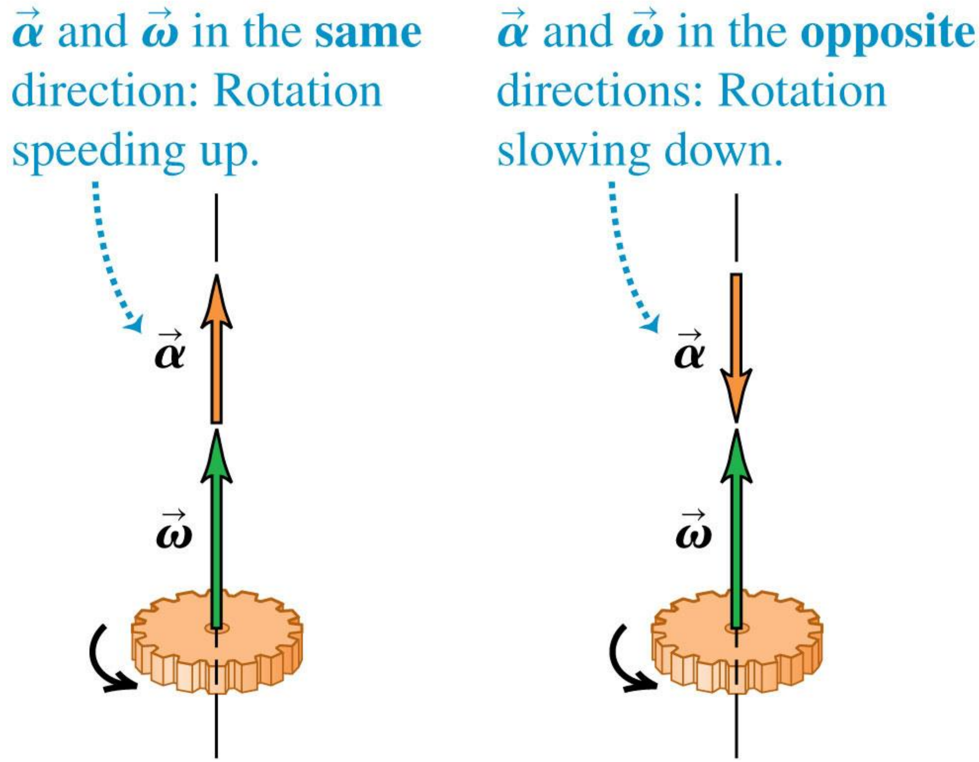
The average angular acceleration is the change in angular velocity divided by the time interval and given by

$$\alpha_{\text{av-}z} = \frac{\omega_{2z} - \omega_{1z}}{t_2 - t_1} = \frac{\Delta\omega_z}{\Delta t}$$



Angular Velocity and Acceleration

Angular Acceleration As a Vector



Angular Velocity and Acceleration

Example 7:

The angular position θ of a 0.18 m radius flywheel is given by $\theta = 2\ t^3$ rad. Find the average angular acceleration over that interval from $t_1 = 2s$ to $t_2 = 5s$.

Solution:

(C)

(A) 24 rad/s^2
(B) 60 rad/s^2
(C) 42rad/s^2
(D) 12 rad/s^2

Relating linear & angular kinematics

RELATING LINEAR AND ANGULAR KINEMATICS

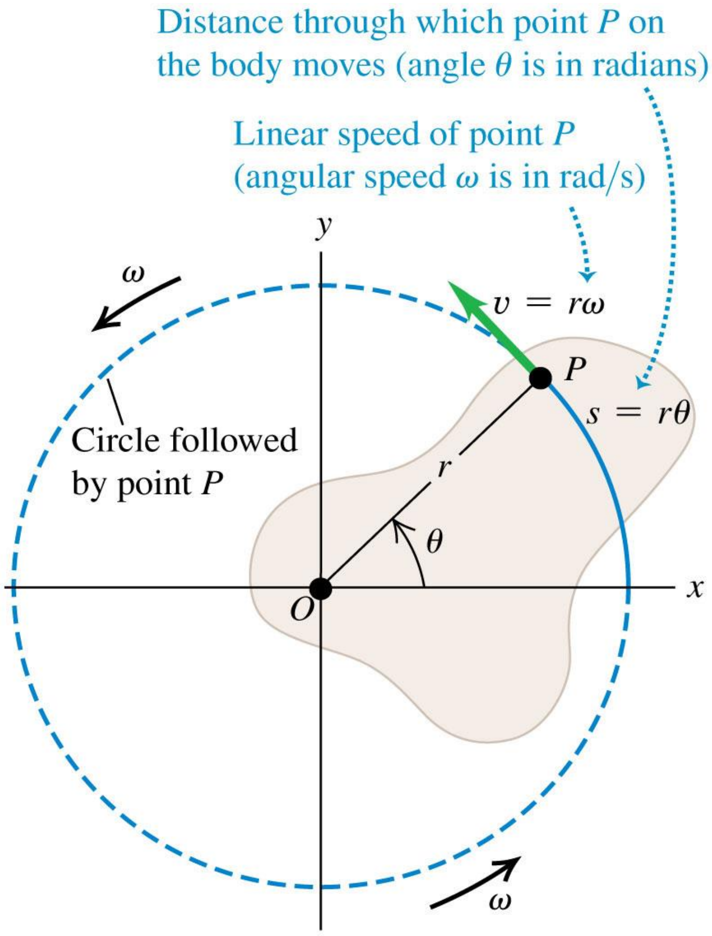
Linear Speed in Rigid-Body Rotation

A point at a distance *r* from the axis of rotation has a linear speed of

$$s = r\theta$$

$$\left|\frac{ds}{dt}\right| = r\left|\frac{d\theta}{dt}\right|$$

$$v = r\omega.$$



Relating linear & angular kinematics

Example 8:

A turbine blade has a radius of 80 cm. At a certain instant, the blade is rotating at 10 rad/s and the angular speed is increasing at 50 rad/s^2. Find the linear speed.

Solution:

(A)

(A) 8 m/s
(B) 4 m/s
(C) 5 m/s
(D) 20 m/s

Relating linear & angular kinematics

Linear Acceleration in Rigid-Body Rotation

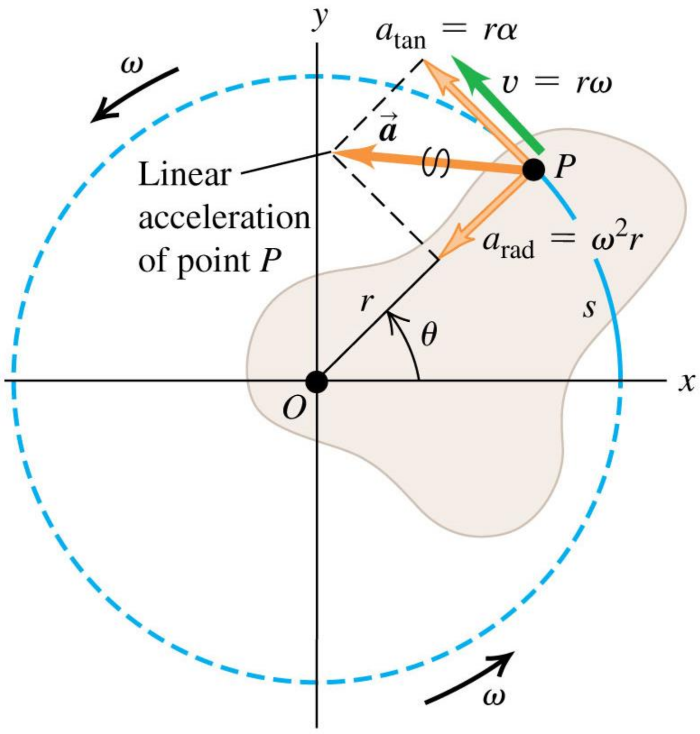
For a point at a distance *r* from the axis of rotation:

– its tangential acceleration is

$$a_{tan} = r\alpha$$

– its centripetal (radial) acceleration is

$$a_{rad} = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$$



Relating linear & angular kinematics

Example 9:

A turbine blade has a radius of 80 cm. At a certain instant, the blade is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s^2. Find the tangential component of the acceleration.

Solution:

(B)

(A) 80 m/s^2
(B) 40 m/s^2
(C) 50 m/s^2
(D) 20 m/s^2

Relating linear & angular kinematics

Example 10:

A turbine blade has a radius of 80 cm. At a certain instant, the blade is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s^2. Find the centripetal component of the acceleration.

Solution:

(A)

(A) 80 m/s^2
(B) 40 m/s^2
(C) 50 m/s^2
(D) 20 m/s^2