ex: If $f(x) = \tan x$, then $f'(x) =$

A) $\lim\limits_{x \to 0} \dfrac{\tan(x+h) - \tan x}{h}$

B) $\lim\limits_{h \to 0} \dfrac{\tan(x-h) + \tan x}{h}$

$\boxed{C)}$ $\lim\limits_{h \to 0} \dfrac{\tan(x+h) - \tan x}{h}$

* Find the eq. of the tangent line

$$y - f(a) = f'(a)(x - a)$$

نحسبها بالقوانين

* Theorem: f is diff $\to$ f is Contin.

f is Contin. $\not\to$ f is diff

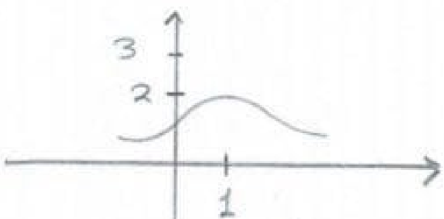
f is disContin. $\to$ f is not diff

* $|x-a|$, $\sqrt{x-a}$ are not diff at $x = a$ قيمة

كل ما بالداخل يساوي صفر

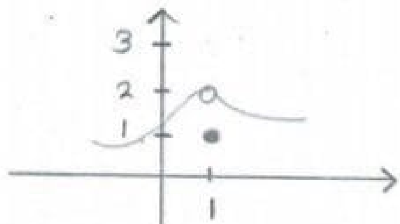
ex: 1) $f(x) = \sqrt{5x}$ not diff at $x = 0$

2) $f(x) = |x+3|$ " " at $x = -3$



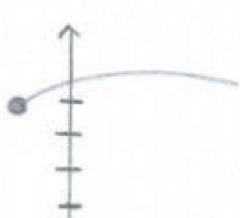
$\lim\limits_{x \to 1} f(x) = 2$

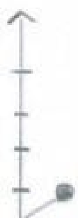
$f(1) = 2$



$\lim\limits_{x \to 1} f(x) = 2$

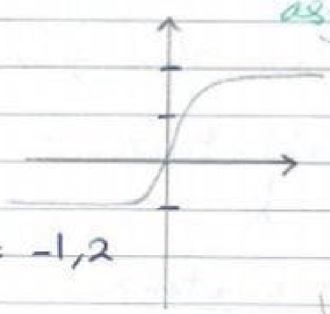
$f(1) = 1$



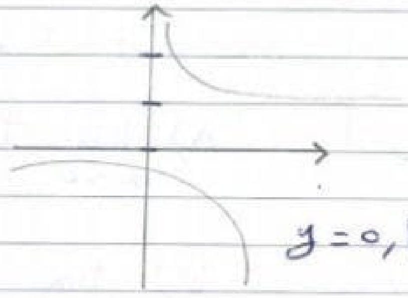


Review 2-6 $\rightarrow$ 4.3

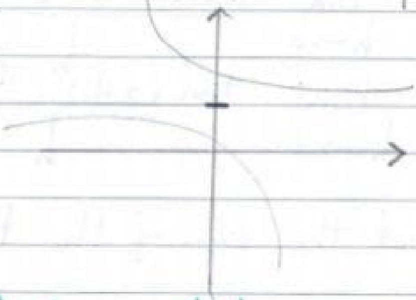
(2.6) from the graph, find the horizontal asymp.



$y = -1, 2$



$y = 0, 1$



$y = 1$

To find horizontal asymp, Evaluate

$\lim_{x \to \infty} f(x)$, $\lim_{x \to -\infty} f(x)$

what if $\lim_{x \to \pm\infty} f(x)$ D.N.E ?

$\Rightarrow$ $f(x)$ has no horizontal asymp.

ex: T or F

$f(x) = \dfrac{2x^4 - 5x + 3}{2x^2 + 4}$ has no horizontal asymp.

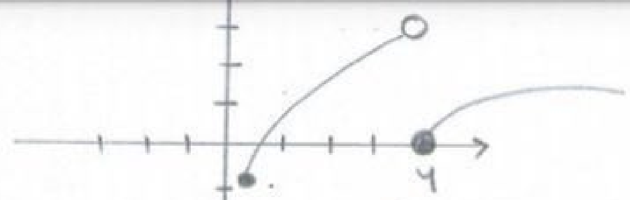
T since $\lim_{x \to \pm\infty} f(x) = \infty$

(2.7 + 2.8)

* Two definitions of m and $f'(a)$

ex: If $f(x) = \tan x$, then $f'(x) =$

A) $\lim_{x \to 0} \dfrac{\tan(x+h) - \tan x}{h}$



$$\lim_{x \to 4^+} f(x) = 0$$

$$\lim_{x \to 4^-} f(x) = 3$$

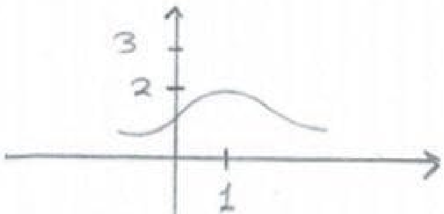
$$f(4) = 0$$

Remark: If a polynomial is of degree n, then any derivative $f^{(m)}(x)$ where $m > n$ is equal to zero

ex: If $f(x) = x^5 + 3x^2 - 3x + 2$, then

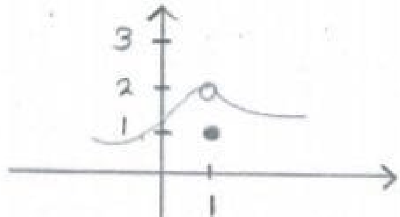
$$f^{(6)} = f^{(7)} = \dots = 0$$

يعني التفاضلات اللي أعلى
من درجة كثيرة الحدود بساوي صفراً



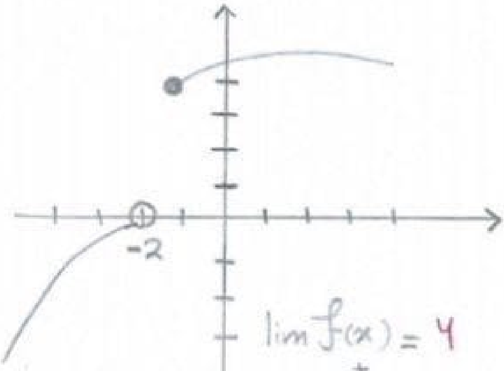
$\lim_{x\to 1} f(x) = 2$

$f(1) = 2$



$\lim_{x\to 1} f(x) = 2$

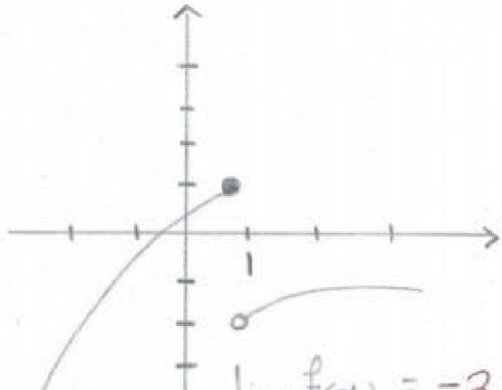
$f(1) = 1$



$\lim_{x\to -2^+} f(x) = 4$

$\lim_{x\to -2^-} f(x) = 0$

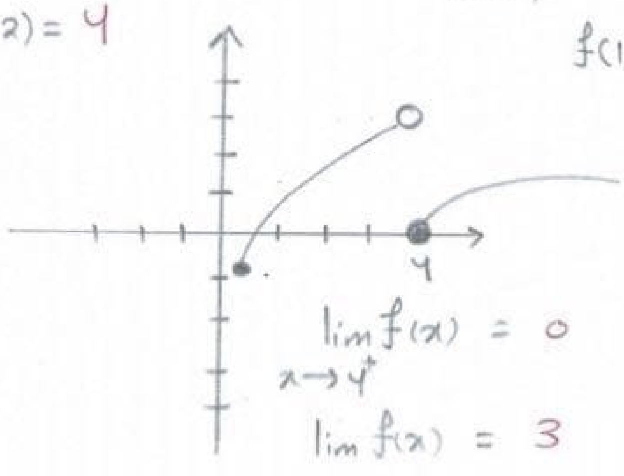
$f(-2) = 4$



$\lim_{x\to 1^+} f(x) = -2$

$\lim_{x\to 1^-} f(x) = 1$

$f(1) = 1$



$\lim_{x\to 4^+} f(x) = 0$

$\lim_{x\to 4^-} f(x) = 3$

$f(4) = 0$

Remark: If a polynomial is of degree n, then any derivative $f^{(m)}(x)$ where $m>n$ is equal to zero

تم النسخ إلى الحافظة.

ex: If $f(x) = x^5 + 3x^2 - 3x + 2$, then

$f^{(6)} = f^{(7)} = \dots = 0$

(2.2)

① Remember all rules for limit.

ex:

a) If $\lim_{x\to 2} f(x) = 3$, $\lim_{x\to 2} g(x) = 2$

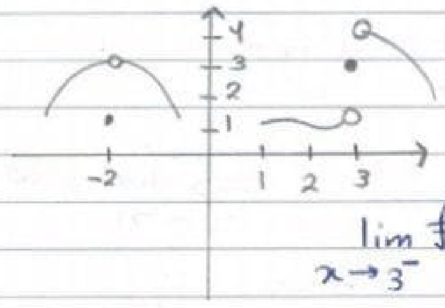
find $\lim_{x\to 2} (f+g)(x) = 3+2 = 5$

b) $\lim_{x\to 1} \frac{(c-4)^2}{5} = \frac{(c-4)^2}{5}$ خرج زي ماهو لأنه ثابت

إحنا بنعوض عن $\underline{x}$ فقط وهنا مافي x

② ex: Find the lim from the graph.

lim الحد الثابت = الحد الثابت



$\lim_{x\to -2} f(x) = 3$

$f(-2) = 1$

$\lim_{x\to 3^-} f(x) = 1$, $\lim_{x\to 3^+} f(x) = 4$

---

③ vertical asymptotes:

from the graph or from fun.

ex:

$f(x) = \frac{x^2-4}{(x-2)(x+3)}$

أولاً أصفار المقام : $(x-2)(x+3) = 0 \Rightarrow (x=2,\ x=-3)$

ثانياً نعوض بقيم x في البسط : $x^2-4\big|_{x=2} = 0$ ⟹ طالما التعويض صفر إذن لن يكون خط تقارب.

هذه الخطوة مهمة لا تنسوها

$x^2-4\big|_{x=-3} = 5 \Rightarrow \underline{\underline{x}} = -3$ is the vertical asymp.

لازم x

(2.3)

How to calculate limit

Direct sub

number

∞ وقتها ندرس الإشارة

$\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty$

$\infty - \infty$

ارجعي للحالات في المحاضرة

Direct sub

number | ∞ وقتها ندرس الإشارة | $\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty$, $\infty - \infty$ ارجعي للحالات في الملزمة

ex: $\lim_{x\to 1} \frac{2x}{(x-1)^2}$

$= \frac{2}{0} = \infty$

ندرس الإشارة من الجهتين

$\lim_{x\to 1^+} \quad \frac{+}{+} = +\infty$

$x>1$

take 1.5

$\Rightarrow \lim_{x\to 1} = \infty$

$\lim_{x\to 1^-} \quad \frac{+}{+} = +\infty$

2) Don't forget squeeze theorem.

ex: $f(x) = \begin{cases} x^2+1 & x\neq 1 \\ x-1 & x=1 \end{cases}$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} (x^2+1) = 2$

ex: $f(x) = \begin{cases} x^2+1 & x\geq 1 \\ x-1 & x<1 \end{cases}$

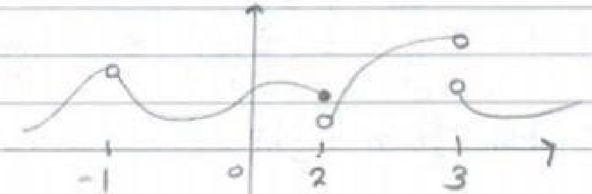
$\lim_{x\to 1} f(x)$ $\begin{cases} \lim_{x\to 1^+} (x^2+1) = 2 \\ \lim_{x\to 1^-} (x-1) = 0 \end{cases}$ لأنه 1 هو اللي بتغير عنده التعريف

but $\lim_{x\to 2} f(x) = \lim_{x\to 2} (x^2+1) = 5$ لأنه $2>1$ أخذنا التعريف الأول

3) $\lim \sqrt{x-3} \rightarrow$ له ٣ حالات في الملزمة

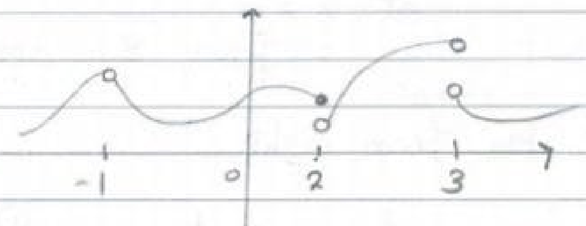
(2.5) Continuity

1) from the graph



(2.5) Continuity

1) from the graph



at $x=-1$ (disContin.)

at $x=0$ (Contin)

at $x=2$ (disContin. but Contin. from left)

at $x=3$ (disContin.)

2) each fun. Contin. on its domain

يعني لمعرفة الدالة متصلة فين نجيب المجال

except : الدالة متعددة التعريف

$$f(x) = \begin{cases} x^2-1 & x \geq 0 \\ x-5 & x < 0 \end{cases}$$

ندرس الاتصال عند الصفر

غير متصلة عند الصفر معناها متصلة على $\mathbb{R}-\{0\}$

لو متصلة عند الصفر معناها متصلة على $\mathbb{R}$

at $x=0$

$f(0) = -1$

$\lim_{x\to 0^+} f(x) = -1$, $\lim_{x\to 0^-} f(x) = -5$

$\Rightarrow$ lim D.N.E $\Rightarrow$ f is disContin. at $x=0$

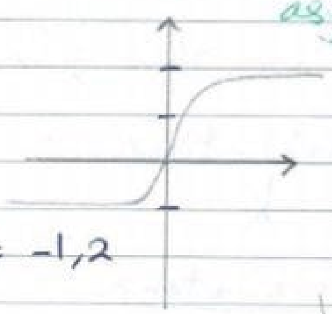
but $\lim_{x\to 0^+} = f(0)$ so we can say it is Contin. from right

$\Rightarrow$ f is Contin. on $\mathbb{R}-\{0\}$

or we can say on $(-\infty,0) \cup [0,\infty)$

Review 2-6 $\longrightarrow$ 4.3

(2.6) from the graph, find the horizontal asymp.



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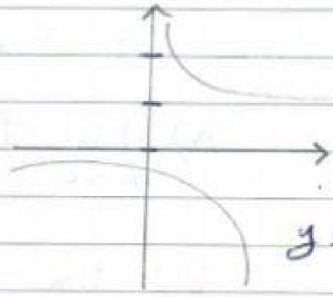
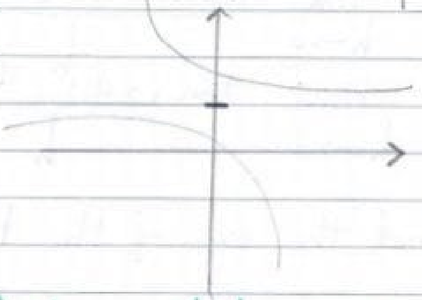


$y = 0, 1$



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