King Saud University

Faculty of Sciences

Department of Mathematics

Second Examination	Math 132 Time: 1H30	Semester I	1439-1440

Exercise 1 : (4+3)

1. Consider the two sets $A := \{1, 2, 3, 4, \{1\}, \{2\}, \{1, 2\}, \{1, \{1\}\}, \{2, \{2\}\}\}$ and $B := \{\emptyset, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$. Determine whether each of the following seven statements is true or false. (Justify your answer).
   (i) S_1: "$\{1, 2\} \in A$". (ii) S_2: "$\{1, 2\} \subseteq A$". (iii) S_3: "$\{1, \{2\}\} \subseteq A$".
   (iv) S_4: "$A \cap \{1, 2, \{\{1\}, \{2\}\}\} = \{1, 2\}$". (v) S_5: "$\emptyset \in B$".
   (vi) S_6: "$\{\{\emptyset\}\} \subseteq B$". (vii) S_7: "$(\{\emptyset, \{\emptyset\}\} \cap \{\emptyset, \{\emptyset, \{\emptyset\}\}\}) \subseteq B$".
2. Consider the following three sets $C := \{a, b, c\}$, $D := \{1, 2, a\}$, and $E := \{(a, 1), (1, a), (b, b), (2, 2), (c, 2), (c, a)\}$. Find the following sets:
   (i) $(C \cap D) \times C$. (ii) $E \setminus (C \times D)$. (iii) $\{\emptyset\} \times E$. 0

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Exercise 2 : (2+2+(2+1+2))
Let R be the relation from the set $A := \{1, 3, 5, 7\}$ to the set $B := \{0, 1, 2, 3, 4\}$ defined as follows: for $a \in A$ and $b \in B$, $[(aRb) \Leftrightarrow (a \leq b)]$.
In this exercise, the elements of each of the sets A and B are listed in icreasing order.

1. List all the ordered pairs in the relation R.
2. Represent the relation R with a matrix.
3. Let S be the relation from B to A defined by $S := \{(0, 1), (1, 1), (2, 1), (3, 5)\}$.
   (a) Find the following relations: $S^{-1} \cap R$ and $S \circ R$.
   (b) Draw the digraph of the relation $S \circ R$.
   (c) Represent the relation $(S \circ R)^2$ with a matrix. 2

Exercise 3 : ((2+2+2)+(2+1))

1. Consider the relation $R := \{(a, a), (c, c), (d, d), (e, e), (f, f), (a, f), (f, a), (b, e)\}$, defined on the set $A := \{a, b, c, d, e, f\}$.
   (a) For the relation R, determine whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive. (Justify your answer).
   (b) Prove that the relation E, defined on the set A by $E := R \cup \{(b, b), (e, b)\}$, is an equivalence relation on A.
   (c) Find the equivalence classes of the equivalence relation E. 2 $\boxed{5}$
2. Consider the partial ordering P of divisibility on the set $B := \{1, 2, 3, 6, 7, 8\}$ (that is, $P := \{(a, b) \in B \times B : a \text{ divides } b\}$).
   (a) Draw the Hasse diagram of P.
   (b) Is P a total ordering?

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$S \circ R = \{(1,1), (1,5), (3,5)\}$ 20/25

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Exercise 1 : (4+3+(4+2))

1. Use a truth table to verify that the following conditional statement is a tautology:

$$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$$

2. Without using truth tables, prove that the following statement is a contingency:

$$p \wedge (p \to q) \wedge (p \to \neg q)$$

3. (a) Without using truth tables, prove the following logical equivalence:

$$\neg[p \vee (\neg p \wedge q)] \equiv (\neg p \wedge \neg q)$$

(b) Deduce from a) the following logical equivalence:

$$\neg[(r \to w) \vee ((r \wedge \neg w) \wedge q)] \equiv ((r \wedge \neg w) \wedge \neg q)$$

Exercise 2 : (3+3+3+3)

1. Given an integer n, prove by contraposition the following statement:
if the integer $5n+7$ is even, then the integer n is odd.

2. Given real numbers x, y, and z, prove by contradiction the following statement:
if $(x+y+z=58)$, then $(x \geq 16$ or $y \geq 38$ or $z \geq 4)$.

3. Use mathematical induction to prove the following statement:

$$\sum_{k=0}^{n-1}(2k+1) = n^2, \quad \text{for each integer } n \text{ with } n \geq 1.$$

4. Consider the sequence $\{a_n\}_{n=0}^{\infty}$ defined as follows:
$a_0 = 6, a_1 = 9$, and $a_n = 2a_{n-1} - a_{n-2}; \forall n \geq 2$.
Use mathematical induction to prove the following statement:

$$a_n = 3n+6, \quad \text{for each integer } n, \text{ with } n \geq 0.$$

$a_k = 3k+6$

$a_{k-1} = 3(k-1)+6$

$= 3k+3$

$2a_k = 6k+12$

$\boxed{a_{k+1} = 3k+9}$

$a_{k+1} = 2a_k - a_{k-1}$

$= 6k+12-(3k+3)$

$= 3k+9$

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$$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$$

2. Without using truth tables, prove that the following statement is a contingency:

$$p \wedge (p \rightarrow q) \wedge (p \rightarrow \neg q)$$

3. (a) Without using truth tables, prove the following logical equivalence:

$$\neg[p \vee (\neg p \wedge q)] \equiv (\neg p \wedge \neg q)$$

(b) Deduce from a) the following logical equivalence:

$$\neg[(r \rightarrow w) \vee ((r \wedge \neg w) \wedge q)] \equiv ((r \wedge \neg w) \wedge \neg q)$$

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$= 6k+12-(3k+3)$

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