المحاضرة الأولى في مساند

(1.1) Four Ways to Represent a Function

الهدف الأول من الفصل الأول

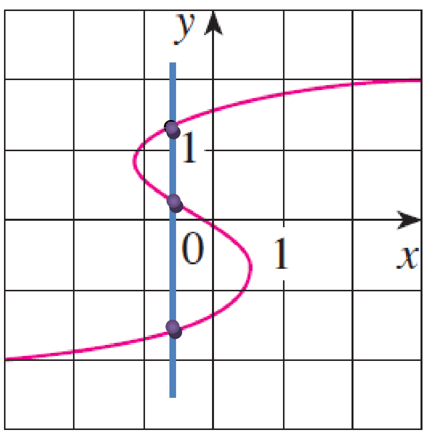
كيفية معرفة المنحنى أنه دالة (function) ام لا (not function)

الهدف الثاني من الفصل الأول

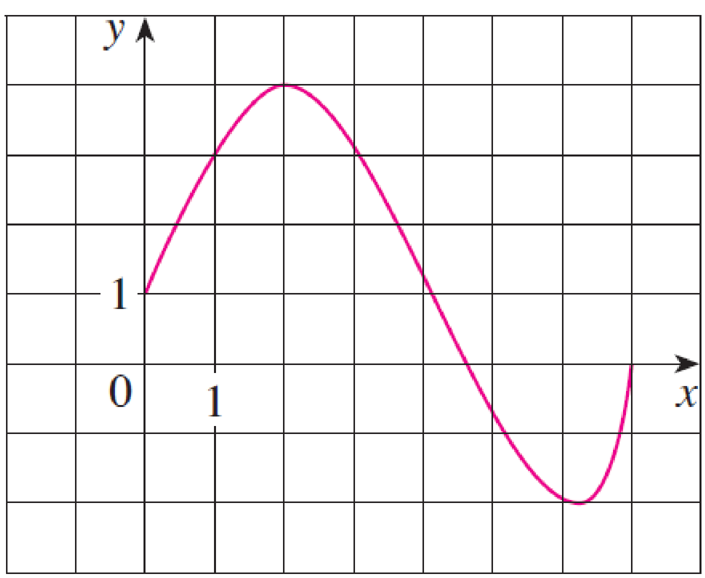
كيفية ايجاد المجال والمدى من الرسم البياني

Exercise (1)

Determine whether the curve is the graph of a function of x. if it is, state the domain and range of the function.



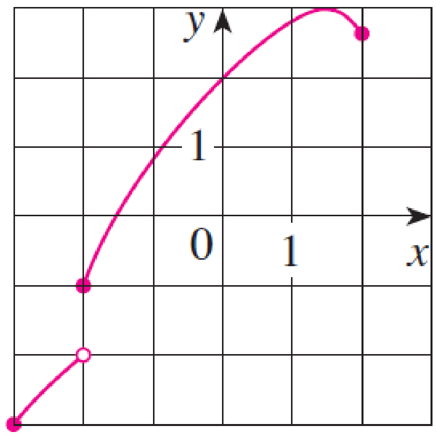
This curve does not represent a function because, as you can see, there are vertical lines that intersect the curve more than once



This curve represents a function

The Domain: [0 , 7]

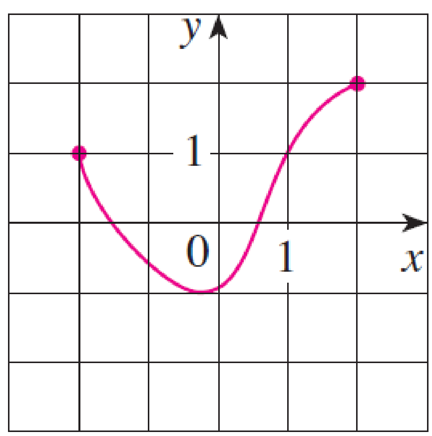
The Range: [-2 , 4]



This curve represents a function

The Domain: $[\,-3\,,2\,]$

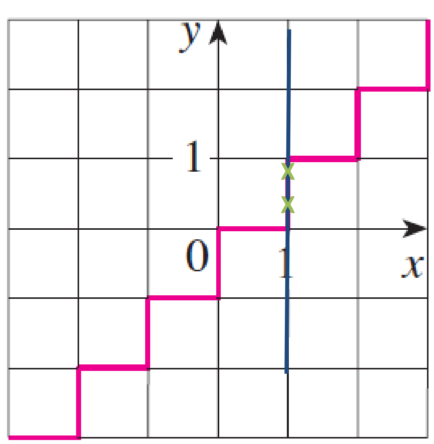
The Range: $[\,-3\,,-2) \cup [-1,3]$



This curve represents a function

The Domain: $[\,-2\,,2\,]$

The Range: $[-1,2]$



This curve does not represent a function because, as you can see, there are vertical lines that intersect the curve more than once

الهدف الثالث من الفصل الأول

كيفية ايجاد مجال الدالة الخطية ومداها وتمثيلها بيانياً

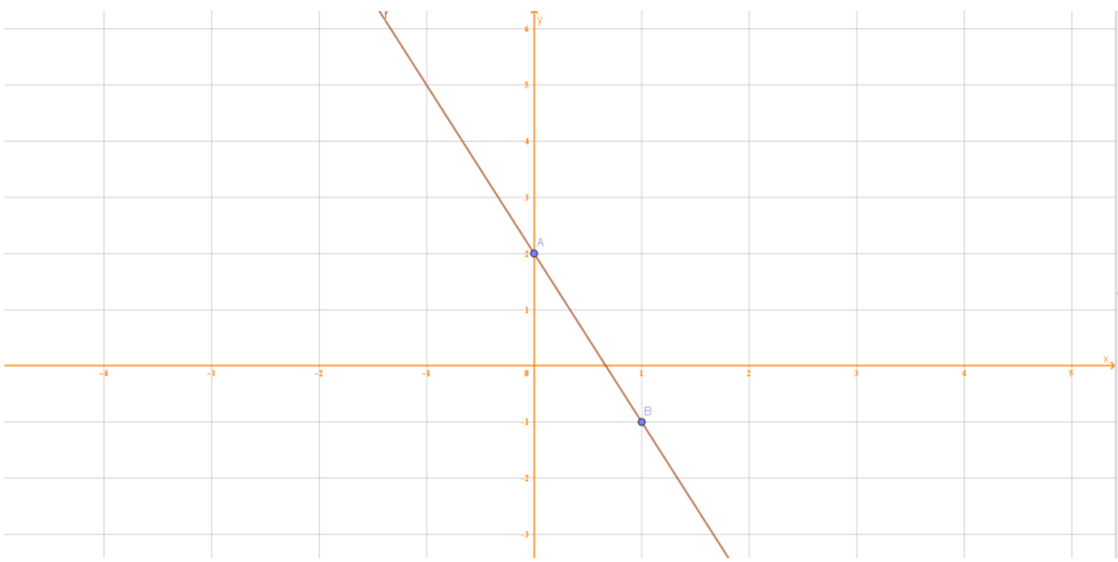
Exercise (2)

Find the domain , range and sketch the graph of the function.

$f(x) = 2 - 3x$

$f(x) = 2 - 3x$ is a linear function

x	0	1
$y = 2 - 3x$	$y(0) = 2 - 3(0)$ $= 2$	$y(1) = 2 - 3(1)$ $= 2 - 3 = -1$
(x, y)	$(0, 2)$	$(1, -1)$



Domain : $R = (-\infty, \infty)$

Range : $R = (-\infty, \infty)$

الهدف الرابع من الفصل الأول

كيفية ايجاد مجال الدالة الكسرية

Exercise (3)

Find the domain of the function.

1) $f(x) = \frac{x+3}{x+1}$

- نوجد اصفار المقام

  $x + 1 = 0$

  $x = -1$

- $D_{f(x)} = R - \{-1\}$

  Or $= \{\ x \ |\ x \neq -1\ \}$

  Or $= (-\infty, -1) \cup (-1, \infty)$

2) $f(x) = \frac{x}{2x-6}$

- نوجد اصفار المقام

  $2x - 6 = 0$

  $2x = 6$

  $\frac{2x}{2} = \frac{6}{2}$

  $x = 3$

- $D_{f(x)} = R - \{3\}$

  Or $= \{\ x \ |\ x \neq 3\ \}$

  Or $= (-\infty, 3) \cup (3, \infty)$

$3)\ f(x) = \frac{x+4}{x^2-9}$

- نوجد اصفار المقام

$x^2 - 9 = 0 \Rightarrow x^2 = 9$

$\sqrt{x^2} = \sqrt{9} \ \ \Rightarrow |x| = 3 \Rightarrow x = \pm 3$

- $D_{f(x)} \ = R - \{-3, 3\}$

Or $\quad = \{\ x \ \mid \ \ x \neq -3\ , x \neq \ 3\ \}$

Or $\quad = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

$4)\ f(x) = \frac{1}{x^2+5}$

- نوجد اصفار المقام

$x^2 + 5 = 0 \Rightarrow x^2 = -5$

$\sqrt{x^2} = \sqrt{-5} \ \ \Rightarrow |x| = \sqrt{-5} \notin R$

إذن لا يوجد أصفار مقام

- $D_{f(x)} \ = R$

$5)\ f(x) = \frac{2x^3-5}{x^2+x-6}$

- نوجد اصفار المقام

$x^2 + x - 6 = 0$

$(x-2)(x+3) = 0 \begin{cases} \boxed{x - 2 = 0 \ \Longrightarrow x = 2} \\ \boxed{x + 3 = 0 \ \Longrightarrow x = -3} \end{cases}$

- $D_{f(x)} \ = R - \{-3, 2\}$

Or $\quad = \{\ x \ \mid \ \ x \neq -3\ , x \neq \ 2\ \}$

Or $\quad = (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

$6)\ f(x) = \frac{4x}{x^2-5x+6}$

- نوجد اصفار المقام

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0 \quad \begin{cases} \boxed{x - 2 = 0 \implies x = 2} \\ \\ \boxed{x - 3 = 0 \implies x = 3} \end{cases}$$

- $D_{f(x)} = R - \{2, 3\}$

Or $= \{\ x \ \mid \ x \neq 2\ , x \neq\ 3\ \}$

Or $= (-\infty, 2) \cup (2, 3) \cup (3, \infty)$

المحاضرة الثانية في مساند

(1.1) Four Ways to Represent a Function

الهدف الخامس من الفصل الأول

كيفية ايجاد مدى بعض الدوال الكسرية ورسمها (Rational function)

Exercise (1)

find the domain , range and sketch the graph of the function

1) $f(x) = \frac{x^2-1}{x+1}$

The domain of $f(x)$:

- نوجد اصفار المقام

  $x + 1 = 0$

  $x = -1$

- $D_{f(x)} = R - \{-1\}$

  Or $= \{\ x \mid x \neq -1\ \}$

  Or $= (-\infty, -1) \cup (-1, \infty)$

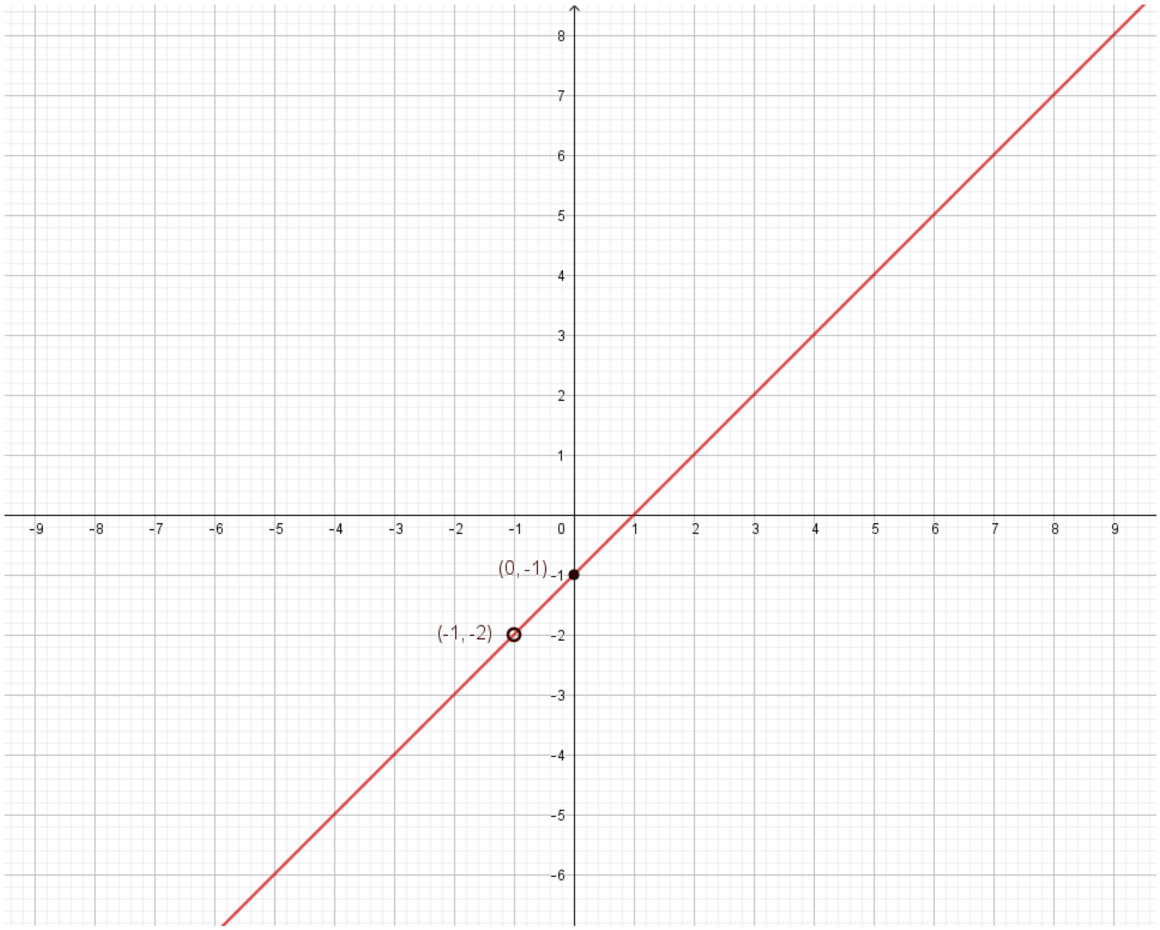
The graph of $f(x)$:

$f(x) = \frac{x^2-1}{x+1}$ for all $x \neq -1$

$f(x) = \frac{(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$ for all $x \neq -1$

$f(x) = x - 1$ for all $x \neq -1$

x	0	-1
$y = x - 1$	$y(0) = 0 - 1$ $= -1$	$y(1) = -1 - 1$ $= -2$
(x, y)	$(0, -1)$	$(-1, -2)$



The range of $f(x) = R - \{-2\}$

الهدف السادس من الفصل الأول

كيفية ايجاد مجال الدالة الجذرية $(f(x) = \sqrt[n]{g(x)})$

١) لو كان n عدد فردي فإن مجال الدالة الجذرية R
٢) لو كان n عدد زوجي فإنه يجب ما بداخل الجذر أكبر من أو يساوي الصفر

Exercise (2)

Find the domain of the function.

1)
$$\left.\begin{array}{l} f(x) = \sqrt[3]{2x-1} \\ f(x) = \sqrt[5]{x^2-4} \\ f(x) = \sqrt[7]{3-7x-x^2} \\ f(x) = \sqrt[9]{4x^5+3x} \end{array}\right\}$$

The domain of $f(x)$:R

2) $f(x) = \sqrt[4]{15-3x}$

$15 - 3x \geq 0$

$-3x \geq -15$

$\frac{-3x}{-3} \leq \frac{-15}{-3}$

$x \leq 5$

The domain of $f(x)$ $=\{x|\ x \leq 5\} = (-\infty, 5]$

3) $f(x) = \sqrt[6]{2x+3}$

$2x + 3 \geq 0$

$2x \geq -3$

$\frac{2x}{2} \geq -\frac{3}{2}$

$x \geq -\frac{3}{2}$

The domain of $f(x)$ $=\left\{x|\ x \geq -\frac{3}{2}\right\} = \left[-\frac{3}{2}, \infty\right)$

4) $f(x) = \sqrt{4 - x^2}$

الطريقة الجبرية

$4 - x^2 \geq 0$

$-x^2 \geq -4$

$\frac{-1x^2}{-1} \leq \frac{-4}{-1}$

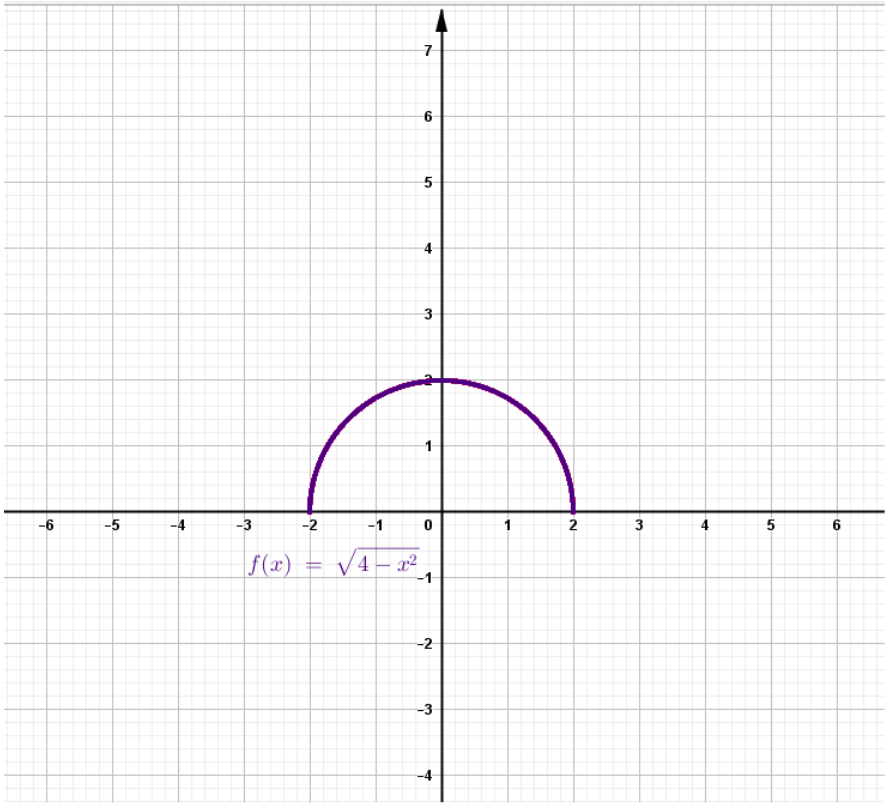
$x^2 \leq 4$

$\sqrt{x^2} \leq \sqrt{4}$

$|x| \leq 2 \quad \Leftrightarrow \quad -2 \leq x \leq 2$

The domain of $f(x)$ $=\{x|-2 \leq x \leq 2\} = [-2, 2]$

الطريقة البيانية



The domain of $f(x) = [-2\ ,2]$
The range of $f(x) = [0\ ,2]$

5) $\boldsymbol{f(x) = \sqrt{x^2 - 4}}$

الطريقة الجبرية

$$x^2 - 4 \geq 0$$

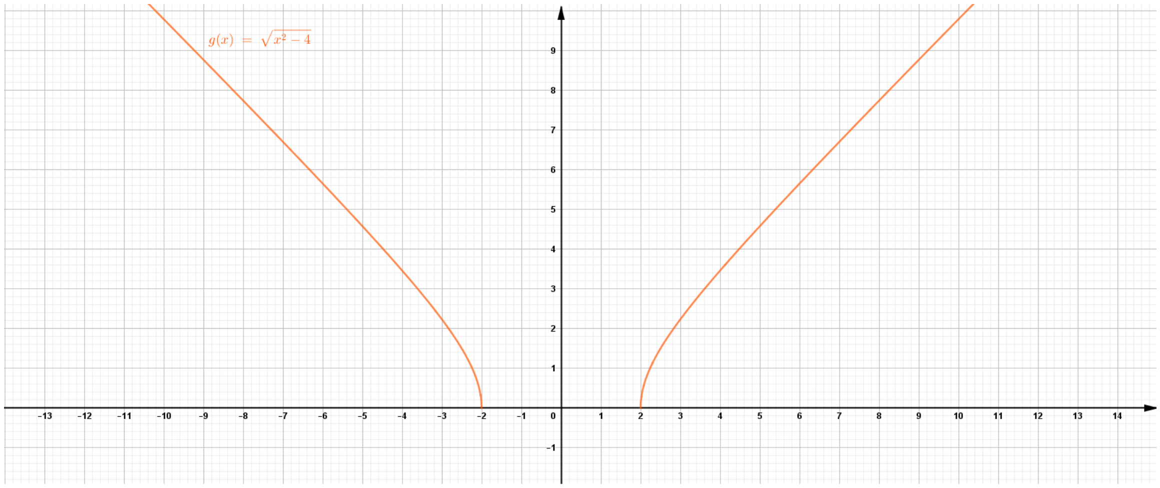
$$x^2 \geq 4$$

$$\sqrt{x^2} \geq \sqrt{4}$$

$$|x| \geq 2 \quad \Leftrightarrow \quad x \leq -2 \quad or \;\; x \geq 2$$

$$D_{f(x)} = \{x|\ x \leq -2 \ or\ x \geq 2\ \} = (-\infty, -2] \cup [2, \infty) = R - (-2, 2)$$

الطريقة البيانية

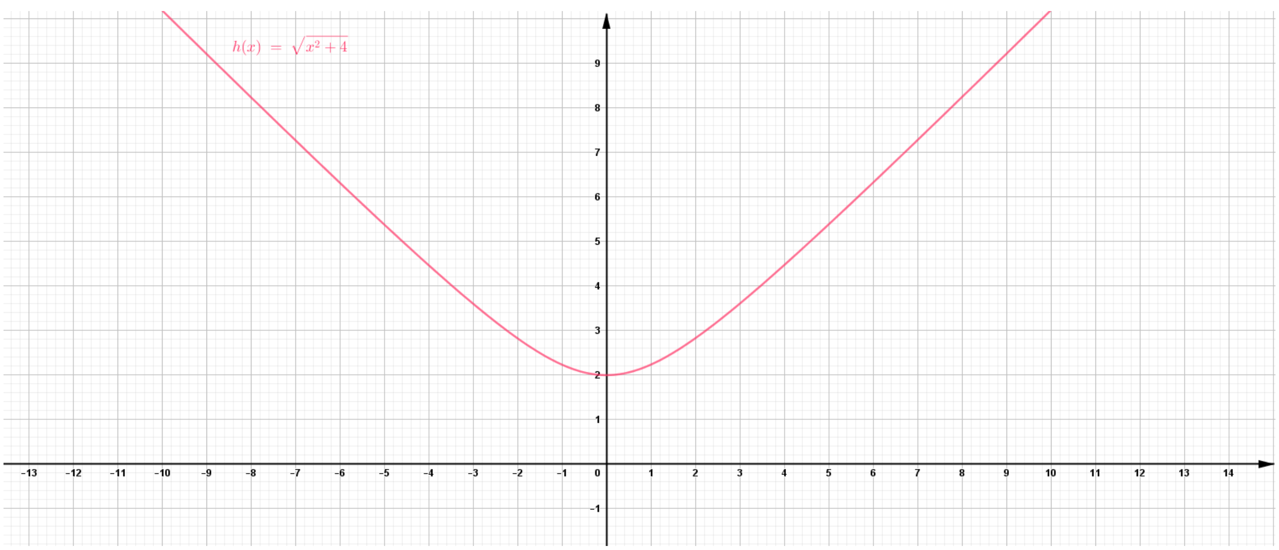


$$D_{f(x)} = (-\infty, -2] \cup [2, \infty) = R - (-2, 2)$$

$$R_{f(x)} = [0, \infty)$$

6) $f(x) = \sqrt{x^2 + 4}$ *or* $\sqrt{4 + x^2}$

الطريقة البيانية



$$D_{f(x)} = (-\infty, \infty)$$

$$R_{f(x)} = [2, \infty)$$

الطريقة الجبرية

$$x^2 + 4 \geq 0$$

$$x^2 \geq -4$$

$$\sqrt{x^2} \geq \sqrt{-4}$$

$$|x| \geq \sqrt{-4}$$

$$x \geq \pm\sqrt{-4} \notin R$$

$$D_{f(x)} = (-\infty, \infty)$$

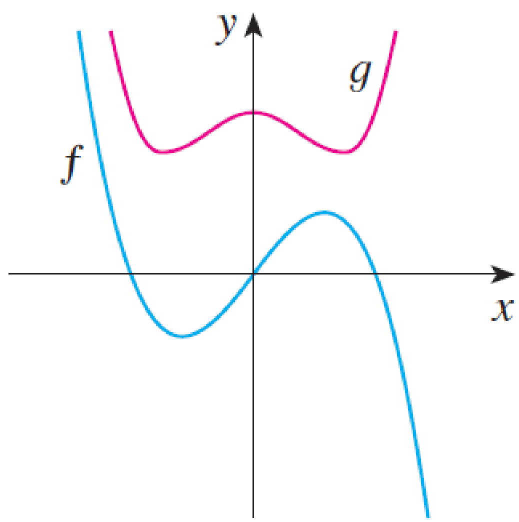
الهدف السابع من الفصل الأول

كيفية تحديد انه الدالة زوجية ام فردية ام ليست فردية او زوجية بيانيا او جبريا

Exercise (3)

Graphs of f and g are shown. Decide whether each function is even , odd , or neither.

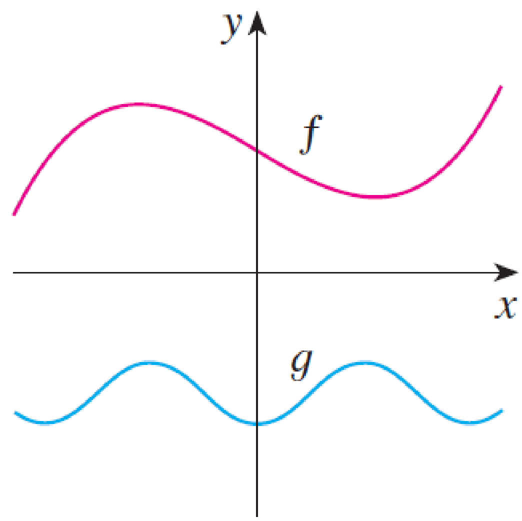
1)



$f(x)$ is an odd function and symmetric about $(0, 0)$ or the origin

$g(x)$ is an even function and symmetric about y −axis

2)

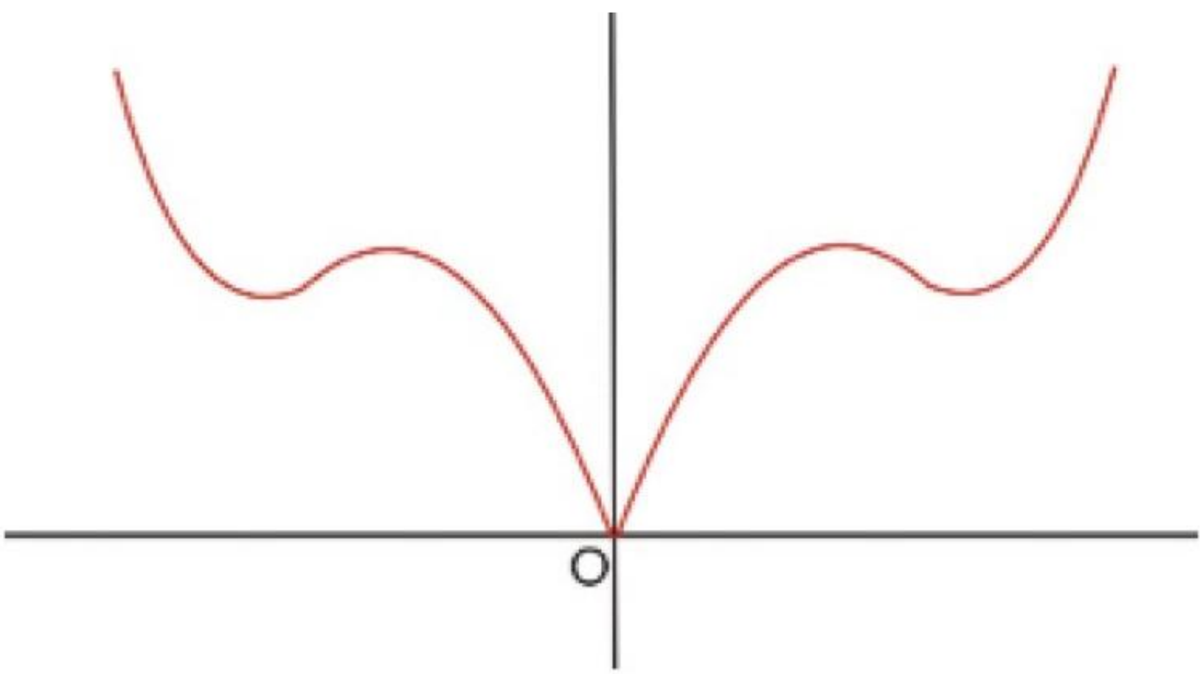


$f(x)$ **is a neither**

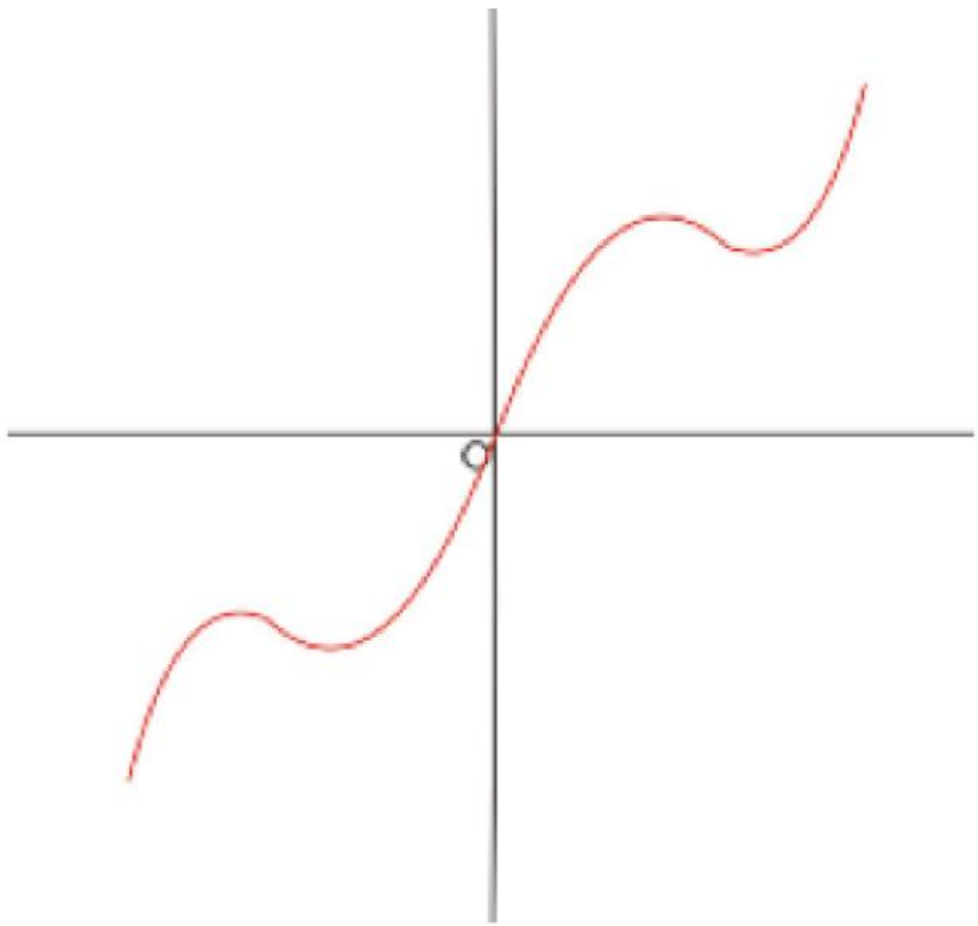
$g(x)$ **is an even function and symmetric about** y **−axis**

Exercise (4)

Graphs are shown. Decide whether each function is even , odd , or neither.



The graph is an even function and symmetric about y **−axis**



The graph is an odd function and symmetric about $(0,0)$ or the origin

Exercise (5)

Determine whether f is even , odd , or neither.

1) $f(x) = 3$

$f(x) = 3(1)$

$f(x) = 3x^{0 \to \text{even number}}$

$\therefore f(x)$ is an even function and symmetric about y-axis

2) $f(x) = 2x$

$f(x) = 2x^{1 \to \text{odd number}}$

$\therefore f(x)$ is an odd function and symmetric about the origin

3) $f(x) = 5x^{100}$

$f(x) = 5x^{100 \to \text{even number}}$

$\therefore f(x)$ is an even function and symmetric about y-axis

4) $f(x) = 4x^{21}$

$f(x) = 2x^{21\to\text{odd number}}$

$\therefore$ $f(x)$ is an odd function and symmetric about the origin

5) $f(x) = \frac{7}{x^{10}}$

$$f(x) = \frac{7}{x^{10\to\text{even number}}}$$

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

6) $f(x) = \frac{6}{x^{5}}$

$$f(x) = \frac{6}{x^{5\to\text{odd number}}}$$

$\therefore$ $f(x)$ is an odd function and symmetric about the origin

7) $f(x) = \sin(\theta)$
$f(x) = \csc(\theta)$
$f(x) = \tan(\theta)$
$f(x) = \cot(\theta)$

Are an odd functions

Symmetric about (0,0)

8) $f(x) = \cos(\theta)$
$f(x) = \sec(\theta)$

Are an even functions

Symmetric about y-axis

9) $f(x) = |x|$

$f(x)$ is an even function and symmetric about y-axis

Note :

odd $\pm$ odd $=$ odd , even $\pm$ even $=$ even , odd $\pm$ even $=$ neither

10) $f(x) = 1 + 3x^2 - x^4$

$$f(x) = \underbrace{1x^0 + 3x^2}_{\boxed{\text{Are an even functions}}} - x^4$$

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

11) $f(x) = |x| + \cos(x) - 9x^4$

$$f(x) = \underbrace{|x| + \cos(x) - 9x^4}_{\boxed{\text{Are an even functions}}}$$

$\therefore\ f(x)$ is an even function and symmetric about y-axis

12) $f(x) = 7 - 3x^2$

$$f(x) = \underbrace{7x^0 - 3x^2}_{\boxed{\text{Are an even functions}}}$$

$\therefore\ f(x)$ is an even function and symmetric about y-axis

13) $f(x) = x + \csc(x) - \frac{2}{x^3}$

$$f(x) = \underbrace{x + \csc(x) - \frac{2}{x^3}}_{\boxed{\text{Are an odd functions}}}$$

$\therefore\ f(x)$ is an odd function and symmetric about (0,0)

14) $f(x) = 1 + 3x^2 - x^4$

$$f(x) = \underbrace{1x^0 + 3x^2}_{\boxed{\text{Are an even functions}}} - \underbrace{x^5}_{\boxed{\text{is an odd function}}}$$

$\therefore\ f(x)$ is a neither even nor odd

15) $f(x) = x \pm 3$

$$f(x) = x^1 \pm 3x^0$$

$$\boxed{\text{odd}} \pm \boxed{\text{even}}$$

$\therefore\ f(x)$ is a neither even nor odd

Note :

odd $\times$ odd $=$ even	even $\times$ even $=$ even
odd $\times$ even $=$ odd	odd $\times$ neither $=$ neither
even $\times$ neither $=$ neither	neither $\times$ neither $=$ neither
odd $\div$ odd $=$ even	even $\div$ even $=$ even
odd $\div$ even $=$ odd	odd $\div$ neither $=$ neither
even $\div$ neither $=$ neither	neither $\div$ neither $=$ neither

16) $f(x) = x|x|$

$f(x) = x \times |x|$

odd $\times$ even $=$ odd

$\therefore$ $f(x)$ is an odd function and symmetric about (0,0)

17) $f(x) = x\sin(x)$

$f(x) = x \times \sin(x)$

odd $\times$ odd $=$ even

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

18) $f(x) = |x|\sec(x)$

$f(x) = |x| \times \sec(x)$

even $\times$ even $=$ even

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

19) $f(x) = (x+5)\cos(x)$

$f(x) = (x^1 + 5x^0) \times \cos(x)$

neither $\times$ even $=$ neither

$\therefore$ $f(x)$ is a neither even nor odd

20) $f(x) = \frac{x}{\cot(x)}$

$f(x) = \frac{x}{\cot(x)} = \frac{\text{odd}}{\text{odd}} = \text{even}$

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

21) $f(x) = \frac{|x|}{\sin(x)}$

$$f(x) = \frac{|x|}{\sin(x)} = \frac{\text{even}}{\text{odd}} = \text{odd}$$

$\therefore$ $f(x)$ is an odd function and symmetric about (0,0)

22) $f(x) = \frac{x^4+\cos(x)}{\sec(x)}$

$$f(x) = \frac{x^4+\cos(x)}{\sec(x)} = \frac{\text{even}+\text{even}}{\text{even}} = \frac{\text{even}}{\text{even}} = \text{even}$$

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

23) $f(x) = \frac{x^2}{x^4+1}$

$$f(x) = \frac{x^2}{x^4+1x^0} = \frac{\text{even}}{\text{even}+\text{even}} = \frac{\text{even}}{\text{even}} = \text{even}$$

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

24) $f(x) = \frac{x}{x^2+1}$

$$f(x) = \frac{x^1}{x^2+1x^0} = \frac{\text{odd}}{\text{even}+\text{even}} = \frac{\text{odd}}{\text{even}} = \text{odd}$$

$\therefore$ $f(x)$ is an odd function and symmetric about (0,0)

25) $f(x) = \frac{x}{x+1}$

$$f(x) = \frac{x^1}{x^1+1x^0} = \frac{\text{odd}}{\text{odd}+\text{even}} = \frac{\text{odd}}{\text{neither}} = \text{nither}$$

$\therefore$ $f(x)$ is a neither even nor odd

Note :

$|\text{odd}| = \text{even}$, $|\text{even}| = \text{even}$, $|\text{neither}| = \text{neither}$

$\sqrt[\text{even}]{\text{even}} = \text{even}$, $\sqrt[\text{even}]{\text{odd}} = \text{neither}$, $\sqrt[\text{even}]{\text{neither}} = \text{neither}$

$\sqrt[\text{odd}]{\text{odd}} = \text{odd}$, $\sqrt[\text{odd}]{\text{even}} = \text{even}$, $\sqrt[\text{odd}]{\text{neither}} = \text{neither}$

26) $f(x) = \left|x^3 - \sin(x)\right|$

$$|\text{odd} - \text{odd}| = |\text{odd}| = \text{even}$$

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

27) $f(x) = \left|\, x^4 + |x| \,\right|$

$$|\text{even} + \text{even}| = |\text{even}| = \text{even}$$

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

28) $f(x) = |\ (x - 1)\tan(x)\ |$

$$|\text{neither} \times \text{odd}| = |\text{neither}| = \text{neither}$$

$\therefore$ $f(x)$ is a neither even nor odd

29) $f(x) = \sqrt[3]{x^2 - 4}$

$$f(x) = \sqrt[3]{\text{even} - \text{even}} = \sqrt[3]{\text{even}} = \text{even}$$

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

30) $f(x) = \sqrt[3]{x^7 + \csc(x)}$

$$f(x) = \sqrt[3]{\text{odd} + \text{odd}} = \sqrt[3]{\text{odd}} = \text{odd}$$

$\therefore$ $f(x)$ is an odd function and symmetric about (0,0)

31) $f(x) = \sqrt[3]{x^3 + |x|}$

$$f(x) = \sqrt[3]{\text{odd} + \text{even}} = \sqrt[3]{\text{neither}} = \text{neither}$$

$\therefore$ $f(x)$ is a neither even nor odd

32) $f(x) = \sqrt[4]{x^2 - 4}$

$$f(x) = \sqrt[4]{\text{even} - \text{even}} = \sqrt[4]{\text{even}} = \text{even}$$

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

33) $f(x) = \sqrt[4]{x^7 + \csc(x)}$

$$f(x) = \sqrt[4]{\text{odd} + \text{odd}} = \sqrt[4]{\text{odd}} = \text{neither}$$

$\therefore$ $f(x)$ is a neither even nor odd

34) $f(x) = \sqrt[4]{x^3 + |x|}$

$f(x) = \sqrt[4]{\text{odd} + \text{even}} = \sqrt[4]{\text{neither}} = \text{neither}$

$\therefore$ $f(x)$ is a neither even nor odd

Note :

$(\text{odd})^{\text{even}} = \text{even}$, $(\text{even})^{\text{even}} = \text{even}$, $(\text{neither})^{\text{even}} = \text{neither}$

$(\text{odd})^{\text{odd}} = \text{odd}$, $(\text{even})^{\text{odd}} = \text{even}$, $(\text{neither})^{\text{odd}} = \text{neither}$

35) $f(x) = \sin^2(x)$

$f(x) = (\sin(x))^2 = (odd)^2 = \text{even}$

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

36) $f(x) = \tan^3(x)$

$f(x) = (\tan(x))^3 = (odd)^3 = \text{odd}$

$\therefore$ $f(x)$ is an odd function and symmetric about (0,0)

37) $f(x) = \cos^4(x)$

$f(x) = (\cos(x))^4 = (\text{even})^2 = \text{even}$

$\therefore$ $f(x)$ is an even function and symmetric about y-axis

38) $f(x) = \sec^5(x)$

$f(x) = (\sec(x))^5 = (\text{even})^5 = \text{even}$

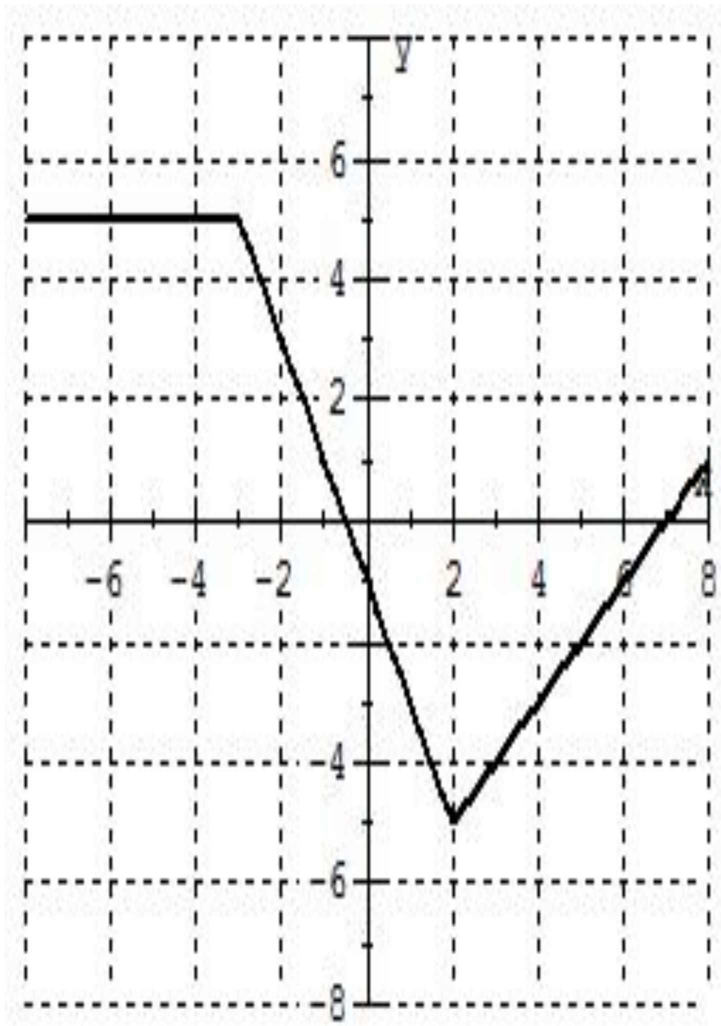
$\therefore$ $f(x)$ is an even function and symmetric about y-axis

الهدف الثامن من الفصل الأول

كيفية تحديد الفترات التزايدية والتناقصية للدوال بيانيا

Exercise (6)

The graph of a function f is given



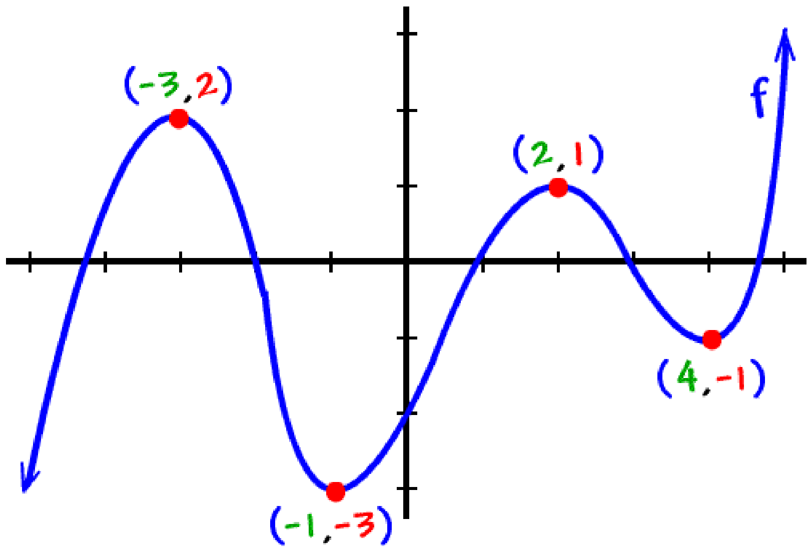
a) $f(x)$ is decreasing on $[-3, 2]$ or $(-3, 2)$

b) $f(x)$ is increasing on $[2, 8]$ or $(2, 8)$

c) $f(x)$ is constant on $[-7, -3]$ or $(-7, -3)$

Exercise (7)

The graph of a function f is given



a) $f(x)$ is increasing on $(-\infty,-3] \cup [-1,2] \cup [4,\infty)$

or $f(x)$ is increasing on $(-\infty,-3) \cup (-1,2) \cup (4,\infty)$

b) $f(x)$ is decreasing on $[-3,-1] \cup [2,4]$

Or $f(x)$ is decreasing on $(-3,-1) \cup (2,4)$

المحاضرة الثالثة في مساند

(1.1) Four Ways to Represent a Function

الهدف التاسع من الفصل الأول

كيفية ايجاد مجال و مدى ورسم وصور العناصر لدوال المتعددة التعريف

(Piecewise Defined Functions)

Exercise (1)

A) If $f(x) = \begin{cases} x+2 & \text{if} \quad x<0 \\ 1-x & \text{if} \quad x \geq 0 \end{cases}$

1) Find $f(-3)$, $f(2)$ and $f(0)$

(a $f(-3) = -3+2 = -1$

عوضنا عن 3- في الدالة الأولى لأن $-3 \in (-\infty, 0)$

(b $f(2) = 1-2 = -1$

عوضنا عن 2 في الدالة الثانية لأن $2 \in [0, \infty)$

(c $f(0) = 1-0 = 1$

عوضنا عن 0 في الدالة الثانية لأن $0 \in [0, \infty)$

2) Find the domain of the function $f(x)$

$$D_{f(x)} = \{x \mid x<0\} \cup \{x \mid x \geq 0\}$$

$$= (-\infty, 0) \cup [0, \infty)$$

$$= (-\infty, \infty) = \mathbb{R}$$

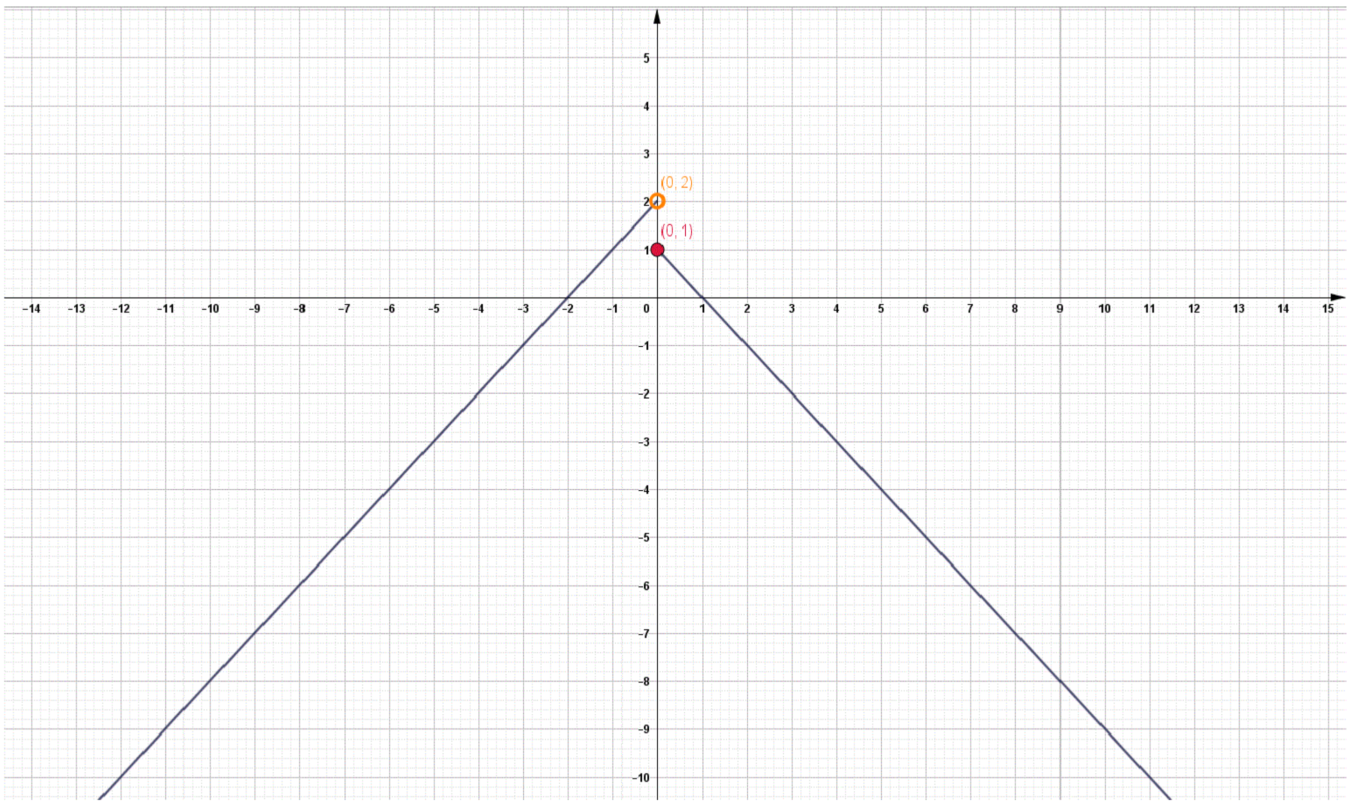
3) Sketch the graph of the function $f(x)$ and find the range

let $f_1(x) = x+2$ for all $x \in (-\infty, 0)$

x	0	-1
$y = f_1(x)$	2	1
(x, y)	(0,2)	(-1,1)

let $f_2(x) = 1 - x$ for all $x \in [0, \infty)$

x	0	1
$y = f_2(x)$	1	0
(x, y)	(0,1)	(1,0)



The range : $(-\infty, 2)$

B) If $f(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if} \quad x < 2 \\ 2x - 5 & \text{if} \quad x \geq 2 \end{cases}$

1) Find $f(-3)$, $f(2)$ and $f(0)$

a) $f(-3) = 3 - \frac{1}{2}(-3) = \frac{3}{1} + \frac{3}{2} = \frac{3\times2}{1\times2} + \frac{3\times1}{2\times1} = \frac{6}{2} + \frac{3}{2} = \frac{6+3}{2} = \frac{9}{2}$

عوضنا عن 3- في الدالة الأولى لأن $-3 \in (-\infty, 2)$

b) $f(2) = 2(2) - 5 = 4 - 5 = -1$

عوضنا عن 2 في الدالة الثانية لأن $2 \in [2, \infty)$

c) $f(0) = 3 - \frac{1}{2}(0) = 3 - 0 = 3$

عوضنا عن 0 في الدالة الأولى لأن $0 \in (-\infty, 2)$

2) Find the domain of the function $f(x)$

$$D_{f(x)} = \{x \mid x < 2\} \cup \{x \mid x \geq 2\}$$

$$= (-\infty, 2) \cup [2, \infty)$$

$$= (-\infty, \infty) = \mathbb{R}$$

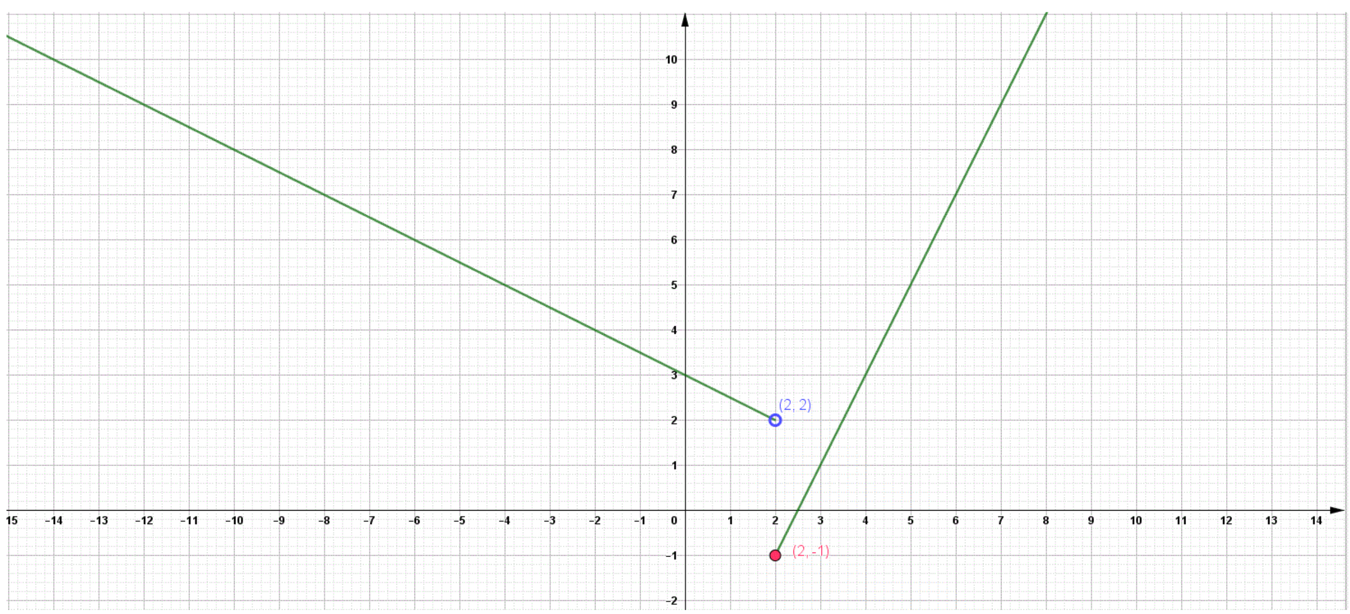
3) Sketch the graph of the function $f(x)$ and find the range

let $f_1(x) = 3 - \frac{1}{2}x$ for all $x \in (-\infty, 2)$

x	2	0
$y = f_1(x)$	2	3
(x, y)	(2,2)	(0,3)

let $f_2(x) = 2x - 5$ for all $x \in [2, \infty)$

x	2	3
$y = f_2(x)$	-1	1
(x, y)	(2,-1)	(3,1)



The range of $f(x) = [-1, \infty)$

C) If $f(x) = \begin{cases} x+1 & \text{if} \quad x \leq -1 \\ x^2 & \text{if} \quad x > -1 \end{cases}$

4) Find $f(-3)$, $f(2)$ and $f(0)$

(a $\quad f(-3) = -3 + 1 = -2$

عوضنا عن 3- في الدالة الأولى لأن $-3 \in (-\infty, -1]$

(b $\quad f(2) = (2)^2 = 4$

عوضنا عن 2 في الدالة الثانية لأن $2 \in (-1, \infty)$

(c $\quad f(0) = (0)^2 = 0$

عوضنا عن 0 في الدالة الثانية لأن $0 \in (-1, \infty)$

5) Find the domain of the function $f(x)$

$$D_{f(x)} = \{x \mid x \leq -1\} \cup \{x \mid x > -1\}$$
$$= (-\infty, -1] \cup (-1, \infty)$$
$$= (-\infty, \infty) = \mathbb{R}$$

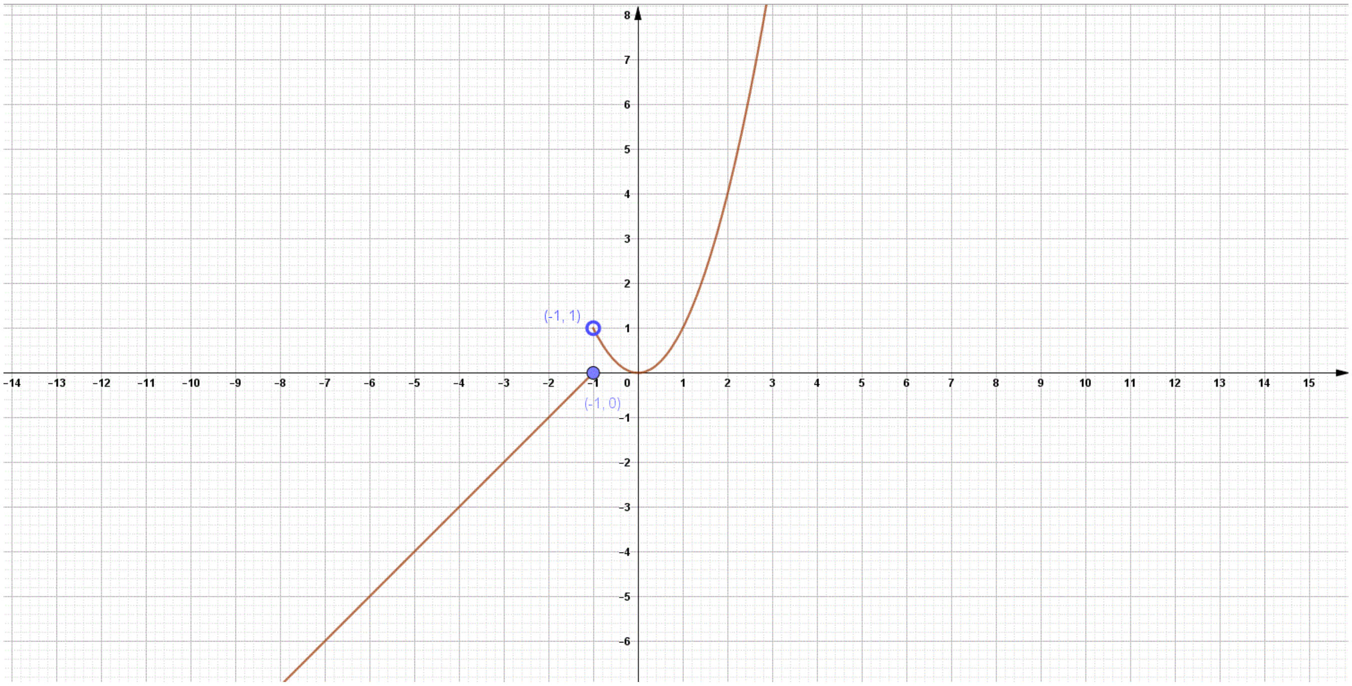
6) Sketch the graph of the function $f(x)$ and find the range

let $f_1(x) = x + 1$ for all $x \in (-\infty, -1]$

x	-1	-2
$y = f_1(x)$	0	-1
(x, y)	(-1,0)	(-2,-1)

let $f_2(x) = x^2$ for all $x \in \cup (-1, \infty)$

x	-1	0	1	2
$y = f_2(x)$	1	0	1	4
(x, y)	(-1,1)	(0,0)	(1,1)	(2,4)



The range of $f(x) = \mathbb{R}$

D) If $f(x) = \begin{cases} -1 & \text{if} \quad x \leq 1 \\ 7 - 2x & \text{if} \quad x > 1 \end{cases}$

1) Find $f(-3)$, $f(2)$ and $f(0)$

d) $f(-3) = -1$

عوضنا عن 3- في الدالة الأولى لأن $-3 \in (-\infty, 1]$

e) $f(2) = 7 - 2(2) = 7 - 4 = 3$

عوضنا عن 2 في الدالة الثانية لأن $2 \in (1, \infty)$

f) $f(0) = -1$

عوضنا عن 0 في الدالة الأولى لأن $0 \in (-\infty, 1]$

2) Find the domain of the function $f(x)$

$$D_{f(x)} = \{x \mid x \leq 1\} \cup \{x \mid x > 1\}$$
$$= (-\infty, 1] \cup (1, \infty)$$
$$= (-\infty, \infty) = \mathbb{R}$$

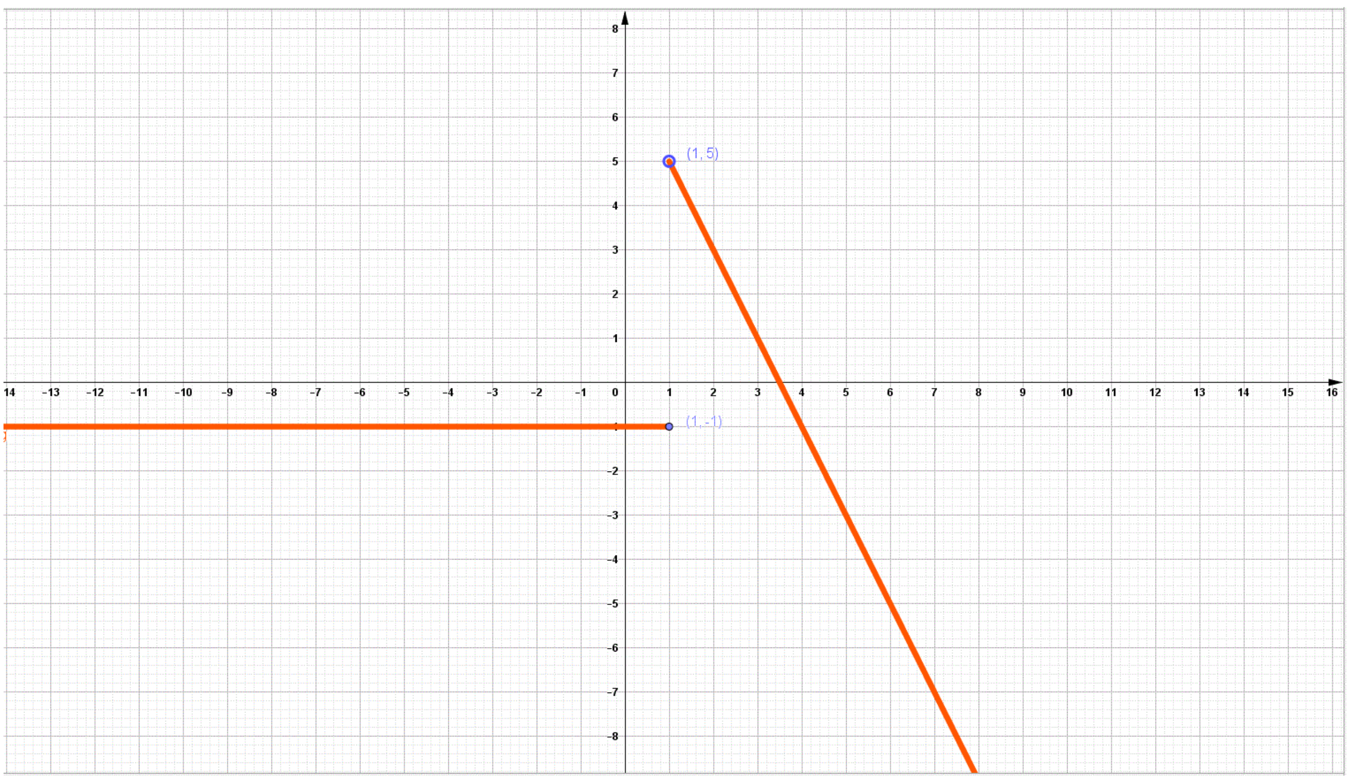
3) Sketch the graph of the function $f(x)$ and find the range

let $f_1(x) = -1$ for all $x \in (-\infty, 1]$

x	1	0
$y = f_1(x)$	-1	-1
(x, y)	(1,-1)	(0,-1)

let $f_2(x) = 7 - 2x$ for all $x \in \cup (1, \infty)$

x	1	2
$y = f_2(x)$	5	3
(x, y)	(1,5)	(2,3)



The range of $f(x) = (-\infty, 5)$

الهدف العاشر من الفصل الأول

كيفية ايجاد مجال و مدى ورسم وفك دالة القيمة المطلقة

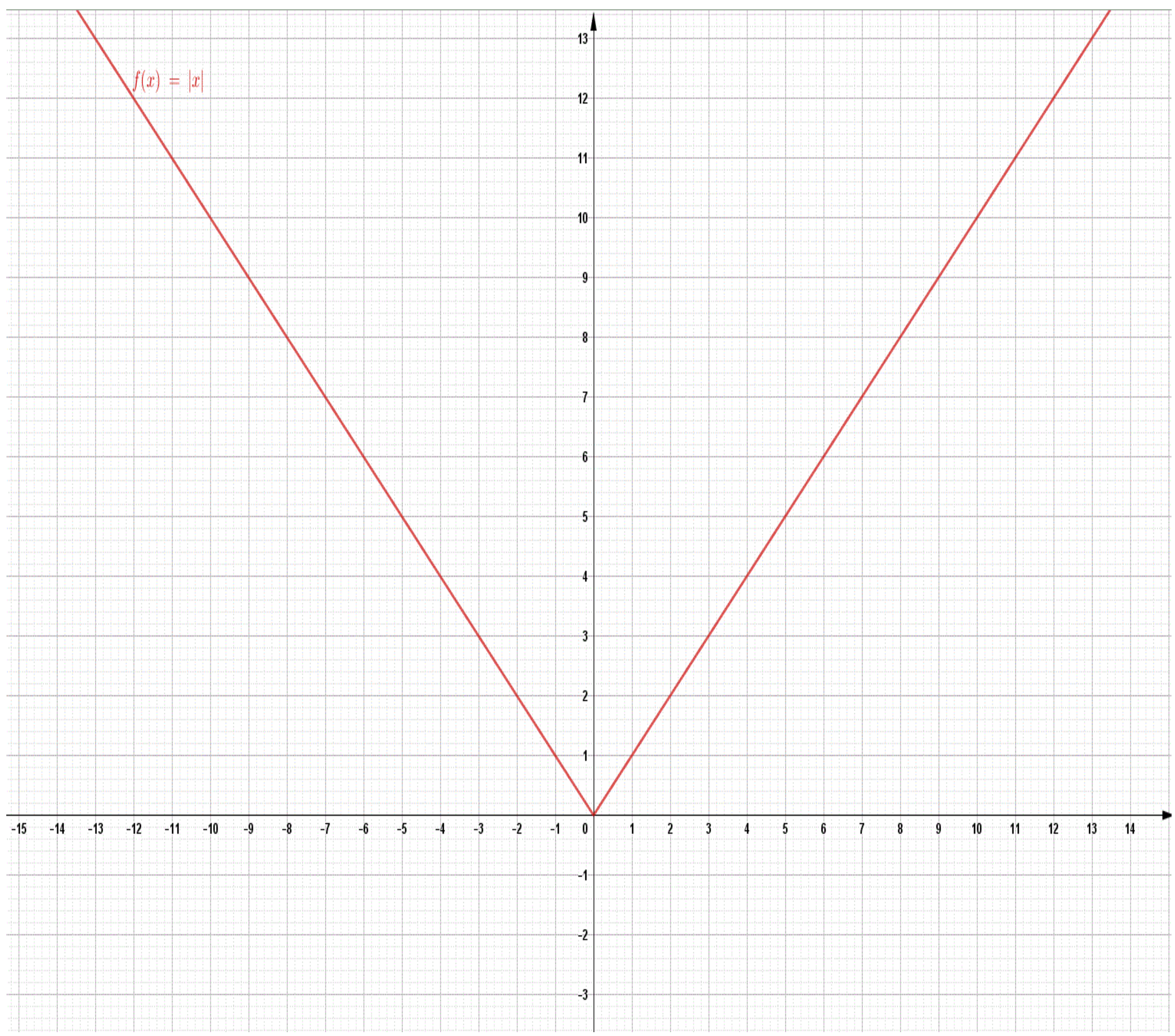
(absolute value Function)

Exercise (1)

A) If $f(x) = |x|$

then sketch the graph of the function and find the domain and range

$$f(x) = |x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$$



The domain of $f(x) = (-\infty, \infty)$

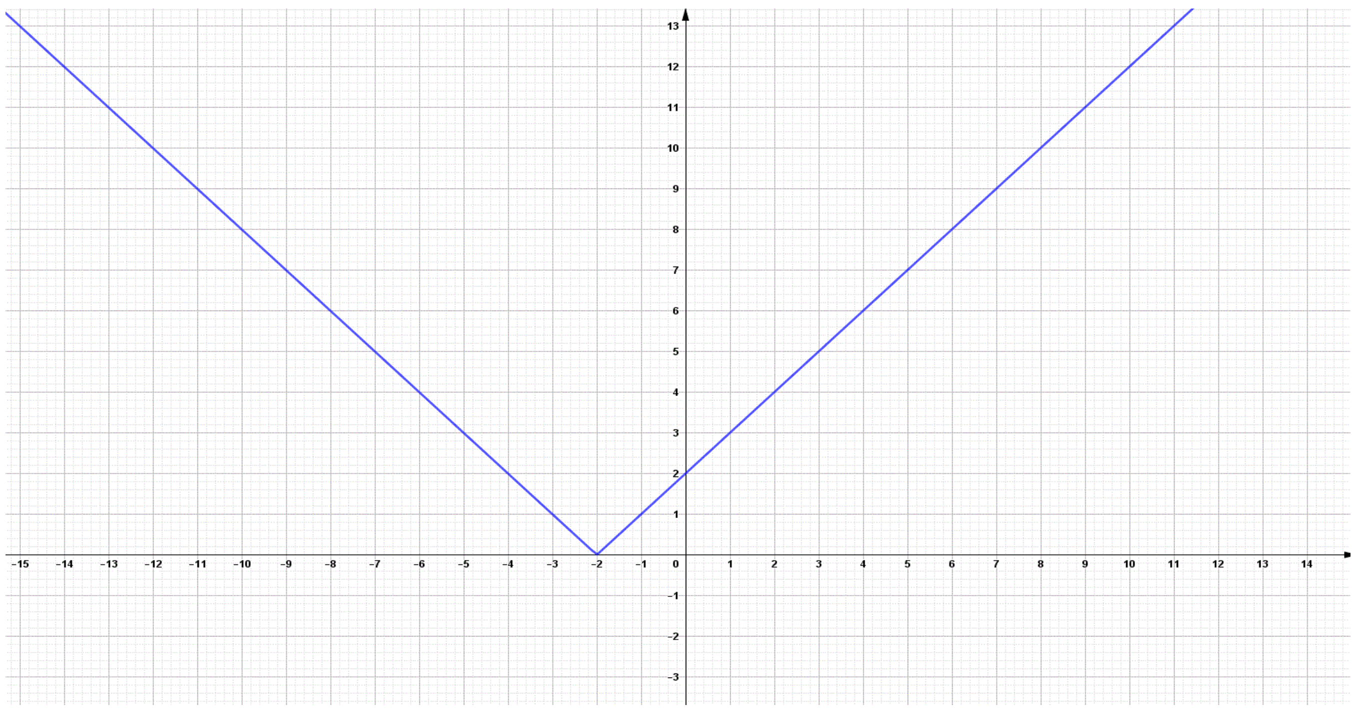
The range of $f(x) = [0, \infty)$

B)If $f(x) = |x+2|$

then sketch the graph of the function and find the domain and range

$$f(x) = |x+2| = \begin{cases} x+2 & \text{if } \quad x+2 \ge 0 \\ -(x+2) & \text{if } \quad x+2 < 0 \end{cases}$$

$$= \begin{cases} x+2 & \text{if } \quad x \ge -2 \\ -x-2 & \text{if } \quad x < -2 \end{cases}$$



The domain of $f(x) = (-\infty, \infty)$

The range of $f(x) = [0, \infty)$

C)If $f(x) = |6-2x|$

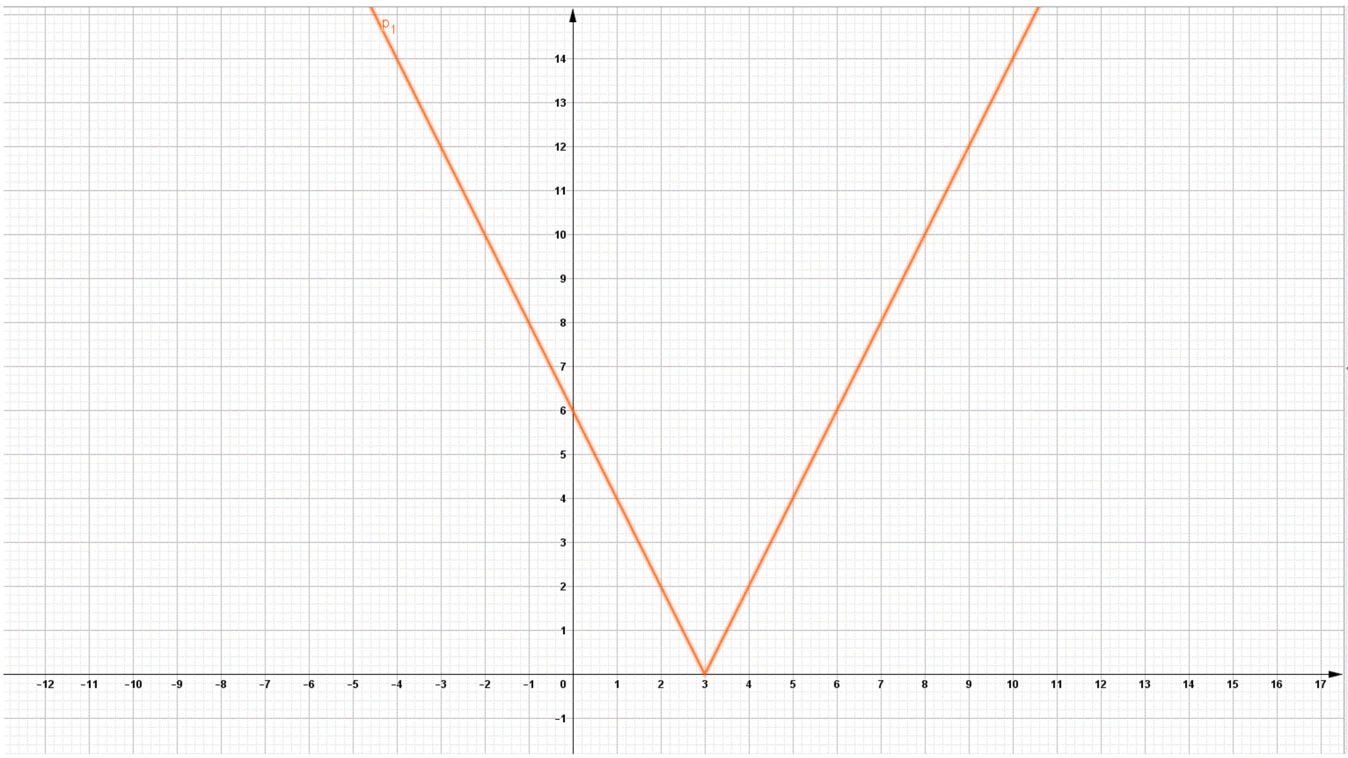
then sketch the graph of the function and find the domain and range

$$f(x) = |6-2x| = \begin{cases} 6-2x & \text{if } \quad 6-2x \ge 0 \\ -(6-2x) & \text{if } \quad 6-2x < 0 \end{cases}$$

$$= \begin{cases} 6-2x & \text{if } \quad -2x \ge -6 \\ -6+2x & \text{if } \quad -2x < -6 \end{cases}$$

$$= \begin{cases} 6-2x & \text{if } \quad \dfrac{-2x}{-2} \le \dfrac{-6}{-2} \\ 2x-6 & \text{if } \quad \dfrac{-2x}{-2} > \dfrac{-6}{-2} \end{cases}$$

$$= \begin{cases} 6-2x & \text{if } \quad x \le 3 \\ 2x-6 & \text{if } \quad x > 3 \end{cases}$$



The domain of $f(x) = (-\infty, \infty)$

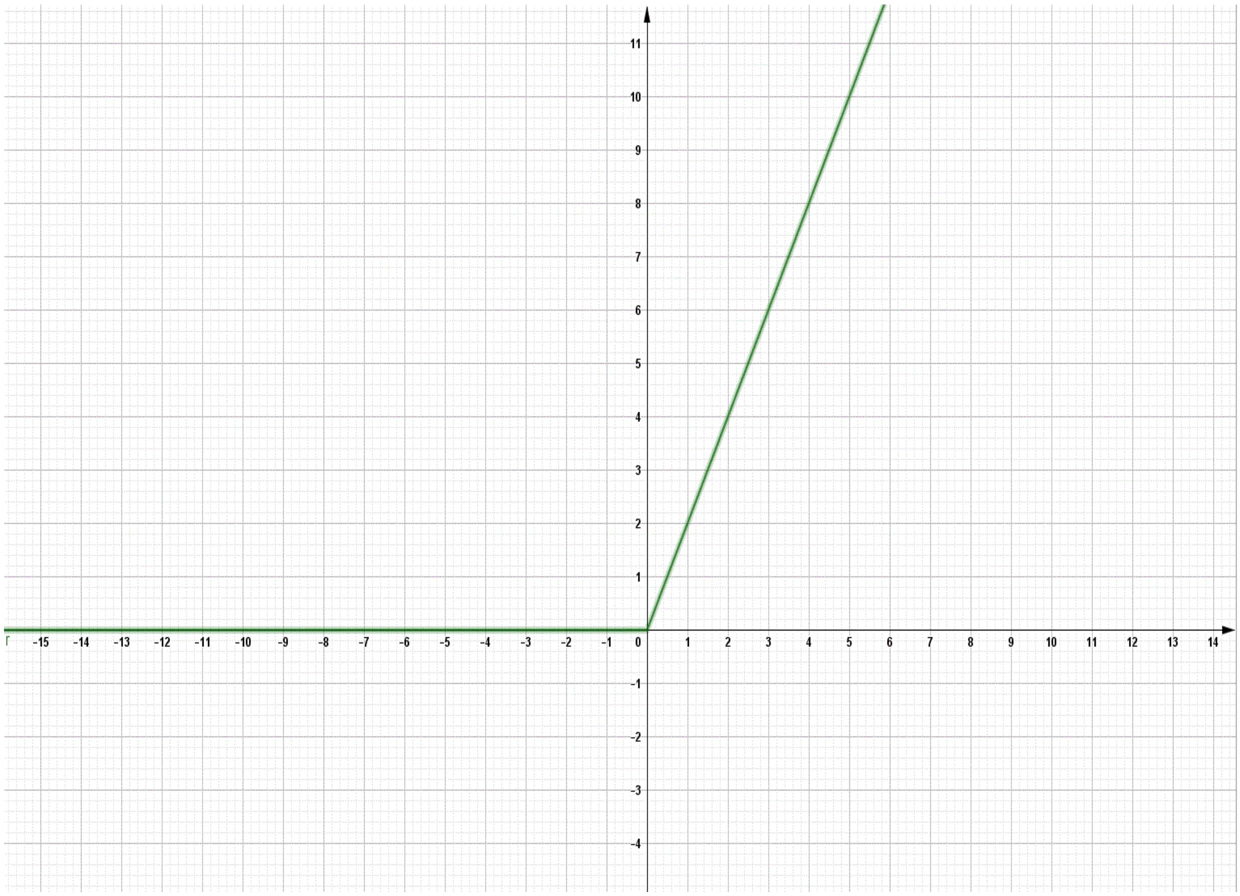
The range of $f(x) = [0, \infty)$

D) If $f(x) = x + |x|$

then sketch the graph of the function and find the domain and range

$$f(x) = x + |x| = \begin{cases} x + x & \text{if} \quad x \geq 0 \\ x - x & \text{if} \quad x < 0 \end{cases}$$

$$= \begin{cases} 2x & \text{if} \quad x \geq 0 \\ 0 & \text{if} \quad x < 0 \end{cases}$$



The domain of $f(x) = (-\infty, \infty)$

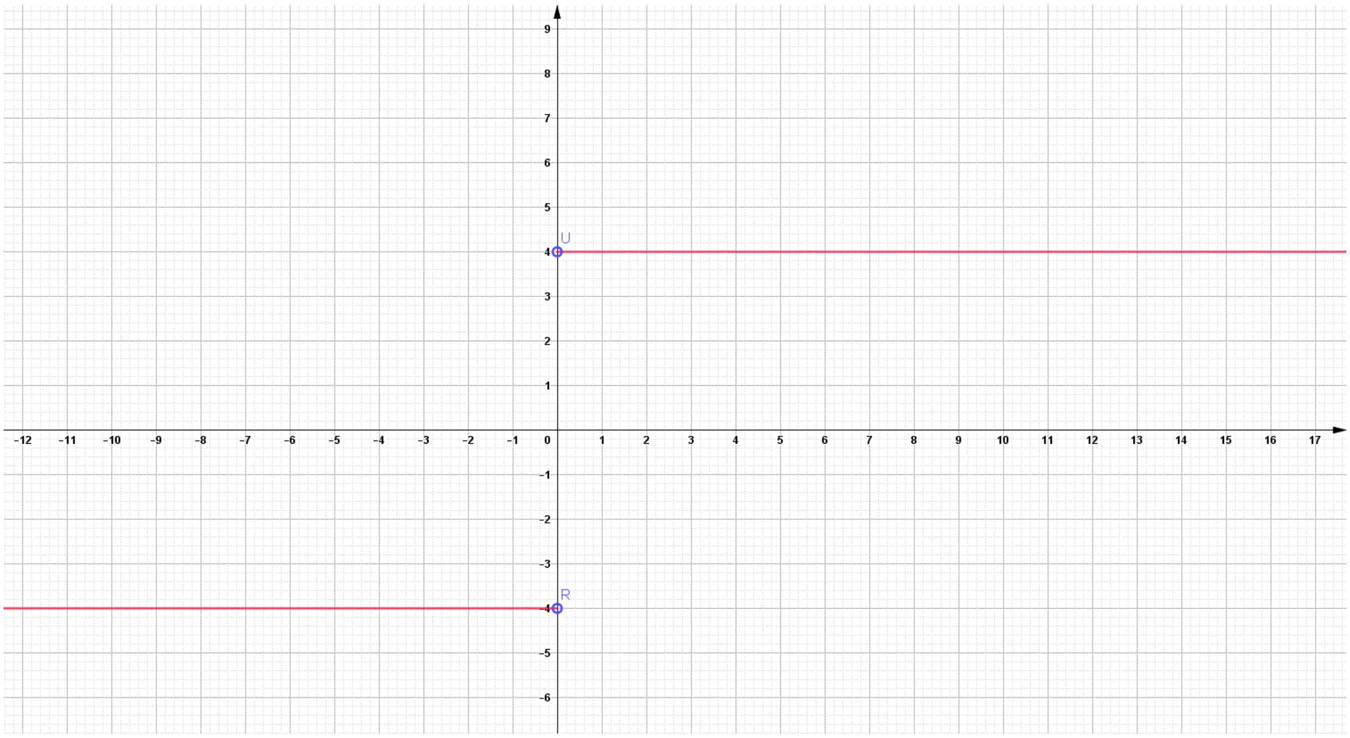
The range of $f(x) = [\mathbf{0}, \infty)$

E)If $f(x) = \frac{4x}{|x|}$

then sketch the graph of the function and find the domain and range

$$f(x) = \frac{4x}{|x|} = \begin{cases} \frac{4x}{x} & \text{if} \quad x > 0 \\ \frac{4x}{-x} & \text{if} \quad x < 0 \end{cases}$$

$$= \begin{cases} 4 & \text{if} \quad x > 0 \\ -4 & \text{if} \quad x < 0 \end{cases}$$



The domain of $f(x) = (-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty) = \mathbb{R} - \{\mathbf{0}\}$

The range of $f(x) = \{-4, 4\}$

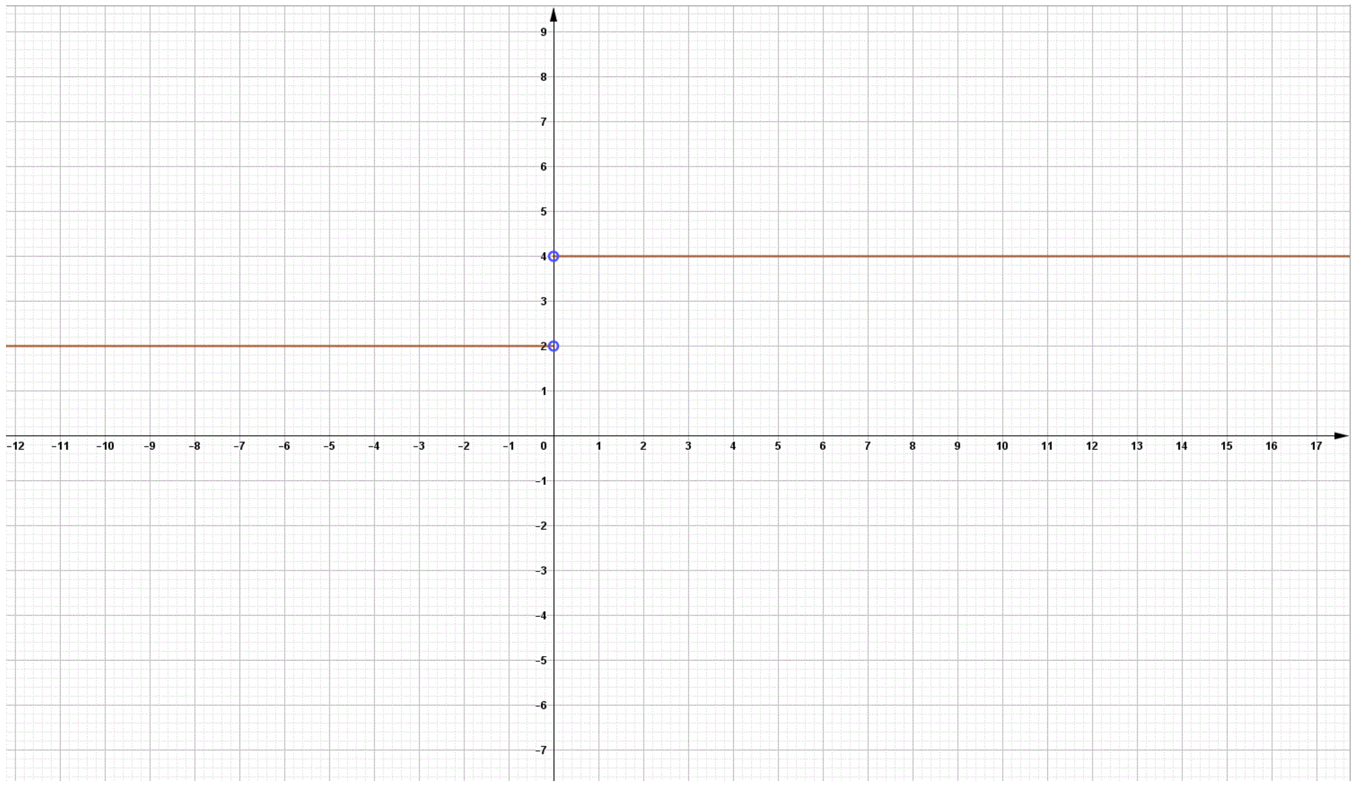
F) If $f(x) = \frac{3x+|x|}{x}$

then sketch the graph of the function and find the domain and range

$$f(x) = \frac{3x+|x|}{x} = \begin{cases} \dfrac{3x+x}{x} & \text{if} \quad x > 0 \\ \dfrac{3x-x}{x} & \text{if} \quad x < 0 \end{cases}$$

$$= \begin{cases} \dfrac{4x}{x} & \text{if} \quad x > 0 \\ \dfrac{2x}{x} & \text{if} \quad x < 0 \end{cases}$$

$$= \begin{cases} 4 & \text{if} \quad x > 0 \\ 2 & \text{if} \quad x < 0 \end{cases}$$



The domain of $f(x) = (-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty) = \mathbb{R} - \{\mathbf{0}\}$

The range of $f(x) = \{2, 4\}$

G) If $f(x) = \begin{cases} |x| & \text{if} \quad |x| \leq 1 \\ 1 & \text{if} \quad |x| > 1 \end{cases}$

then sketch the graph of the function and find the domain and range

$$f(x) = \begin{cases} |x| & \text{if} \quad -1 \leq x \leq 1 \\ 1 & \text{if} \quad x > 1 \text{ or } x < -1 \end{cases}$$

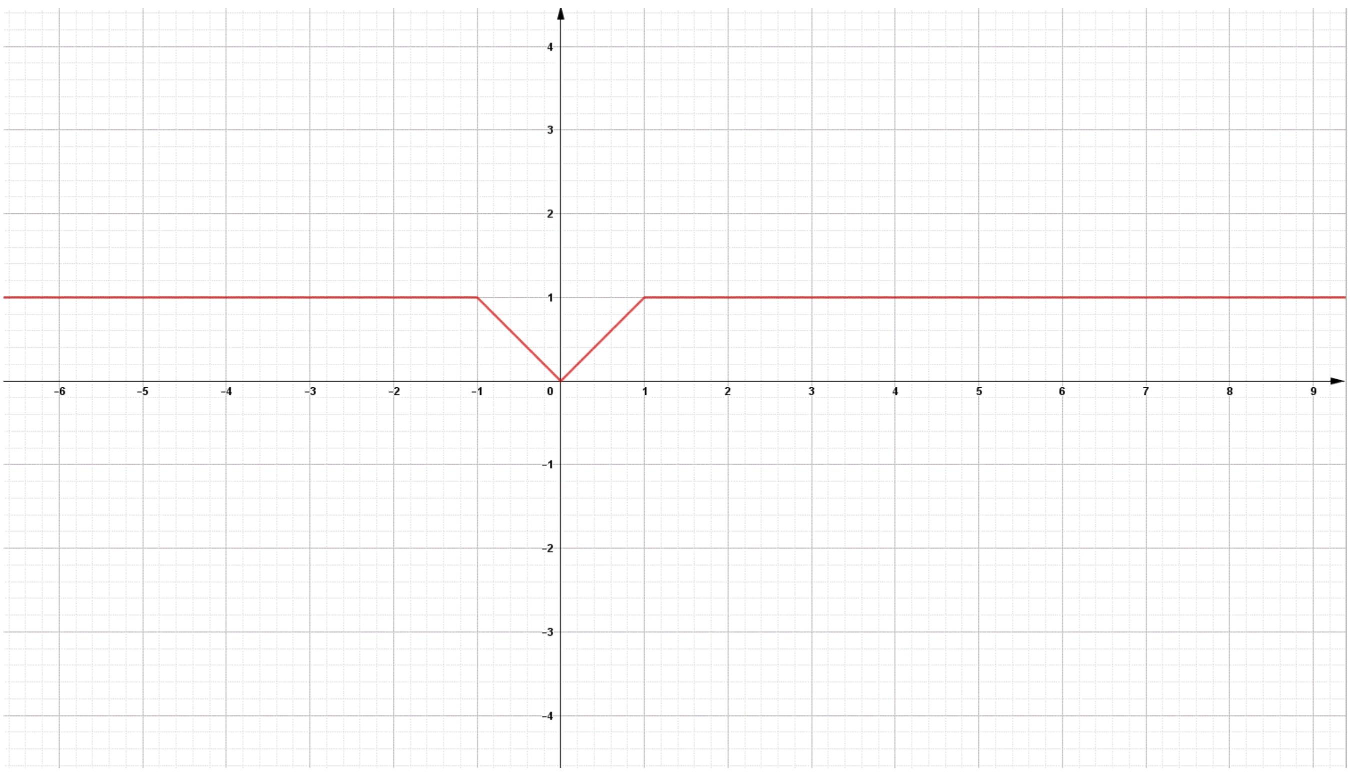
$$\begin{aligned} D_{f(x)} &= \{x \mid -1 \leq x \leq 1\} \cup \{x \mid x > 1 \text{ or } x < -1\} \\ &= [-1, 1] \cup \{(1, \infty) \cup (-\infty, -1)\} \\ &= (-\infty, \infty) = \mathbb{R} \end{aligned}$$

let $f_1(x) = |x|$ for all $x \in [-1, 1]$

x	-1	0	1
$y = f_1(x)$	1	0	1
(x, y)	(-1,1)	(0,0)	(1,1)

let $f_2(x) = 1$ for all $x \in \cup\ (1, \infty) \cup (-\infty, -1)$

x	1	2	-1	-2
$y = f_2(x)$	1	1	1	1
(x, y)	(1,1)	(2,1)	(-1,1)	(-2,1)



The range of $f(x) = [0, 1]$

ملاحظة

1) $\pi \cong 3.14$; $e \cong 2.7$

2) لإيجاد قيمة $\sqrt{2}$

نحصر العدد الذي بداخل الجذر بين أقرب عددين مربعين

$$1 < 2 < 4$$

ثم نأخذ الجذر التربيعي للمتراجحة السابقة

$$\sqrt{1} < \sqrt{2} < \sqrt{4}$$

$$1 < \sqrt{2} < 2$$

إذن قيمة جذر ٢ عدد عشري أكبر من العدد ١ وأصغر من العدد ٢

لإيجاد قيمة $\sqrt{10}$

نحصر العدد الذي بداخل الجذر بين أقرب عددين مربعين

$$9 < 10 < 16$$

ثم نأخذ الجذر التربيعي للمتراجحة السابقة

$$\sqrt{9} < \sqrt{10} < \sqrt{16}$$

$$3 < \sqrt{10} < 4$$

إذن قيمة جذر ١٠ عدد عشري أكبر من العدد ٣ وأصغر من العدد ٤

Exercise (2)

$$|2-\pi| = \left|\underset{\text{ناتج العملية عدد سالب}}{2-\pi}\right| = -(2-\pi) = -2+\pi = \pi-2$$

$$|3-\pi| = \left|\underset{\text{ناتج العملية عدد سالب}}{3-\pi}\right| = -(3-\pi) = -3+\pi = \pi-3$$

$$|4-\pi| = \left|\underset{\text{ناتج العملية عدد موجب}}{4-\pi}\right| = 4-\pi$$

$$|2-e| = \left|\underset{\text{ناتج العملية عدد سالب}}{2-e}\right| = -(2-e) = -2+e = e-2$$

$$|-3-e| = \left|\underset{\text{ناتج العملية عدد سالب}}{-3-e}\right| = -(-3-e) = 3+e$$

$$|4-e| = \left|\underset{\text{ناتج العملية عدد موجب}}{4-e}\right| = 4-e$$

$$|1-\sqrt{2}| = \left|\underset{\text{ناتج العملية عدد سالب}}{1-\sqrt{2}}\right| = -(1-\sqrt{2}) = -1+\sqrt{2}$$

$$= \sqrt{2}-1$$

$$|-4-\sqrt{2}| = \left|\underset{\text{ناتج العملية عدد سالب}}{-4-\sqrt{2}}\right| = -(-4-\sqrt{2}) = 4+\sqrt{2}$$

$$|3-\sqrt{2}| = \left|\underset{\text{ناتج العملية عدد موجب}}{3-\sqrt{2}}\right| = 3-\sqrt{2}$$

1.2 - Types of The Functions.

Type 1: Polynomials.

Example: If $p(x) = 2x^6 - x^4 + \frac{2}{5}x^3 + \sqrt{2}$ then

a) Name: Polynomial

b) The Degree: Polynomial of degree 6

c) Leading Coefficient: 2
or Coefficient of x^6
or a_6

d) Coefficient of x^3: $a_3 = \frac{2}{5}$

e) Constant term: $a_0 = \sqrt{2}$
or Coefficient of: x^0

f) $a_4 = -1$

g) $a_2 = 0$

Example: $P(x) = 5x^{\pi} - \frac{1}{3}x^{e} + 2x^{\sqrt{3}} - 2.1x^{-5} - \sqrt{2}\,x^{\frac{3}{4}} + 3x^{3.5} - x^{\sin(1)} + \sqrt{x} - x^{\log} - \frac{1}{x} + \ln(4)$ is not Polynomial

$\boxed{\text{a) T}}$ b) F

Example: $P(x) = \frac{1}{3}x - \sqrt{2}x^2 + \pi x^3 - x^7 + \frac{3}{e}$ is Polynomial of degree .....

a) 1 b) 2 c) 3 $\boxed{\text{d) } 7}$

H.W: $P(x) = -\frac{2}{3}x^3 + 5x - 8$

a) Name:

b) degree:

c) Leading coefficient:

d) constant term:

e) coefficient of x:

f) $a_2 = \ldots\ldots$

Note: The Domain of any Polynomial is $\mathbb{R}$ or $(-\infty, \infty)$

Example: The Domain of $f(x) = x^2 - 16$ is...

a) $\{4, -4\}$ b) $\mathbb{R} - \{\pm 4\}$ c) $\mathbb{R} - [4, 4]$

D) $[-4, 4]$ E) $(-4, 4)$ $\boxed{\text{F) } \mathbb{R}}$

Type of Polynomials:

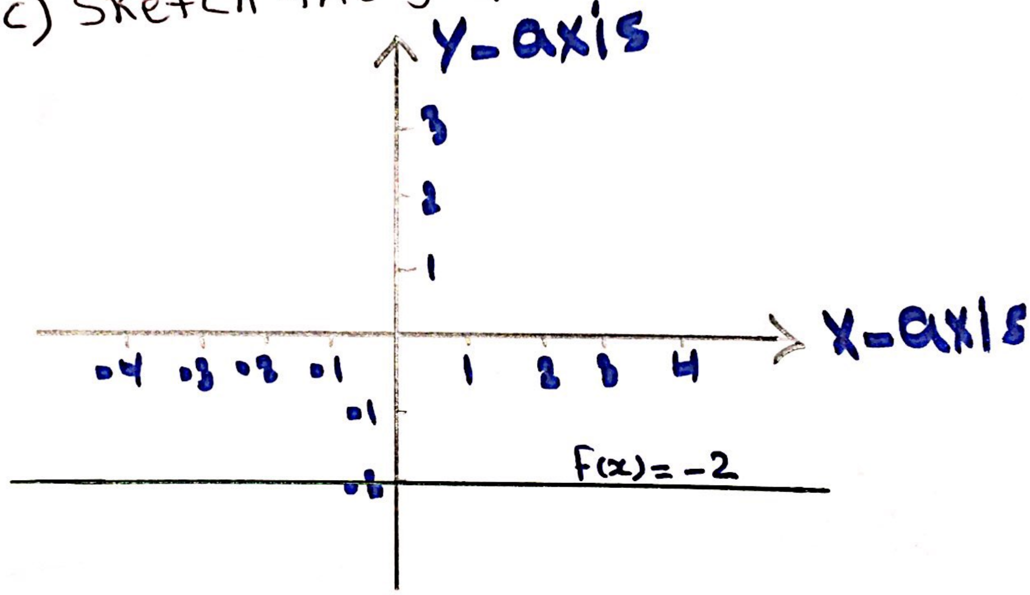
A) Constant function: $P(x) = a$ is Polynomial of degree Zero for all $a \in \mathbb{R}$

Example: If $f(x) = -2$ then...

a) $f(x)$ is polynomial of degree Zero

b) $f(x)$ is called Constant function

c) Sketch the graph of $f(x) = -2$



d) The graph of $f(x) = -2$ is Horizontal line

whith Slope: 0

y-intercept: -2

Parallel to x-axis

E) $D_{f(x)} = \mathbb{R}$ $\quad R_f = \{-2\}$

B) Linear function: $P(x) = ax+b$ is Polynomial of degree 1 whith Slope: a and y-intercept: b

Example: $f(x) = 2-5x$

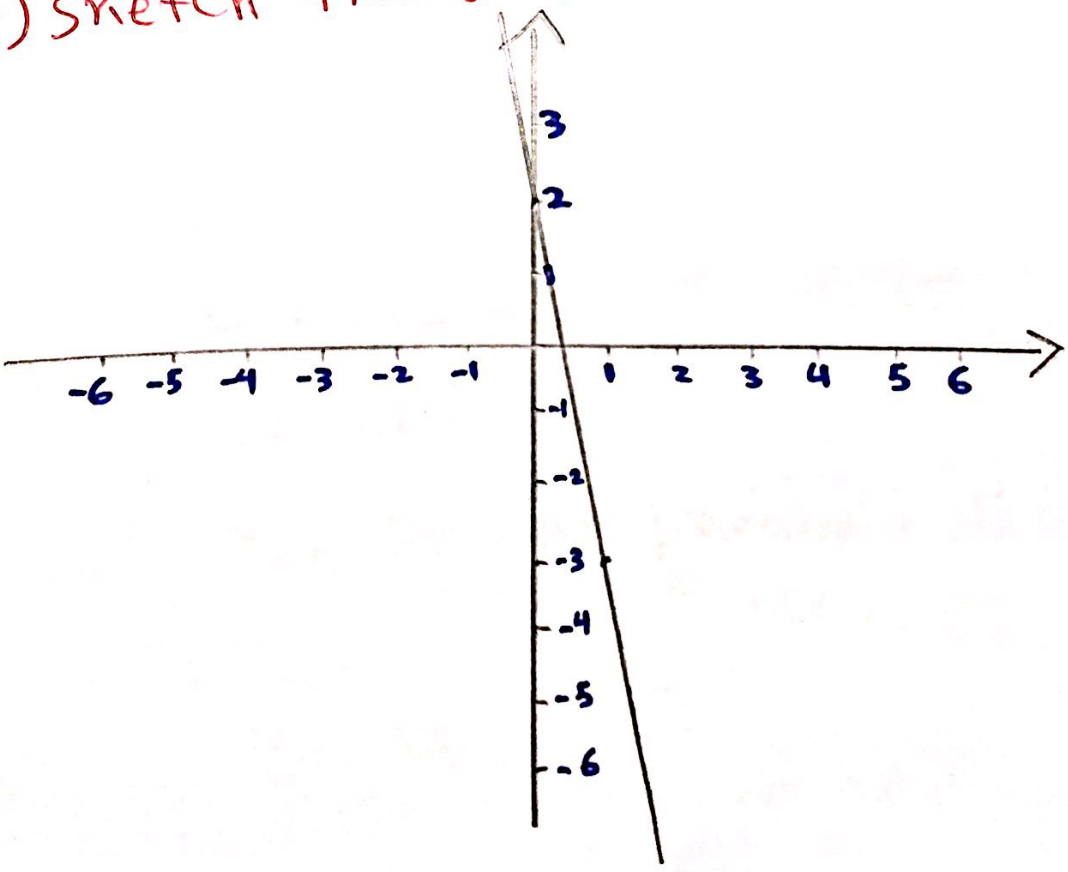
a) $f(x)$ is Polynomial of degree 1

b) $f(x)$ is called a linear Function

c) The Slope: -5 y-intercept: 2

d) $D_{f(x)} = \mathbb{R}$ $R_{f(x)} = \mathbb{R}$

E) sketch the graph of $f(x) = 2-5x$



c) Quadratic function: $P(x) = ax^2 + bx + c$ is Polynomial of degree 2
It's graph is alway a parabola

Note: if $a > 0$ then the parabola opens upward
if $a < 0$ then the parabola opens downward.

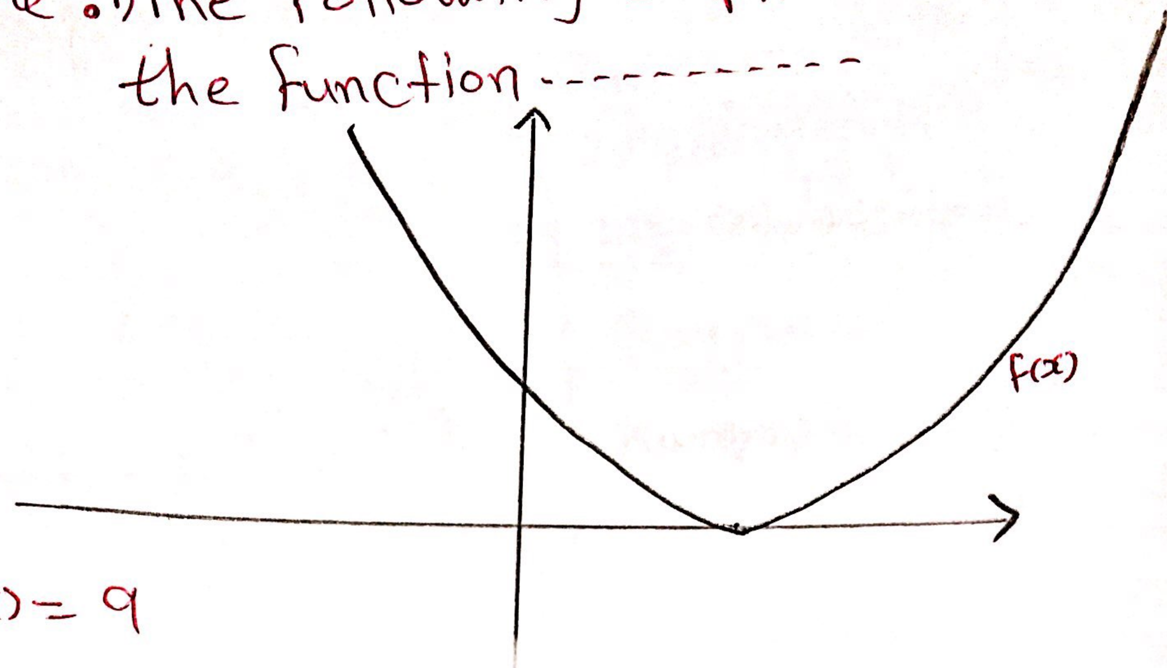
Example: $y = x^2 + x + 1$ is polynomial of degree 2 or Qudratic function
The Graph: Parabola opens up
Since: $a = 1 > 0$

The Domain of $y = x^2 + x + 1$: $(-\infty, \infty)$

Example: $F(x) = 1 - 2x^2$ is polynomial of degree 2 or Qudratic function
The Graph: Parabola opens down
Since: $a = -2 < 0$

The $D_{f(x)} = \mathbb{R}$

Example: 1) The following Graph represents the function ----------



a) $f(x) = 9$

b) $f(x) = 6x + 9$

c) $f(x) = 9 + 6x - x^2$

d) $\boxed{f(x) = x^2 - 6x + 9}$

2) $D_{f(x)} = \mathbb{R}$ $R_{f(x)} = [0, \infty)$

D) The Cubic function: $P(x) = ax^3 + bx^2 + cx + d$

is Polynomial of degree 3

Domain: $\mathbb{R}$ Rang: $\mathbb{R}$

Example:

$f(x) = 3x^3 + 2x + 1$

$y = x^3 + 1$

$H(x) = \frac{1}{2}x^3 - \sqrt{2}\,x^2$

$G(x) = 4x^3$

Polynomial of degree $\underline{\underline{3}}$ or Cubic Function

Domain: $\mathbb{R} = (-\infty, \infty)$

Range: $\mathbb{R} = (-\infty, \infty)$

Type 2: The Power Function

$$f(x) = x^a \text{ for all } a \in \mathbb{R}$$

Example:

$f(x) = x^7$

$f(x) = x^{-8}$

$f(x) = x^{3/2}$

$f(x) = x^{\pi}$

$f(x) = \sqrt[5]{x^{11}} = x^{11/5}$

$f(x) = \frac{1}{x^{-4}} = x^4$

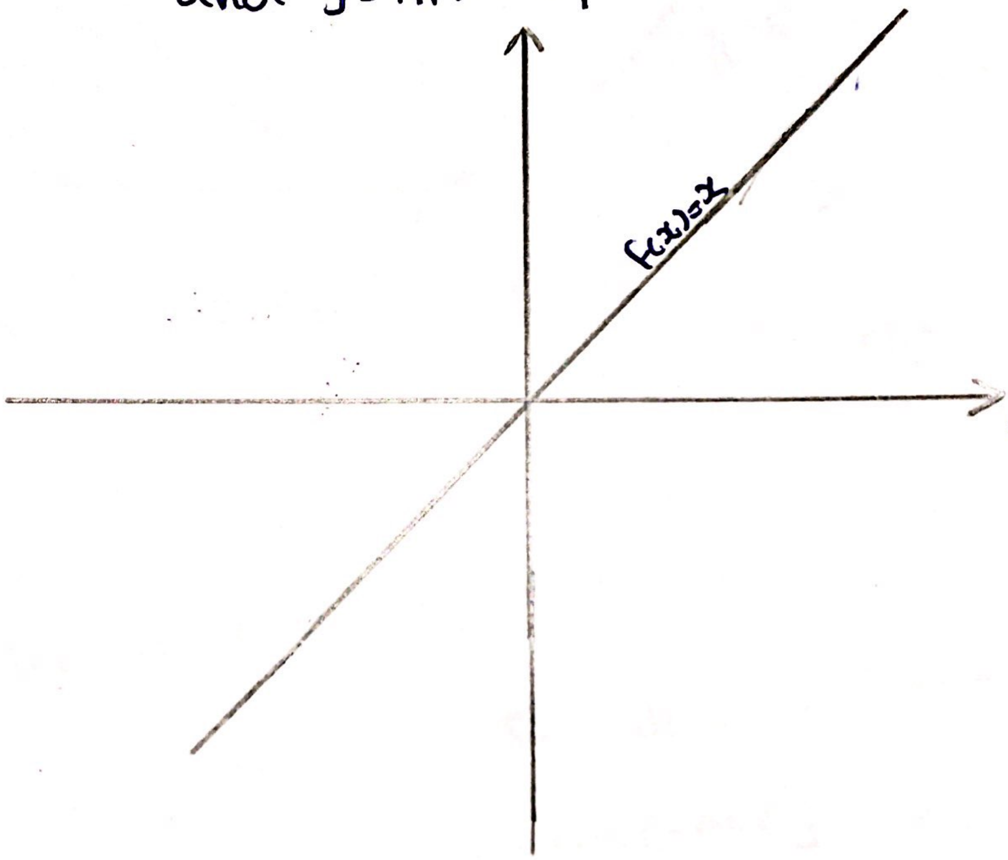
$f(x) = \frac{1}{x^{+10}} = x^{-10}$

The Power Functions

Example : 1) $f(x) = x$

a) Name : Power function or linear function or Polynomial of degree 1 or Unit function

b) Graph : The Straight line with slope : 1 and y-intercept : 0



c) $f(x) = x$ is odd function

d) $f(x) = x$ is Symmetric about The origin or $(0,0)$

e) $f(x) = x$ is increasing forall $x \in \mathbb{R}$

F) $D_{f(x)} = \mathbb{R}$ $\quad R_{f(x)} = \mathbb{R}$

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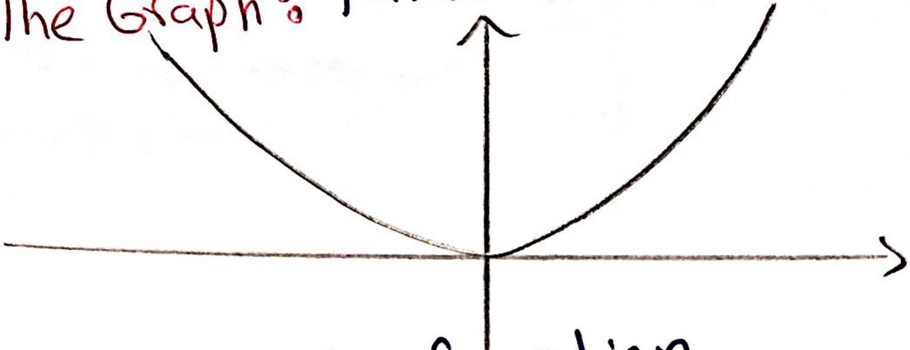
2) $f(x) = x^2$

a) Name: The Power function or Polynomial of Degree 2 or Qudratic function

b) The Graph: Parabola



c) $f(x)$ is even function

d) $f(x)$ is Symmetric about Y_axis

e) $f(x)$ is increasing for all $x \geq 0$

or $f(x)$ is increasing on $[0, \infty)$

f) $f(x)$ is decreasing for all $x \leq 0$

or $f(x)$ is decreasing on $(-\infty, 0]$

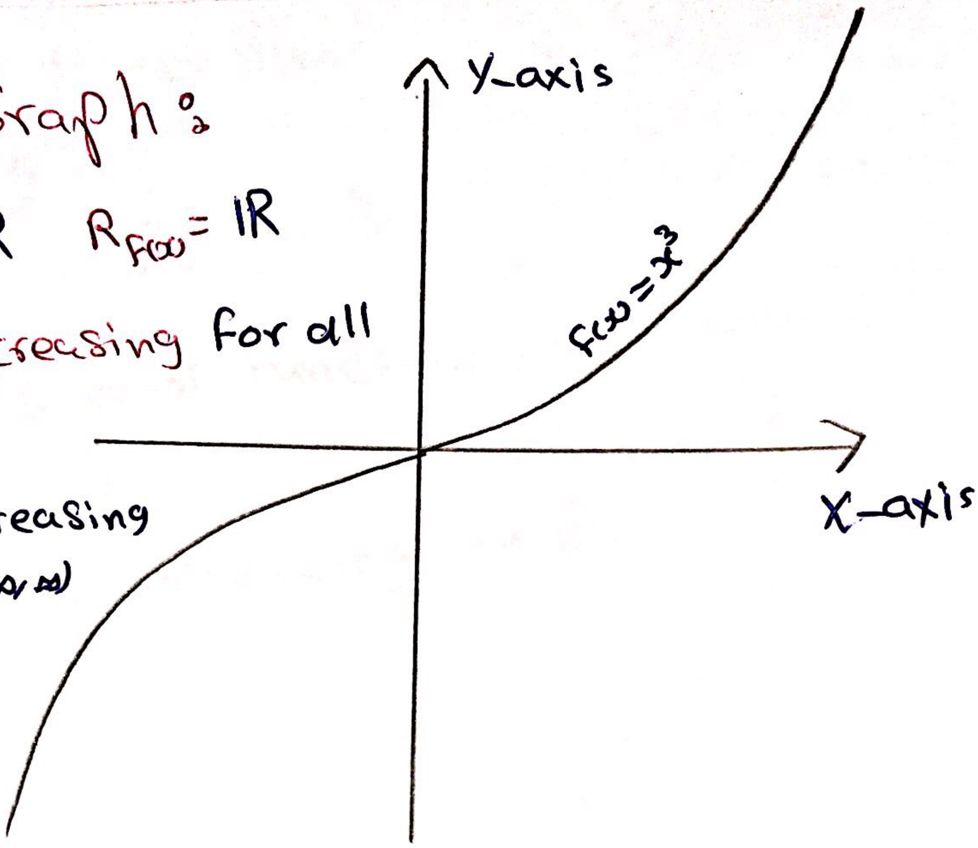
3) $f(x) = x^3$

a) Name: The power function or Cubic function or Polynomail of degree 3

b) $f(x)$ is odd function

c) $f(x)$ is Symmetric about $(0,0)$ or The origin

d) The Graph:

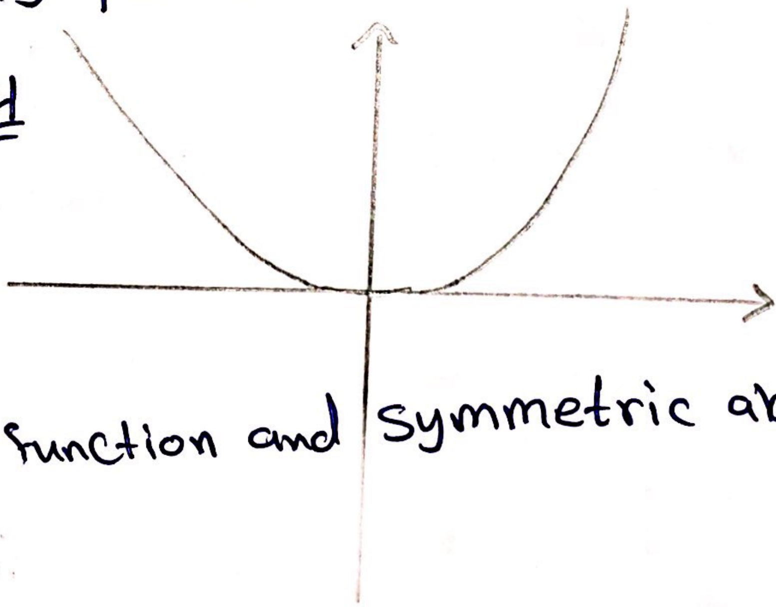


e) $D_{f(x)} = \mathbb{R}$ $\quad R_{f(x)} = \mathbb{R}$

f) $f(x)$ is increasing for all $x \in \mathbb{R}$

or $f(x)$ increasing on $\mathbb{R} = (-\infty, \infty)$

4) $f(x) = x^4$ is Power function or Polynomial of degree $\underline{4}$



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [0, \infty)$

$f(x)$ is even function and symmetric about Y-axis

$f(x)$ is increasing on $[0, \infty)$

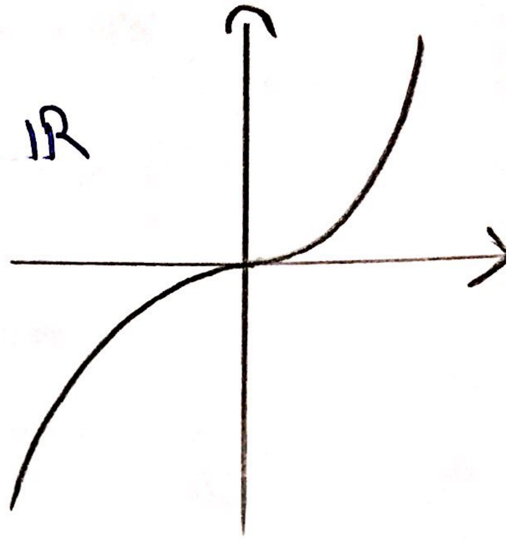
$f(x)$ is decreasing on $(-\infty, 0]$

5) $f(x) = x^5$ is Power function or Polynomial of Degree $\underline{5}$

$D_{f(x)} = \mathbb{R}$; $R_{f(x)} = \mathbb{R}$

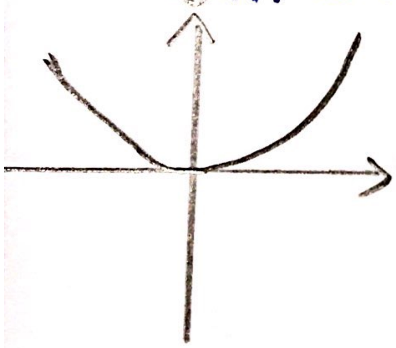
$f(x) = x^5$ is odd function and Symmetric about $(0,0)$

$f(x) = x^5$ is increasing on $\mathbb{R}$



$f(x) = x^n$ for all $n = 2, 3, 4, 5, 6, 7, \dots$

n: even number



Domain: $\mathbb{R}$

Range: $[0, \infty)$

Power function

Polynomial of degree $\underline{n}$

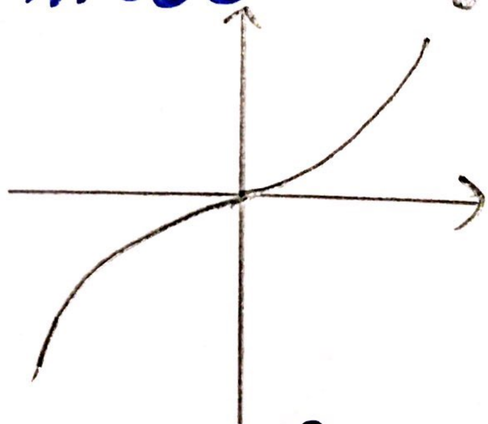
$f(x)$ is even

$f(x)$ is increasing on $[0, \infty)$

$f(x)$ is decreasing on $(-\infty, 0]$

Ex: $f(x) = x^{12}$

n: odd num



Domain: $\mathbb{R}$

Range: $\mathbb{R}$

Power function

Polynomial of Degree $\underline{n}$

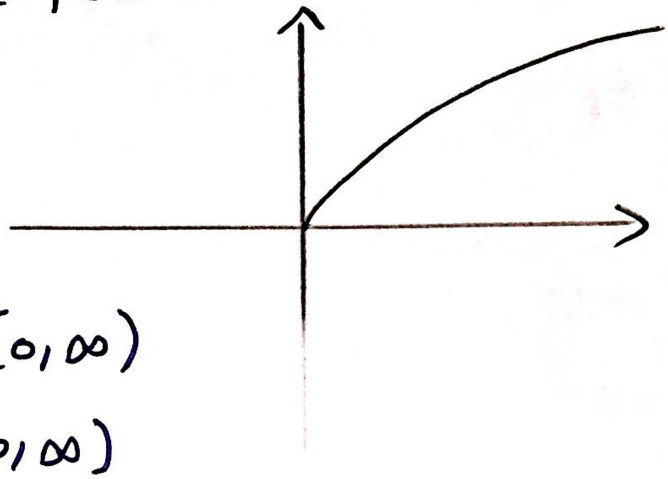
$f(x)$ is odd

$f(x)$ is increasing on $\mathbb{R}$

Ex: $f(x) = x^9$

6) $f(x) = x^{\frac{1}{2}}$ or $f(x) = \sqrt{x}$

Name: The power function or Root function



Domain: $[0, \infty)$

Range: $[0, \infty)$

$f(x) = \sqrt{x}$ is increasing on $[0, \infty)$

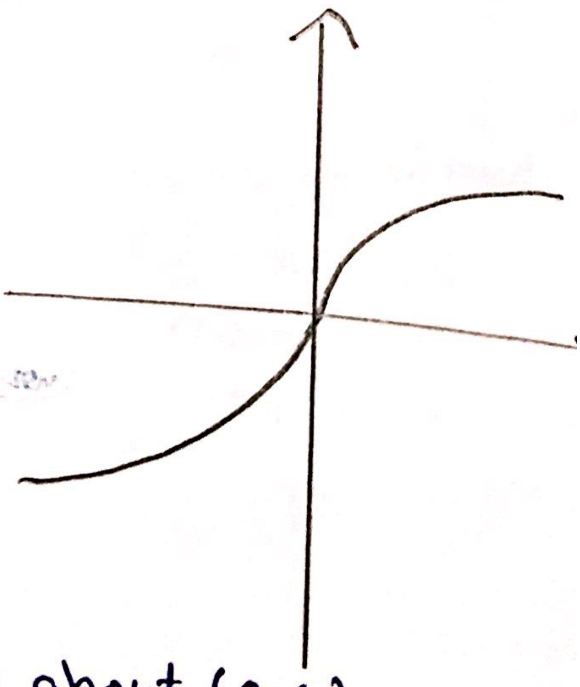
$f(x) = \sqrt{x}$ is neither odd nor even

7) $f(x) = \sqrt[3]{x}$ or $x^{1/3}$

Name: The Power function or Root function

Domain: $\mathbb{R}$

Range: $\mathbb{R}$



$f(x) = x^{1/3}$ is increasing on $\mathbb{R}$

$f(x) = x^{1/3}$ is odd function

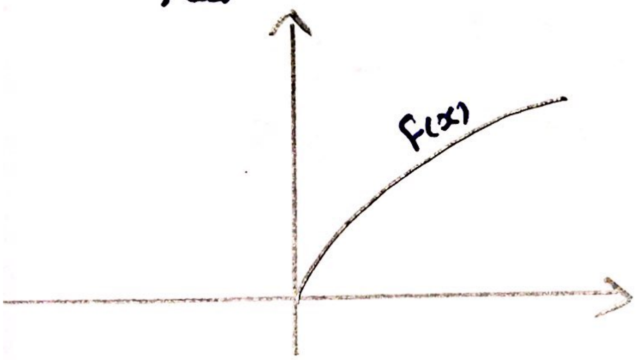
$f(x) = x^{1/3}$ is Symmetric about $(0, 0)$

$f(x) = \sqrt[n]{x}$ or $x^{\frac{1}{n}}$ For all $n = 2, 3, 4, 5, \ldots$

n is even number

$D_{f(x)} = [0, \infty)$

$R_{f(x)} = [0, \infty)$



$f(x)$ is increasing on $[0, \infty)$

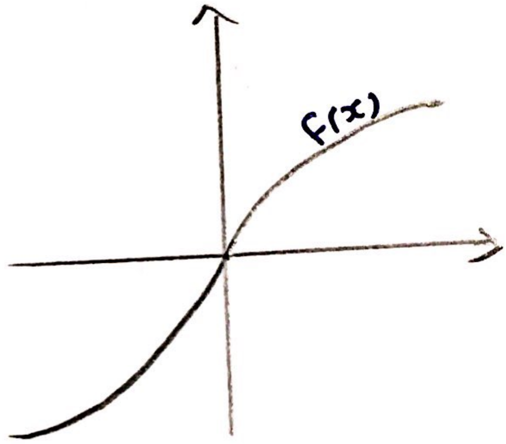
$f(x)$ is neither odd nor even

Ex : $f(x) = \sqrt[6]{x}$

n is odd number

$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = \mathbb{R}$



$f(x)$ is increasing on $\mathbb{R}$

$f(x)$ is odd and Symmetric about the origin

Ex : $f(x) = \sqrt[7]{x}$

⑧ $f(x) = x^{-1}$ or $\frac{1}{x}$

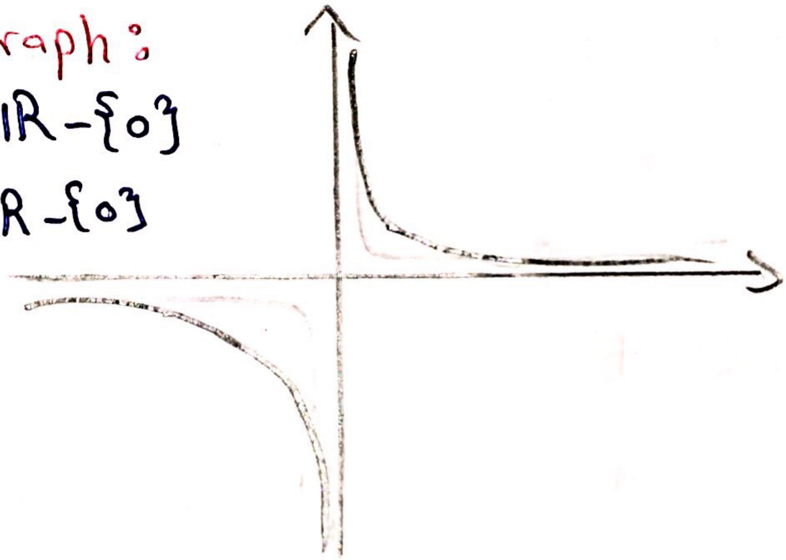
a) Name : The Power function <u>or</u> Rational function <u>or</u> reciprocal function

ⓑ $f(x) = \frac{1}{x}$ is odd function and Symmetric about $(0,0)$

c) The Graph :

$D_{f(x)} = \mathbb{R} - \{0\}$

$R_{f(x)} = \mathbb{R} - \{0\}$



d) $f(x) = \frac{1}{x}$ is decreasing on $\mathbb{R} - \{0\}$

e) $x = 0$ and $y = 0$ are asymbtotes

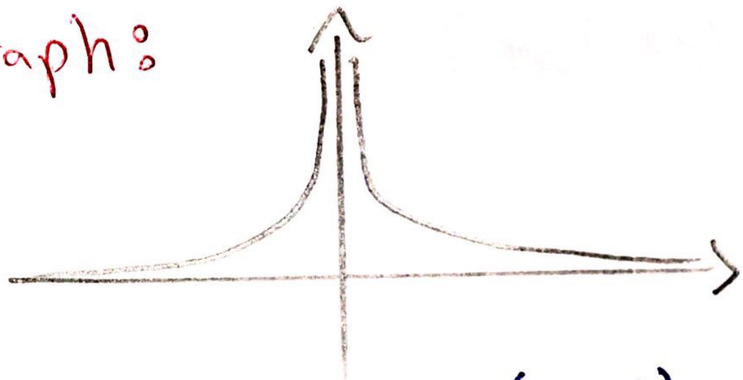
<u>H.W</u> : $f(x) = \frac{1}{x^5}$; $f(x) = \frac{1}{x^3}$

(9) $f(x) = x^{-2}$ or $\frac{1}{x^2}$

Name: The Power function or Rational function

$f(x) = \frac{1}{x^2}$ is even function and Symmetric about y_axis

The Graph:



$D_{f(x)} = \mathbb{R} - \{0\}$ $R_{f(x)} = (0, \infty)$

$x = 0$ and $y = 0$ are asymbtotes.

$f(x)$ is decresing on $(0, \infty)$

$f(x)$ is increasing on $(-\infty, 0)$

H.W $f(x) = x^{-4}$; $f(x) = x^{-10}$

Type 3 : Radical function

$f(x) = \sqrt[n]{p(x)}$ for all $p(x)$ is Polynomial.

Example :

$f(x) = \sqrt[7]{x^3+1}$

$f(x) = \sqrt{x^2+12}$

} The Radical functions

Type 4 : Rational function

$f(x) = \dfrac{P(x)}{Q(x)}$ for all $P(x)$ and $Q(x)$ are polynomials.

$D_{f(x)} = \mathbb{R} - \{$أصفار المقام.$\}$

Example :

$f(x) = \dfrac{x}{x^3+27}$

$F(x) = \dfrac{x-1}{x^2+100}$

} The Rational functions.

Type 5 : Absolute Value Function

$f(x) = | P(x) |$ for all $P(x)$ is Polynomial

$D_{f(x)} = \mathbb{R}$; $R_{f(x)} = [0, \infty)$

Example: $f(x) = |x|$

$f(x) = |x - 4|$

$f(x) = |x^2 - 9|$

$\Bigg\}$ Absolute value functions

$D_f = \mathbb{R}$

$R_f = [0, \infty)$

Type 6 : Algabric function

1. Polynomial
2. Power function
3. Radical function
4. Rational function
5. absolute Value function
6. Add, subtract, multiple, divied and taking for above function.

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Example:

Algabric Function

$g(x) = 3x^7 - 3x^2 - 1$

$g(x) = x^{-20}$

$g(x) = \sqrt{x}$

$g(x) = x^3$

$g(x) = \frac{1}{x^{12}}$

$g(x) = \sqrt{x^2 + 7x + 1}$

$g(x) = \frac{x}{x^2 + x + 7}$

$g(x) = |x + 1|$

$g(x) = x^2 + \sqrt{x} + x^{-3} - |x^2 + 1| + \frac{1}{2}$

$g(x) = \frac{x^4 - 16x^2}{x + \sqrt{x}}$

$g(x) = (x-2)\sqrt[3]{x-1}$

$g(x) = |x-1| + \sqrt{x}$

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Transedental functions: "Not Algabric"

1] Exponential Functions

The Natural Exponetial Function

$f(x) = e^x$

The Genral Exponential Function

$f(x) = a^x$ for all $a \in \mathbb{R}^+$

Example:

$F(x) = 2^x$

$f(x) = \left(\frac{1}{3}\right)^x$

$F(x) = \left(\frac{3}{2}\right)^x$

$f(x) = \pi^x$

$f(x) = (\sqrt{2})^x$

Exponetial Function

$f(x) = x^2$

$f(x) = x^{1/3}$

$f(x) = x^{3/2}$

$f(x) = x^{\pi}$

$f(x) = x^{\sqrt{2}}$

Power function

Example:

✓ $f(x) = (-2)^x$ is a Genral Exponential Function ✗

Since: $a = -2 \notin \mathbb{R}^+$

✓ $f(x) = \pi^{-x}$ is a Genral Exponential function ✓

Since: $F(x) = \pi^{-x} = \frac{1}{\pi^x}$

$F(x) = \left(\frac{1}{\pi}\right)^x$

✓ $f(x) = 5^{\frac{x}{2}}$ is not Exponential function

✓ X

Since: $f(x) = 5^{x/2}$

$= (5^{\frac{1}{2}})^x$

$= (\sqrt{5})^x$ is a Exponential Function

✓ $f(x) = 8^{\frac{x}{3}}$ is a Exponential function

✓ X

Since: $f(x) = 8^{x/3}$

$= (2^3)^{x/3}$

$= 2^{\frac{3x}{3}}$

$= 2^x$ is a Exponential Function.

✓ $f(x) = \sqrt{e^{2x}}$ is not Natural Exponential function

✓ X

Since: $f(x) = \sqrt{e^{2x}} = (e^{2x})^{\frac{1}{2}} = e^{2x \cdot \frac{1}{2}}$

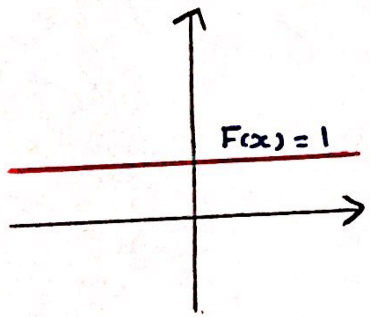
$= e^x$ is natural Exponential Function.

$F(x) = a^x$ For all $a \in \mathbb{R}^+$

$a = 1$

$F(x) = 1$

1) F(x) is constant function
2) F(x) is Polynomial of degree Zero
3)

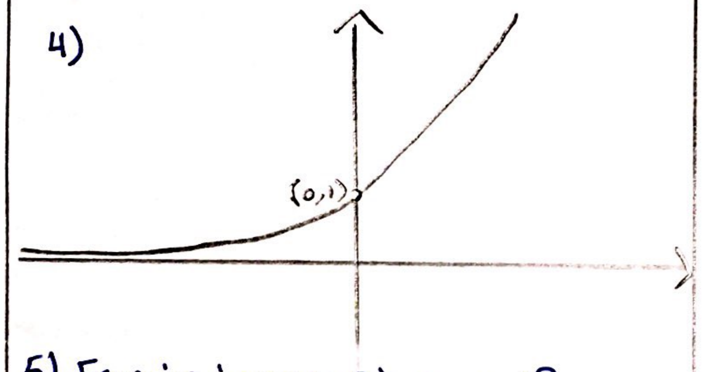


4) F(x) is Horizontal line with slope: 0 and y-intercept: 1
5) $D_{F(x)} = \mathbb{R}$; $R_{F(x)} = \{1\}$
6) F(x) is even function
7) F(x) is Symmetric about y-axis "$x = 0$"

$a > 1$

$F(x) = a^x$

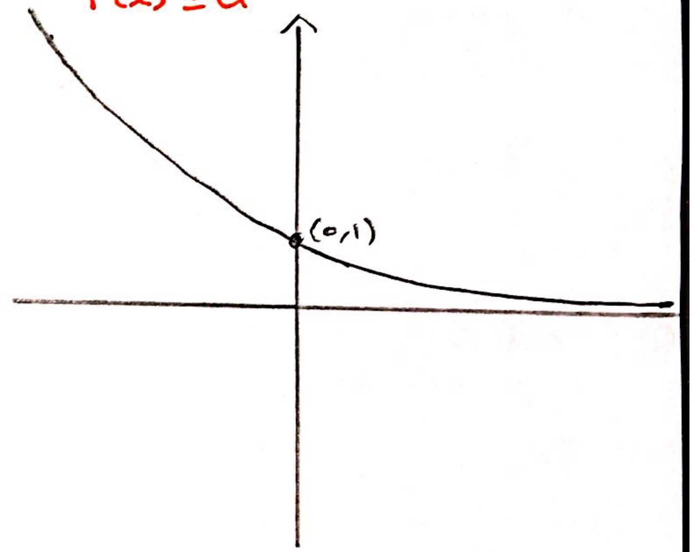
1) F(x) is Exponential Function
2) F(x) is not algabric
3) F(x) is Transedental Function
4)



5) F(x) is increasing on $\mathbb{R}$
6) F(x) is neither even nor odd
7) $D_{F(x)} = \mathbb{R}$; $R_{F(x)} = (0, \infty)$

$0 < a < 1$

$F(x) = a^x$



1) F(x) is decreasing on $\mathbb{R}$
2) F(x) is neither even nor odd
3) $D_{F(x)} = \mathbb{R}$; $R_{F(x)} = (0, \infty)$

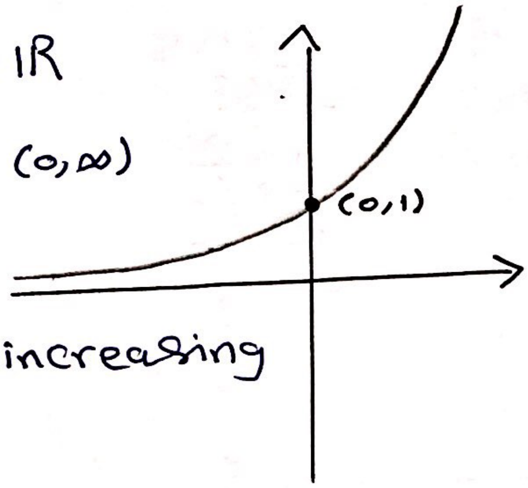
Example:

$F(x) = 2^x$ "$2>1$"

$F(x)$ is a Exponential function

$D_{F(x)} = \mathbb{R}$

$R_{F(x)} = (0,\infty)$



$F(x)$ is increasing on $\mathbb{R}$.

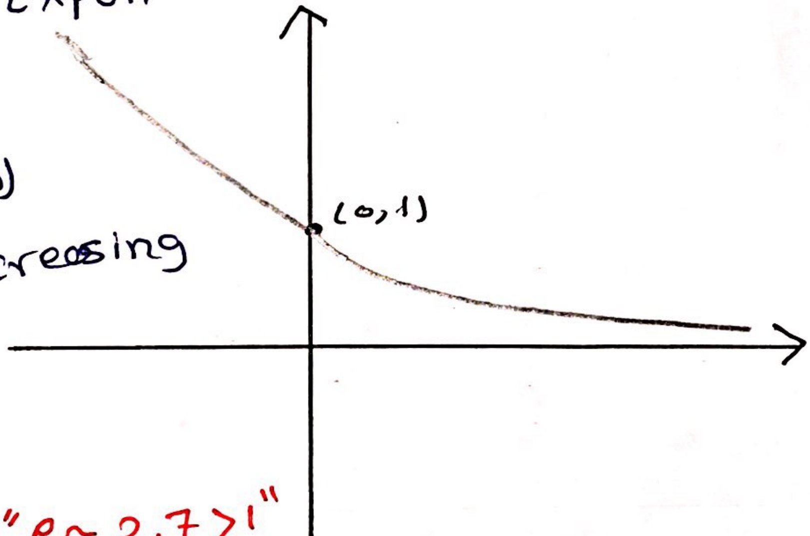
$F(x) = \left(\frac{4}{3}\right)^{-x} = \left(\frac{3}{4}\right)^{x}$ "$0<\frac{3}{4}<1$".

$F(x)$ is a Exponential function

$D_{F(x)} = \mathbb{R}$

$R_{F(x)} = (0,\infty)$

$F(x)$ is decreasing on $\mathbb{R}$

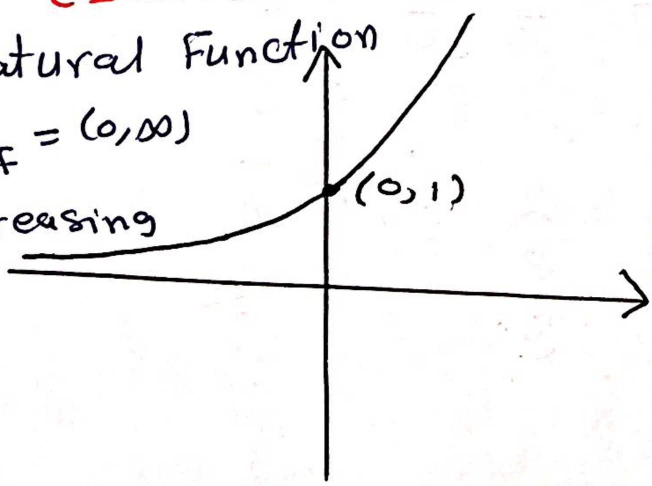


$F(x) = e^x$ "$e \simeq 2.7 > 1$"

$F(x)$ is a Natural Function

$D_F = \mathbb{R}$; $R_F = (0,\infty)$

$F(x)$ is increasing on $\mathbb{R}$



H.W :

1) The Domain of $f(x) = 3^x$ is .....

$\{3\}$ $\quad (0, \infty) \quad (3, \infty) \quad \mathbb{R}$

2) The Range of $f(x) = \left(\frac{5}{6}\right)^x$ is ...

$\{5/6\}$ $\quad (0, \infty) \quad \mathbb{R} \quad (-\infty, 5/6)$

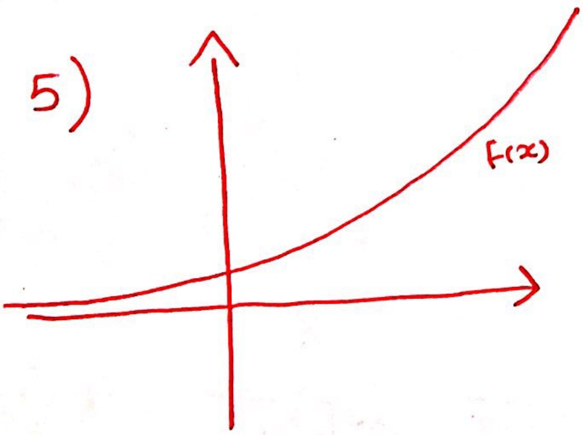
3) $f(x) = \left(\frac{\pi}{3}\right)^x$ is decreasing on $\mathbb{R}$

✓ $\quad$ X

4) $f(x) = e^{-x}$ is increasing on $\mathbb{R}$

✓ $\quad$ X

5)



a) $f(x) = \left(\frac{2}{3}\right)^x$

b) $f(x) = \frac{2}{3}$

c) $f(x) = x^{2/3}$

d) $f(x) = \left(\frac{3}{2}\right)^x$

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Laws of Exponents :-

1) $a^0 = 1$; $e^0 = 1$

2) $a^{-x} = \frac{1}{a^x} = \left(\frac{1}{a}\right)^x$; $e^{-x} = \frac{1}{e^x} = \left(\frac{1}{e}\right)^x$

3) $(a^x)^y = a^{xy}$; $(e^x)^y = e^{xy}$

4) $a^x . a^y = a^{x+y}$; $e^x . e^y = e^{x+y}$

5) $\frac{a^x}{a^y} = a^{x-y}$; $\frac{e^x}{e^y} = e^{x-y}$

6) $(ab)^x = a^x b^x$

7) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

8) $\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x$

9) $a^{p/q} = \sqrt[q]{a^p} = \left(\sqrt[q]{a}\right)^p$

10) $\frac{a^{-x}}{b^{-y}} = \frac{b^y}{a^x}$

11) $\sqrt[n]{ab} = \sqrt[n]{a} . \sqrt[n]{b}$

12) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[n \times m]{a}$

13) $\sqrt[n]{a/b} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

14) $a^n = \underbrace{a.a.a.....a}_{n \text{ time.}}$

15) $a^x > 0$

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Example:

Simplify of the following:

1) a) $\frac{4^{-3}}{2^{-8}} = \frac{2^8}{4^3} = \frac{2^8}{(2^2)^3} = \frac{2^8}{2^6} = 2^{8-6} = 2^2 = 4$

b) $\frac{1}{\sqrt[3]{x^4}} = \frac{1}{x^{4/3}} = x^{-4/3}$

2) a) $8^{4/3} = (2^3)^{4/3} = 2^{3(\frac{4}{3})} = 2^4 = 16$

b) $x(3x^2)^3 = x(3^3 \cdot (x^2)^3)$

$= x(27 \cdot x^6)$

$= 27\, x^1 \cdot x^6$

$= 27\, x^7$

3) a) $b^8(2b)^4 = b^8(2^4 b^4) = b^8(16\, b^4) = 16\, b^4 b^8$

$= 16\, b^{4+8} = 16\, b^{12}$

b) $\frac{(6y^3)^4}{2y^5} = \frac{6^4 (y^3)^4}{2y^5} = \frac{(2^4)(3^4)\, y^{12}}{(2)^1\, y^5} = (2^3)(3^4)\, y^{12-5}$

$= (8)(81)\, y^7 = 648\, y^7$

4) a) $$\frac{x^{2n} \cdot x^{2n-1}}{x^{n+2}} = \frac{x^{2n+2n-1}}{x^{n+2}}$$

$$= \frac{x^{4n-1}}{x^{n+2}}$$

$$= x^{(4n-1)-(n+2)}$$

$$= x^{(4n-1-n-2)}$$

$$= x^{4n-n-1-2}$$

$$= x^{3n-3} = x^{3(n-1)}$$

b) $$\frac{\sqrt{a.\sqrt{b}}}{\sqrt[3]{ab}} = \frac{\sqrt{a} \cdot \sqrt[2]{\sqrt[2]{b}}}{\sqrt[3]{a} \cdot \sqrt[3]{b}} = \frac{\sqrt{a} \cdot \sqrt[4]{b}}{\sqrt[3]{a} \cdot \sqrt[3]{b}}$$

$$= \frac{a^{1/2} \cdot b^{1/4}}{a^{1/3} \cdot b^{1/3}} = a^{\frac{1}{2}-\frac{1}{3}} \cdot b^{\frac{1}{4}-\frac{1}{3}}$$

$$= a^{\frac{1}{6}} \cdot b^{-\frac{1}{12}}$$

$$= \frac{a^{1/6}}{b^{1/12}} = \frac{\sqrt[6]{a}}{\sqrt[12]{b}}$$

c) $$2^x \cdot 8^{2x} = 2^x \cdot (2^3)^{2x} = 2^x \cdot 2^{6x} = 2^{x+6x}$$

$$= 2^{7x} = (2^7)^x$$

3] Logarithmic Functions

The Natural logarithmic Functions	The Genral logarithmic Functions
$f(x) = \log_e x = Ln(x)$	$f(x) = \log_a x$ for all $a \in \mathbb{R}^+$
$f(x) = Ln(x)$ is the inverse function of the $f^{-1}(x) = e^x$	$f(x) = \log_a x$ is the inverse function of the $f^{-1}(x) = a^x$
Domain of $f(x) = Ln(x)$ is $(0, \infty)$	Domain of $f(x) = \log_a x$ is $(0, \infty)$
Range of $f(x) = Ln(x)$ is $\mathbb{R}$	Range of $f(x) = \log_a x$ is $\mathbb{R}$

Example:

✓ $f(x) = \log_2 x$ is logarithmic function

✓ $D_{F(x)} = (0, \infty)$; $R_{F(x)} = \mathbb{R}$

✓ $f(x) = \log_2 x$ is inverse of the function

$f^{-1}(x) = 2^x$

Example

The inverse function of $\left(\frac{3}{2}\right)^x$ is .....

a) $x^{3/2}$ b) $\operatorname{Ln}\left(\frac{3x}{2}\right)$ c) $\left(\frac{2}{3}\right)^x$

$\boxed{\text{d) } \log_{\left(\frac{3}{2}\right)} x}$

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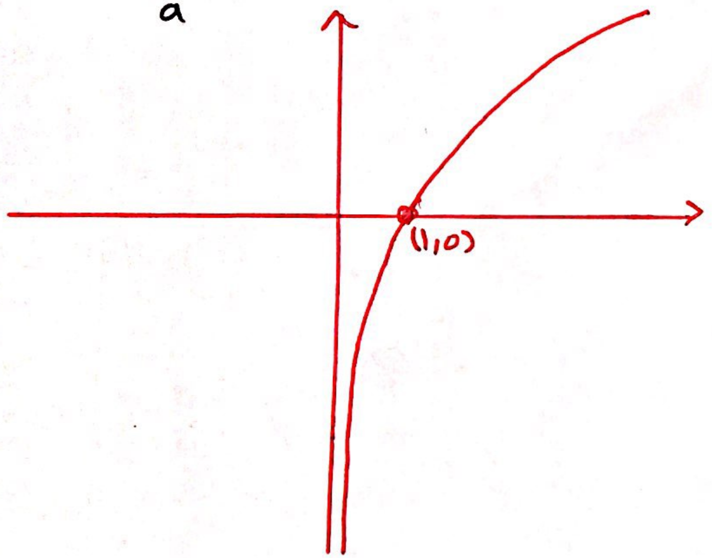
$F(x) = \log_a x \quad \forall a \in \mathbb{R}^+$

$f(x) = \log_a x \quad \forall a > 1$

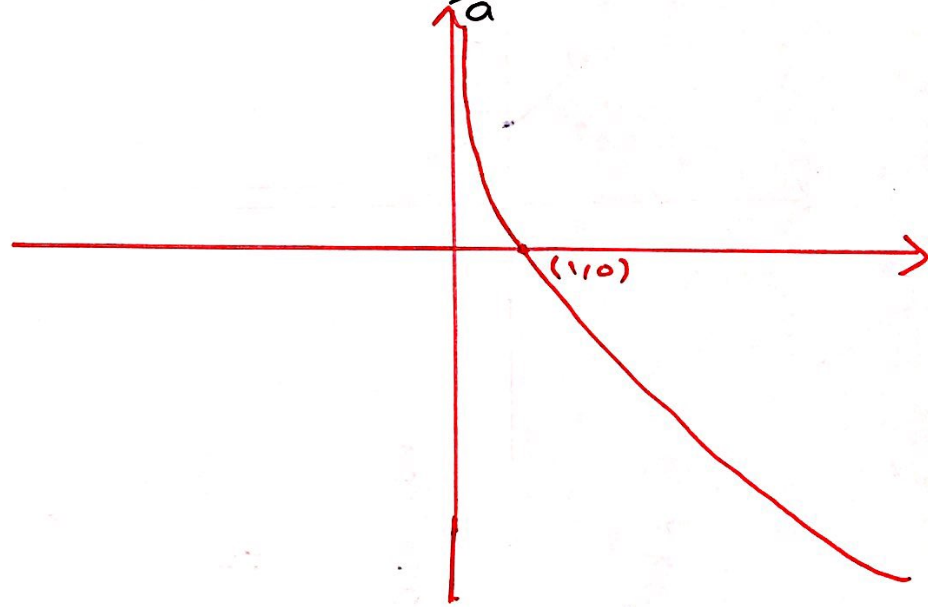


✓ $f(x)$ is increasing function $\forall x \in (0, \infty)$

✓ $f(x)$ is neither even nor odd

✓ $x = 0$ is asymptote of $f(x)$

$f(x) = \log_a x \quad \forall 0 < a < 1$



✓ $f(x)$ is decreasing function $\forall x \in (0, \infty)$

✓ $f(x)$ is neither even nor odd

✓ $x = 0$ is asymptote of $f(x)$.

Example

$f(x) = Ln(x)$

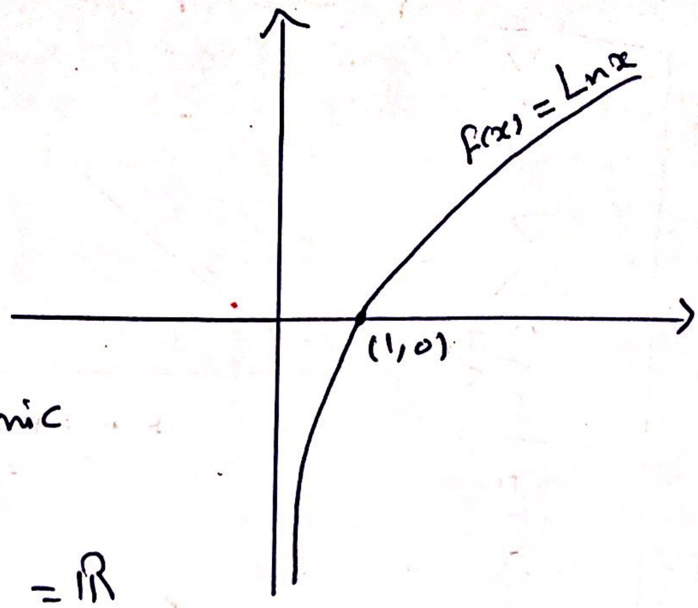
✓ $f(x) = Ln(x) = \log_e x$

$e \simeq 2.7 > 1$

✓ $f(x)$ is the logarithmic function

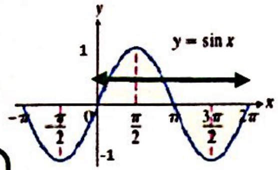
✓ $D_{f(x)} = (0,\infty)$; $R_{f(x)} = \mathbb{R}$

✓ $f(x)$ is increasing on $(0,\infty)$



Trigonometric Functions

$y = \sin x$



Domain: $\mathbb{R}$

Range: $[-1, 1]$

Period: 2π

$\sin(\theta + 2\pi) = \sin\theta$

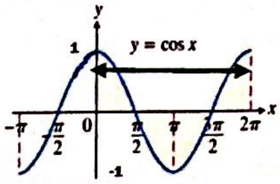
Odd function:

$\sin(-\theta) = -\sin\theta$

$$-1 \le \sin x \le 1 \Leftrightarrow |\sin x| \le 1$$

$$\sin x = 0 \quad when \quad x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots.$$

$y = \cos x$



Domain: $\mathbb{R}$

Range: $[-1, 1]$

Period: 2π

$\cos(\theta + 2\pi) = \cos\theta$

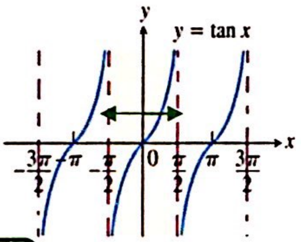
Even function:

$\cos(-\theta) = \cos\theta$

$$-1 \le \cos x \le 1 \Leftrightarrow |\cos x| \le 1$$

$$\cos x = 0 \quad when \quad x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots.$$

$y = \tan x$



Domain:

$\mathbb{R} - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots\right\}$

Range: $\mathbb{R}$

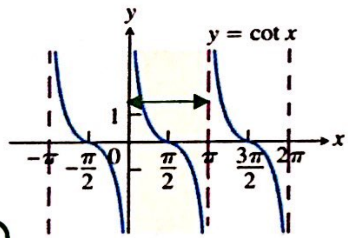
Period: π

$\tan(\theta + \pi) = \tan\theta$

Odd function:

$\tan(-\theta) = -\tan\theta$

$y = \cot x$



Domain:

$\mathbb{R} - \{0, \pm\pi, \pm 2\pi, \ldots\}$

Range: $\mathbb{R}$

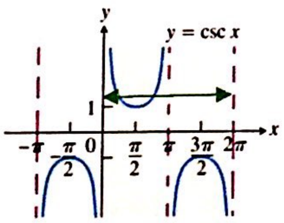
Period: π

$\cot(\theta + \pi) = \cot\theta$

Odd function:

$\cot(-\theta) = -\cot\theta$

$y = \csc x$



Domain:

$\mathbb{R} - \{0, \pm\pi, \pm 2\pi, \ldots\}$

Range:

$\mathbb{R} - (-1, 1)$

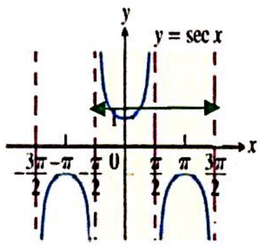
Period: 2π

$\csc(\theta + 2\pi) = \csc\theta$

Odd function:

$\csc(-\theta) = -\csc\theta$

$y = \sec x$



Domain:

$\mathbb{R} - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots.\right\}$

Range:

$\mathbb{R} - (-1, 1)$

Period: 2π

$\sec(\theta + 2\pi) = \sec\theta$

Even function:

$\sec(-\theta) = \sec\theta$

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Example

Classify the following as one types of functions

a) $5^x = f(x)$ exponential function

b) $g(x) = \frac{x^5}{1}$ Power function or Polynomial of degree 5 or rational function

c) $h(x) = \frac{1+x}{1-\sqrt{x}}$ algabric function

d) $u(t) = \frac{1-t+5t^4}{1}$ Polynomial of degree 4 or rational function

e) $y = \pi^x$
exponential function

f) $y = x^{\pi}$
power function

g) $y = x^2(2-x^3)$
$y = 2x^2 - x^5$
Polynomial of degree 5

h) $y = \tan t - \cos t$
trigonometric

I) $y = \frac{s}{1+s}$
rational function

J) $y = \frac{\sqrt{x^3-1}}{1+\sqrt[3]{x}}$ algabric function

k) $\log_2 x = f(x)$
logarithmic function

L) $g(x) = \sqrt[4]{x} = x^{1/4}$
Root function or Power

m) $h(x) = \frac{2x^3}{1-x^2}$
rational function

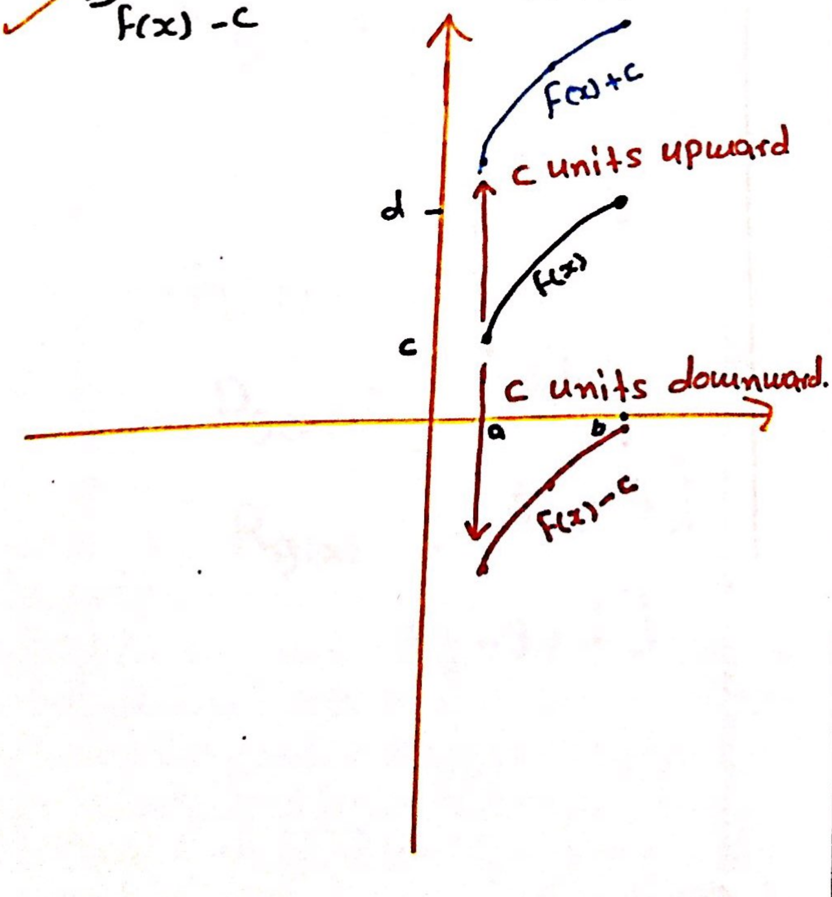
n) $u(t) = 1 - 1.1t + 2.54t^2$
Polynomial of degree 2

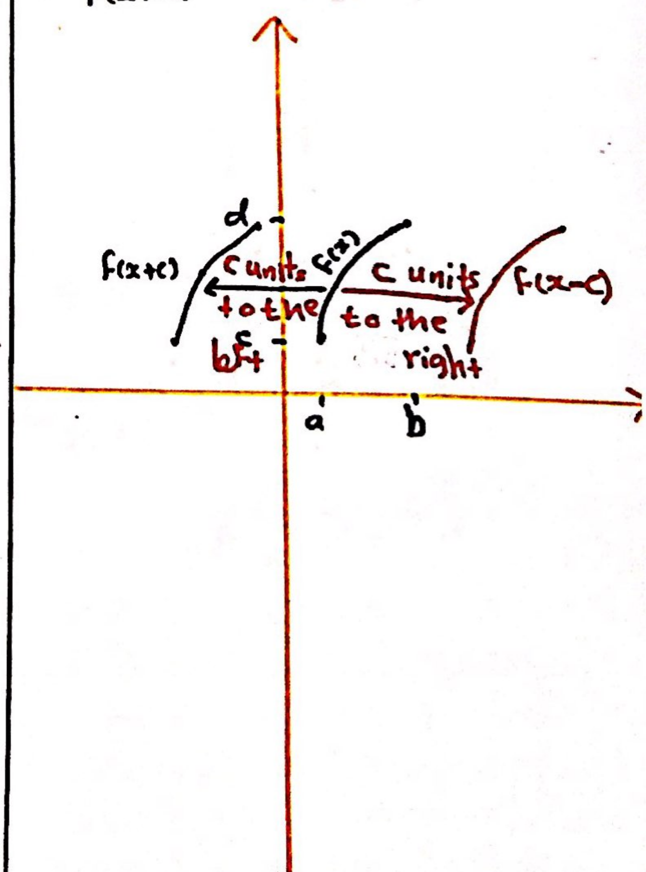
o) $w(\theta) = \sin\theta \cos^2\theta$
trigonometric

1.3 - New Functions From Old Functions

Suppose $f(x)$ is odd Function with $D_{f(x)} = [a,b]$; $R_{f(x)} = [e,d]$ and $c > 0$

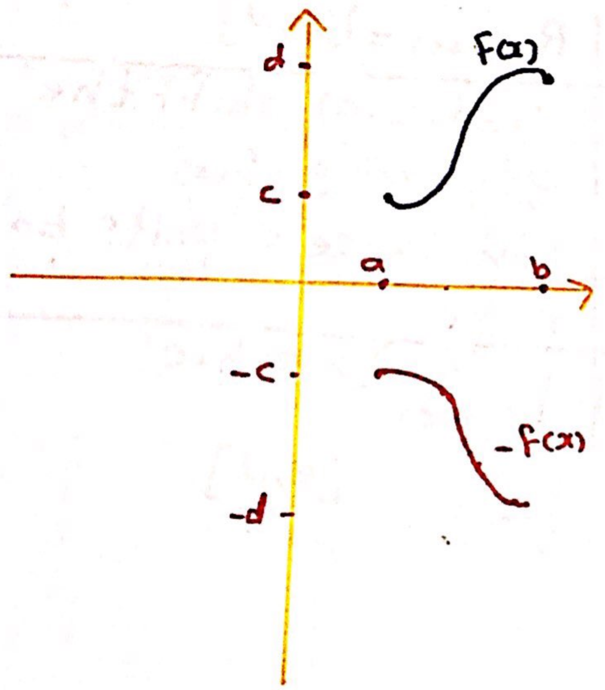
Vertical Shift	Horizontal Shift
$y = f(x) + c$, shif the graph of $y = f(x)$ a distance c units upward $D_{f(x)+c} = [a,b]$; $R_{f(x)+c} = [e+c, d+c]$	$y = f(x+c)$, Shift the graph of $y = f(x)$ a distance c units to left $D_{f(x+c)} = [a-c, b-c]$ $R_{f(x+c)} = [e,d]$
$y = f(x) - c$; Shift the graph of $y = f(x)$ adistance c units downward. $D_{f(x)-c} = [a,b]$; $R_{f(x)+c} = [e-c, d-c]$	$y = f(x-c)$, Shift the graph of $y = f(x)$ adistance c units to right $D_{f(x-c)} = [a+c, b+c]$ $R_{f(x\mp c)} = [e,d]$





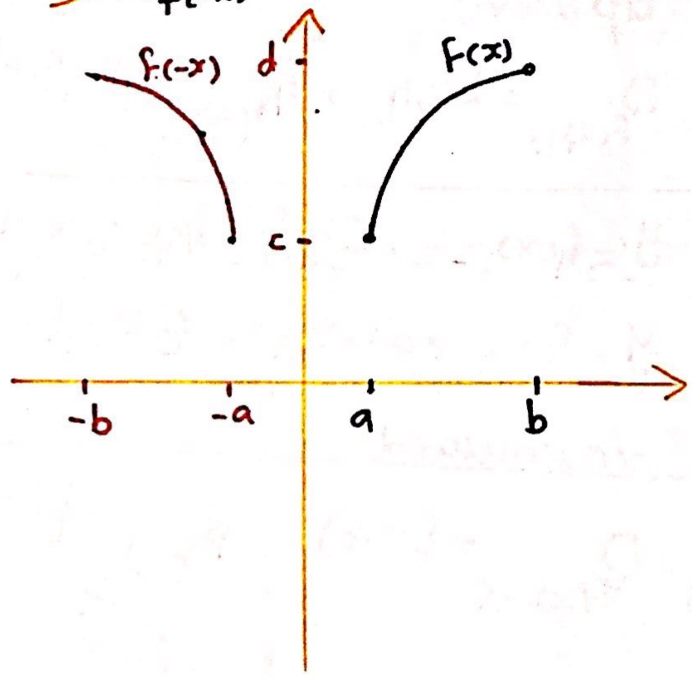
Vertical Reflecting

- $y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis
- $D_{-f(x)} = [a, b]$
- $R_{-f(x)} = [-d, -e]$



Horizontal Reflecting

- $y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis
- $D_{f(-x)} = [-b, -a]$
- $R_{f(-x)} = [e, d]$



Example (1)

If $f(x)$ is a function with Domain: $[-6,5]$ and Range: $[-1,3]$

then find the Domain and Range of the following.

a) $g(x) = f(x)+2$

$D_{g(x)} = [-6,5]$

$R_{g(x)} = [-1+2, 3+2]$

$= [1, 5]$

b) $g(x) = f(x)-3$

$D_{g(x)} = [-6,5]$

$R_{g(x)} = [-1-3, 3-3]$

$= [-4, 0]$

c) $g(x) = -f(x)$

$D_{g(x)} = [-6,5]$

$R_{g(x)} = [-(-1), -3]$

$= [-3, 1]$

d) $g(x) = f(x+2)$

$R_{g(x)} = [-1,3]$

$D_{g(x)} = [-6-2, 5-2]$

$= [-8, 3]$

e) $g(x) = f(x-3)$

$R_{g(x)} = [-1,3]$

$D_{g(x)} = [-6+3, 5+3]$

$= [-3, 8]$

f) $g(x) = f(-x)$

$R_{g(x)} = [-1,3]$

$D_{g(x)} = [-(-6), -5]$

$= [-5, 6]$

h) $g(x) = 3 - f(x)$

$D_{g(x)} = [-6, 5]$

$R_{g(x)}$: i) $R_{-f(x)} = [-(-1), -3]$

$= [-3, 1]$

ii) $R_{3-f(x)} = [-3+3, 1+3]$

$= [0, 4]$

$\Rightarrow R_{g(x)} = [0,4]$ #

I) $g(x) = f(x-1) - 6$

$D_{g(x)} = [-6+1, 5+1] = [-5, 6]$

$R_{g(x)} = [-1-6, 3-6] = [-7, -3]$

<u>H.W</u>

if $D_{f(x)} = [0,6)$ and $R_{f(x)} = [-4, 2)$ then

find ① $D_{1-f(x)}$ and $R_{1-f(x)}$

② $D_{f(x+2)+1}$ and $R_{f(x+2)+1}$

③ $D_{f(-x)-2}$ and $R_{f(-x)-2}$

④ $D_{f(x)+4}$ and $R_{f(x)+4}$

⑤ $R_{f(x+4)}$ and $D_{f(x+4)}$

① $D_{1-f(x)} = [0,6)$

$R_{1-f(x)}$: i) $R_{-f(x)} = [-(-4), -2)$

$= (-2, 4]$

ii) $R_{1-f(x)} = (-2+1, 4+1]$

$= (-1, 5]$

② $D_{f(x+2)+1} = [0-2, 6-2)$

$= [-2, 4)$

$R_{f(x+2)+1} = [-4+1, 2+1)$

$= [-3, 3)$

③ $D_{f(-x)-2} = [-0, -6)$

$= (-6, 0]$

$R_{f(-x)-2} = [-4-2, 2-2)$

$= [-6, 0)$

④ $D_{f(x)+4} = [0, 6)$

$R_{f(x)+4} = [-4+4, 2+4)$

$= [0, 6)$

⑤ $D_{f(x+4)} = [0-4, 6-4)$

$= [-4, 2)$

$R_{f(x+4)} = [-4, 2)$

Example (1)

Find the Domain and Range of the following:

1 - $y = \sqrt{x} + 2$

The old function : $f(x) = \sqrt{x}$

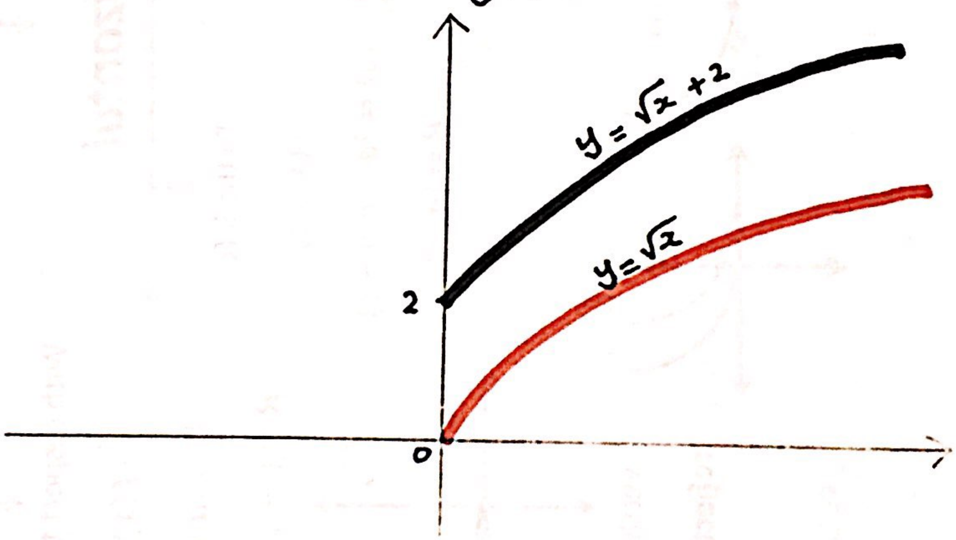
$D_{f(x)} = [0, \infty)$

$R_{f(x)} = [0, \infty)$

The New function : $y = \sqrt{x} + 2$

$D_{y(x)} = [0, \infty)$

$R_{y(x)} = [0+2, \infty+2) = [2, \infty)$



Type of the Shift : Vertical Shift

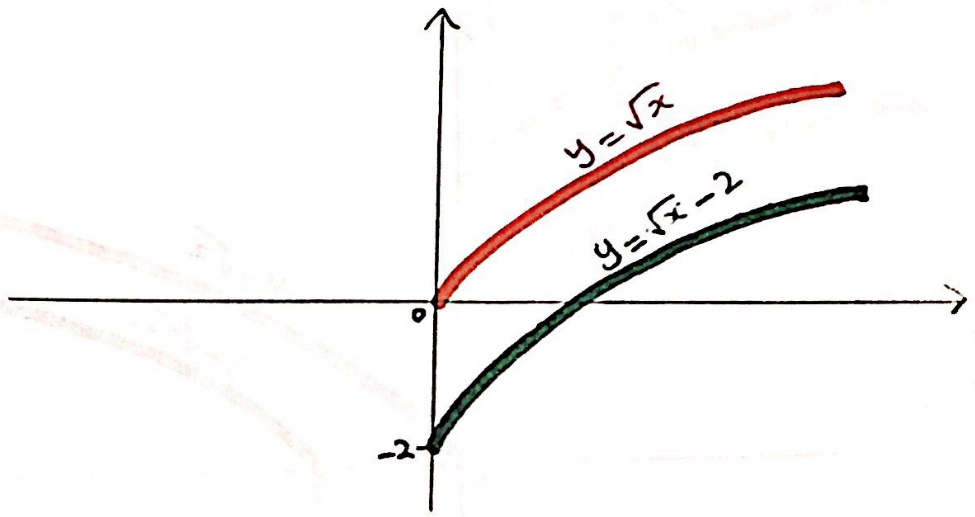
$y = \sqrt{x} + 2$, Shift the graph of $y = \sqrt{x}$ a distance 2 units upward.

2- $y=\sqrt{x}-2$

The old function: $f(x)=\sqrt{x}$ → $D_{f(x)}=[0,\infty)$, $R_{f(x)}=[0,\infty)$

The new function: $y=\sqrt{x}-2$ → $D_{f(x)}=[0,\infty)$, $R_{f(x)}=[0-2,\infty-2)$

$=[-2,\infty)$



Type of the Shift: Vertical shift.

$y=\sqrt{x}-2$, shift the graph of $y=\sqrt{x}$ a distance 2 units downward.

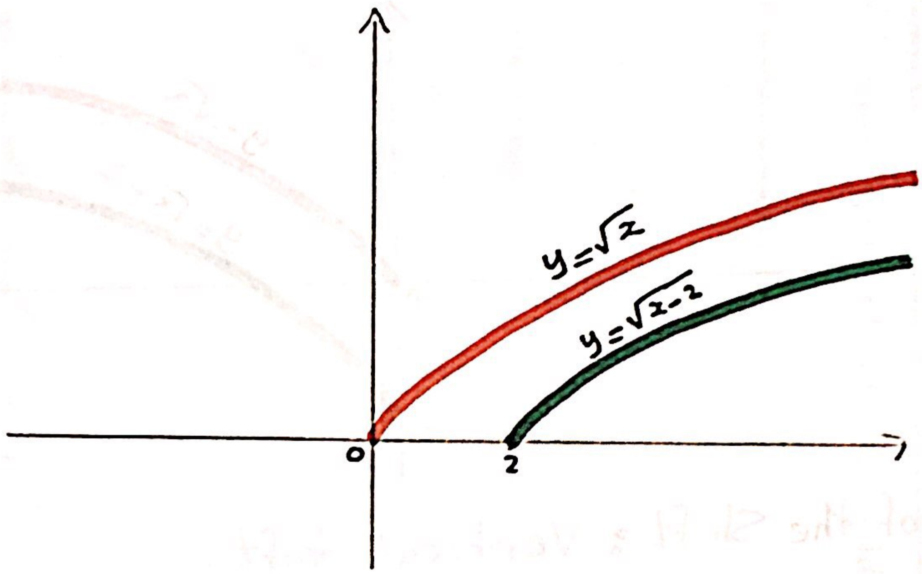
3) $y = \sqrt{x-2}$

The new function: $f(x) = \sqrt{x}$ $\nearrow$ $D_{f(x)} = [0, \infty)$

$\searrow$ $R_{f(x)} = [0, \infty)$

The new function: $y = \sqrt{x-2}$ $\nearrow$ $D_{y(x)} = [0+2, \infty+2) = [2, \infty)$

$\searrow$ $R_{y(x)} = [0, \infty)$



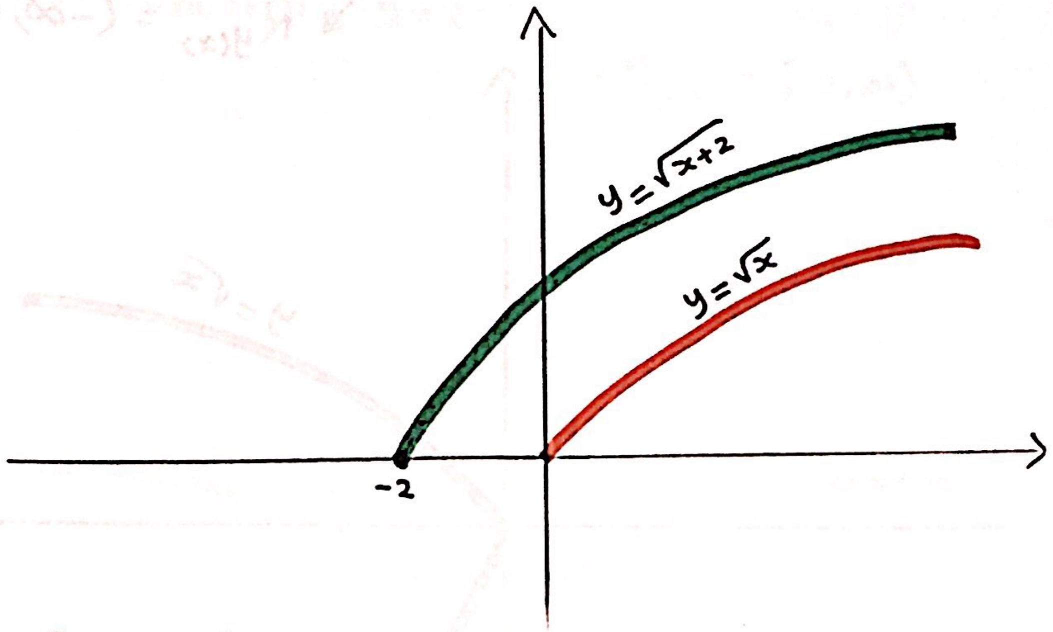
Type of the Shift: Horizontal Shift

$y = \sqrt{x-2}$, Shift the graph of $y = \sqrt{x}$ a distance 2 units to the right

4) $y = \sqrt{x+2}$

The old function : $f(x) = \sqrt{x}$ $\nearrow D_{f(x)} = [0, \infty)$ $\searrow R_{f(x)} = [0, \infty)$

The new function : $y = \sqrt{x+2}$ $\nearrow D_{y(x)} = [0-2, \infty-2) = [-2, \infty)$ $\searrow R_{y(x)} = [0, \infty)$



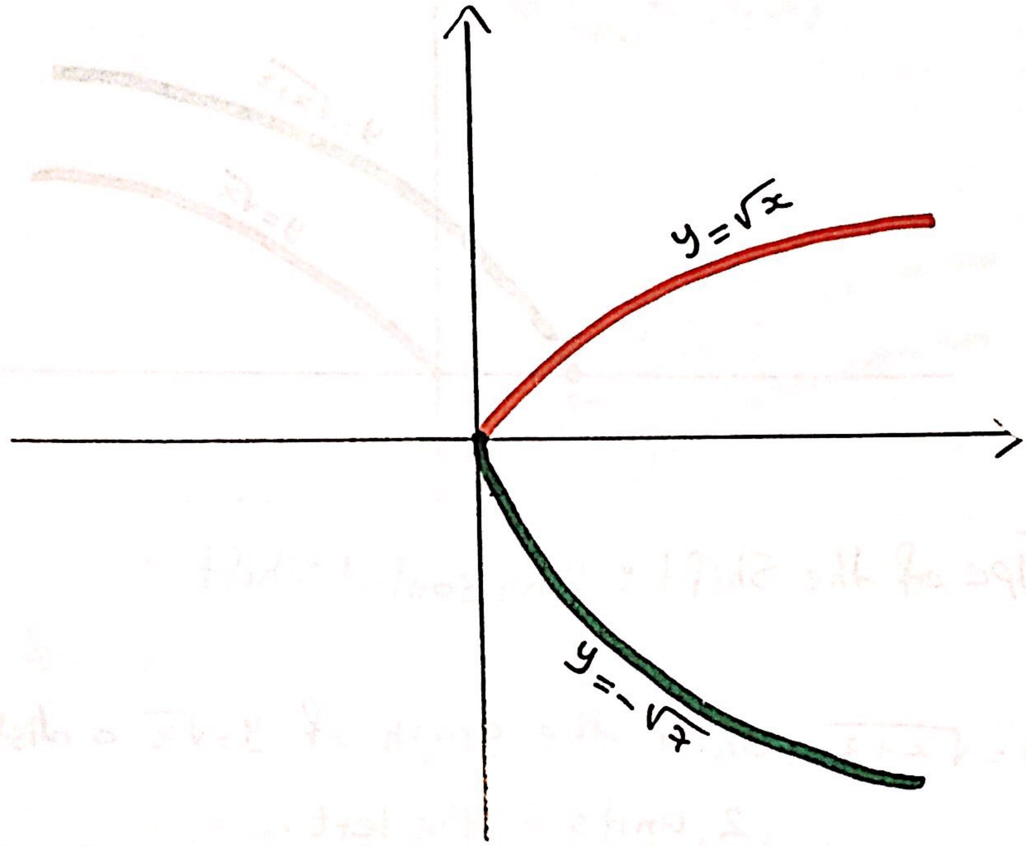
Type of the Shift : Horizontal Shift

$y = \sqrt{x+2}$, shift the graph of $y = \sqrt{x}$ a distance 2 units to the left.

5) $y = -\sqrt{x}$

The old function: $f(x) = \sqrt{x}$ $\nearrow D_{f(x)} = [0, \infty)$ $\searrow R_{f(x)} = [0, \infty)$

The new function: $y = -\sqrt{x}$ $\nearrow D_{y(x)} = [0, \infty)$ $\searrow R_{y(x)} = (-\infty, 0]$



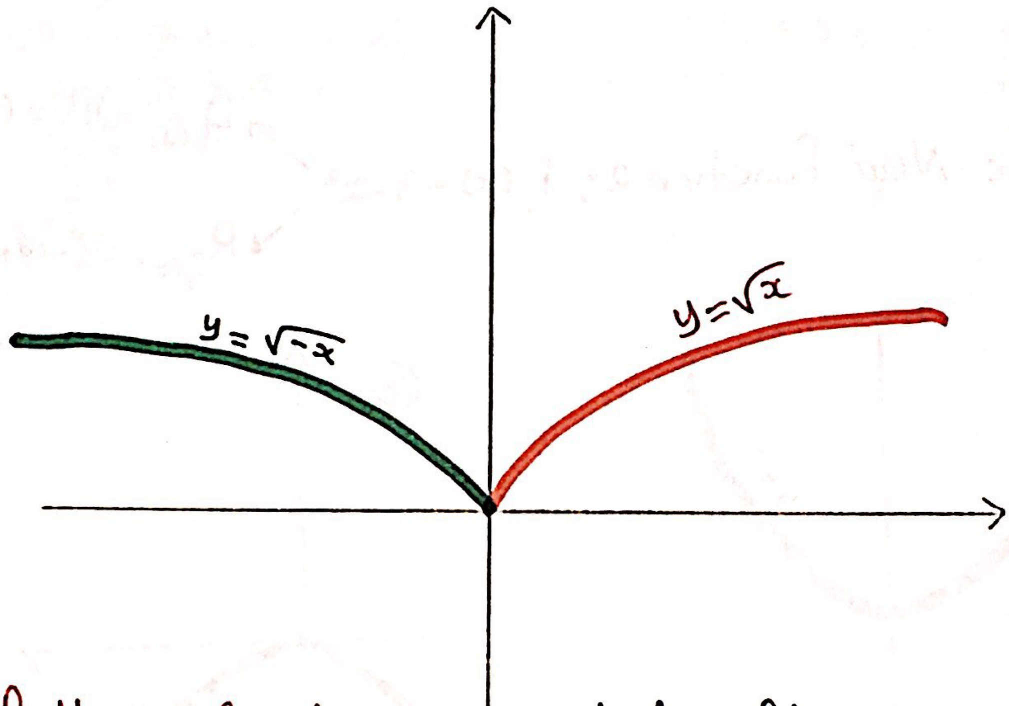
Type of the reflect: Vertical reflecting

$y = -\sqrt{x}$, reflect the graph of $y = \sqrt{x}$ about x_axis "$y = 0$"

6) $y = \sqrt{-x}$

The old function: $f(x) = \sqrt{x}$ → $D_{f(x)} = [0, \infty)$, $R_{f(x)} = [0, \infty)$

The new function: $y = \sqrt{-x}$ → $D_{y(x)} = (-\infty, 0]$, $R_{y(x)} = [0, \infty)$

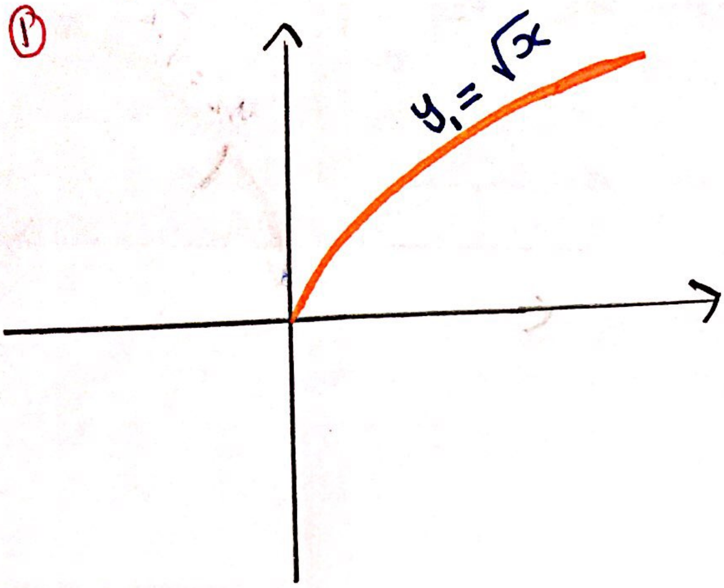


Type of the reflect: Horizontal reflecting.

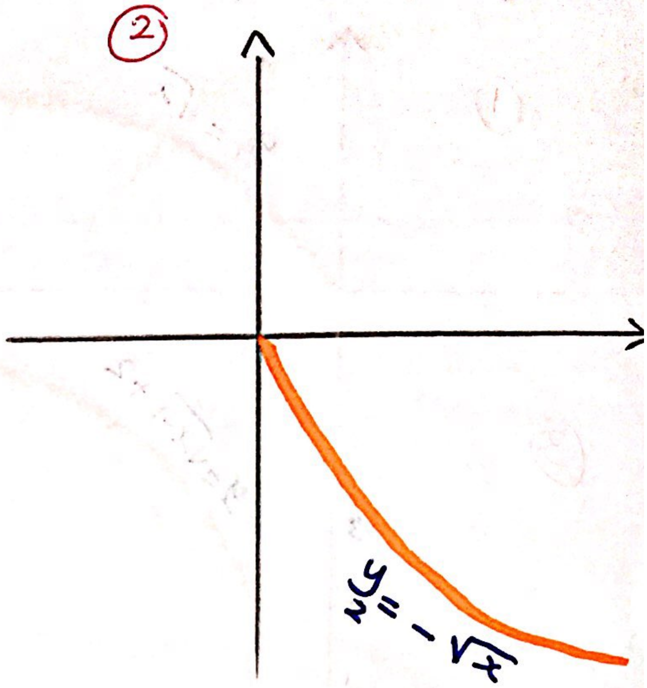
$y = \sqrt{-x}$, reflect the graph of $y = \sqrt{x}$ about y-axis "$x = 0$"

7) $y = 2 - \sqrt{x}$

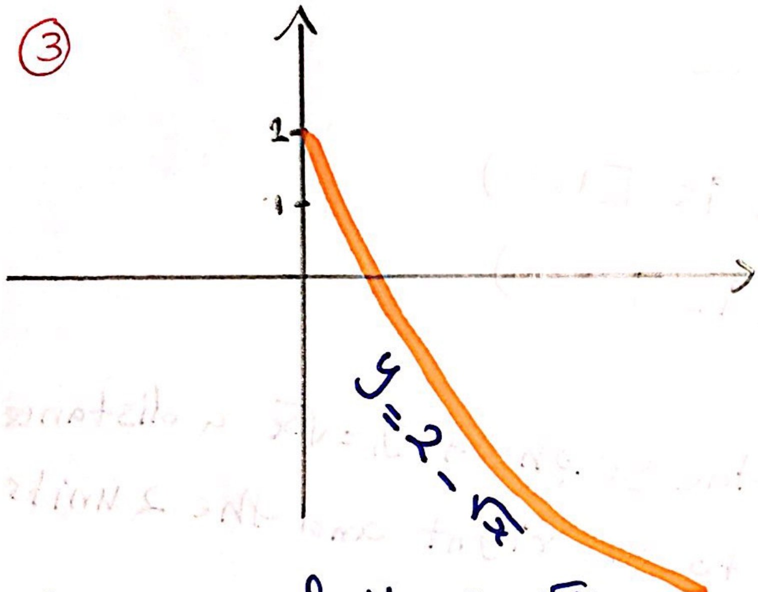
①



②



③



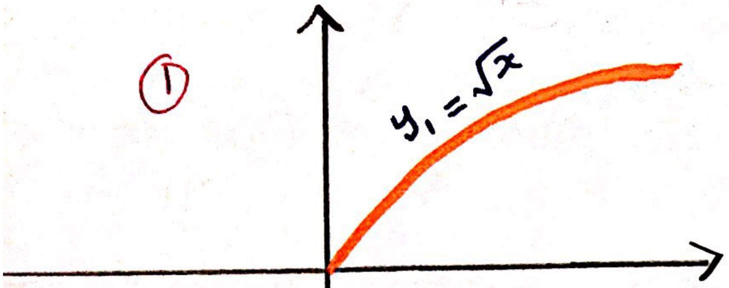
Domain of $y = 2 - \sqrt{x}$ is $[0, \infty)$

Range of $y = 2 - \sqrt{x}$ is $(-\infty, 2]$

$y = 2 - \sqrt{x}$, reflect the graph of $f_1(x) = \sqrt{x}$ about x-axis and Shift the graph of $f_2(x) = -\sqrt{x}$ a distance 2 units upward.

8) $y = \sqrt{x-1} + 2$

①



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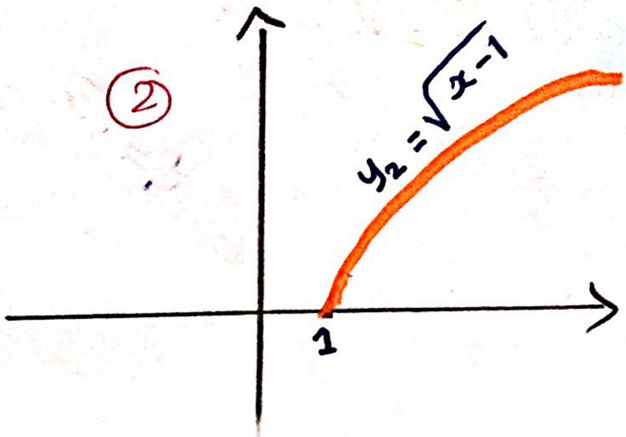
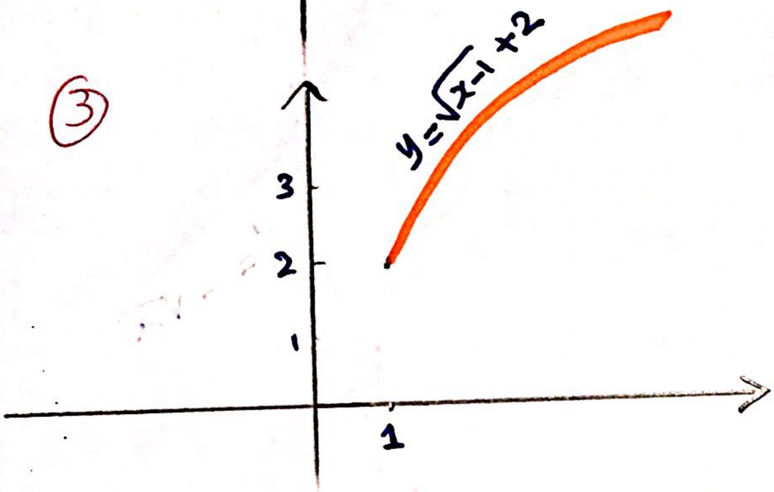


③

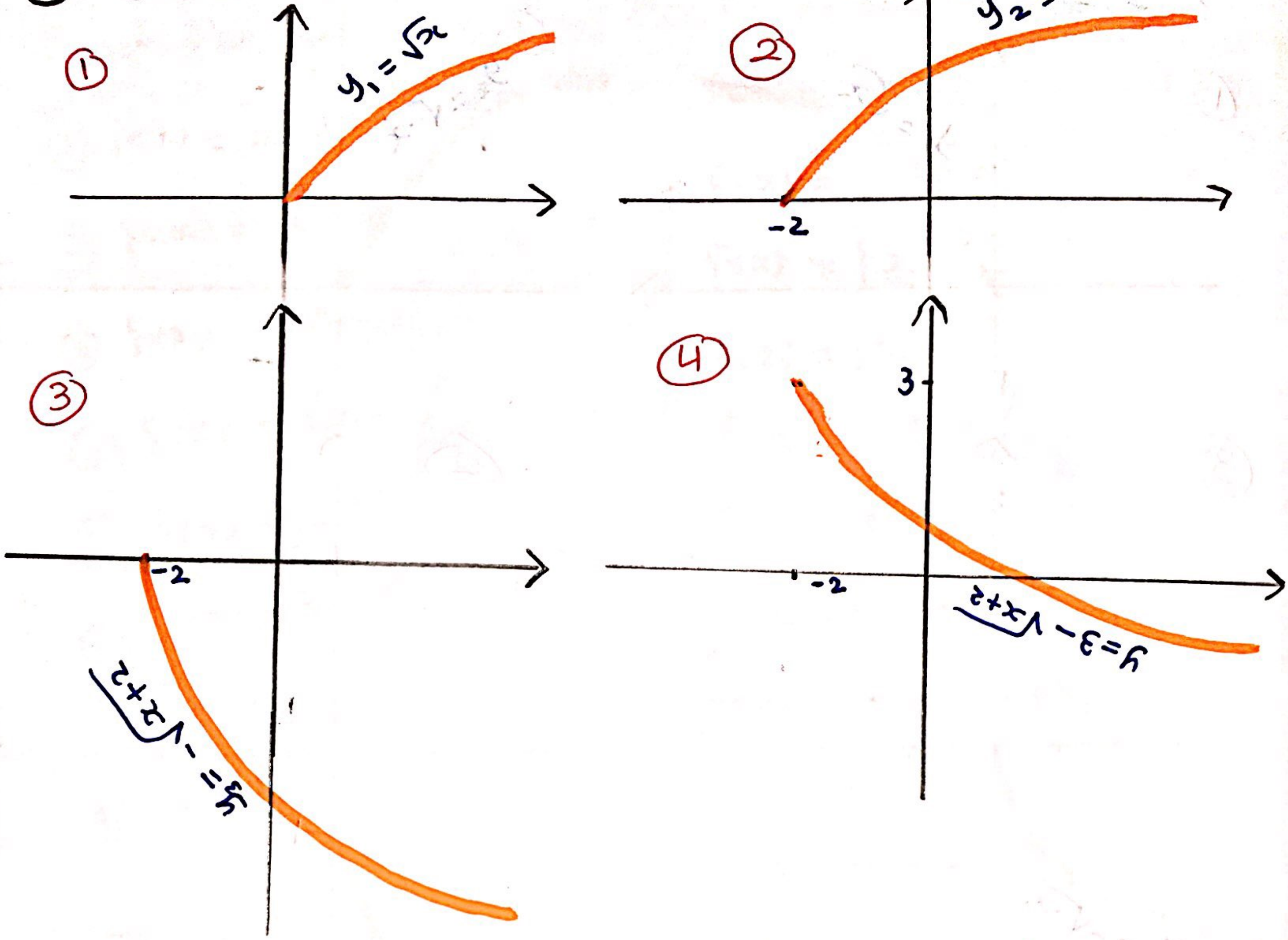


Domain of $y = \sqrt{x-1} + 2$ is $[1, \infty)$

Range of $y = \sqrt{x-1} + 2$ is $[2, \infty)$

$y = \sqrt{x-1} + 2$, Shift the graph of $y_1 = \sqrt{x}$ a distance 1 unit to the right and the 2 units upward.

⑨ $y = 3 - \sqrt{x+2}$



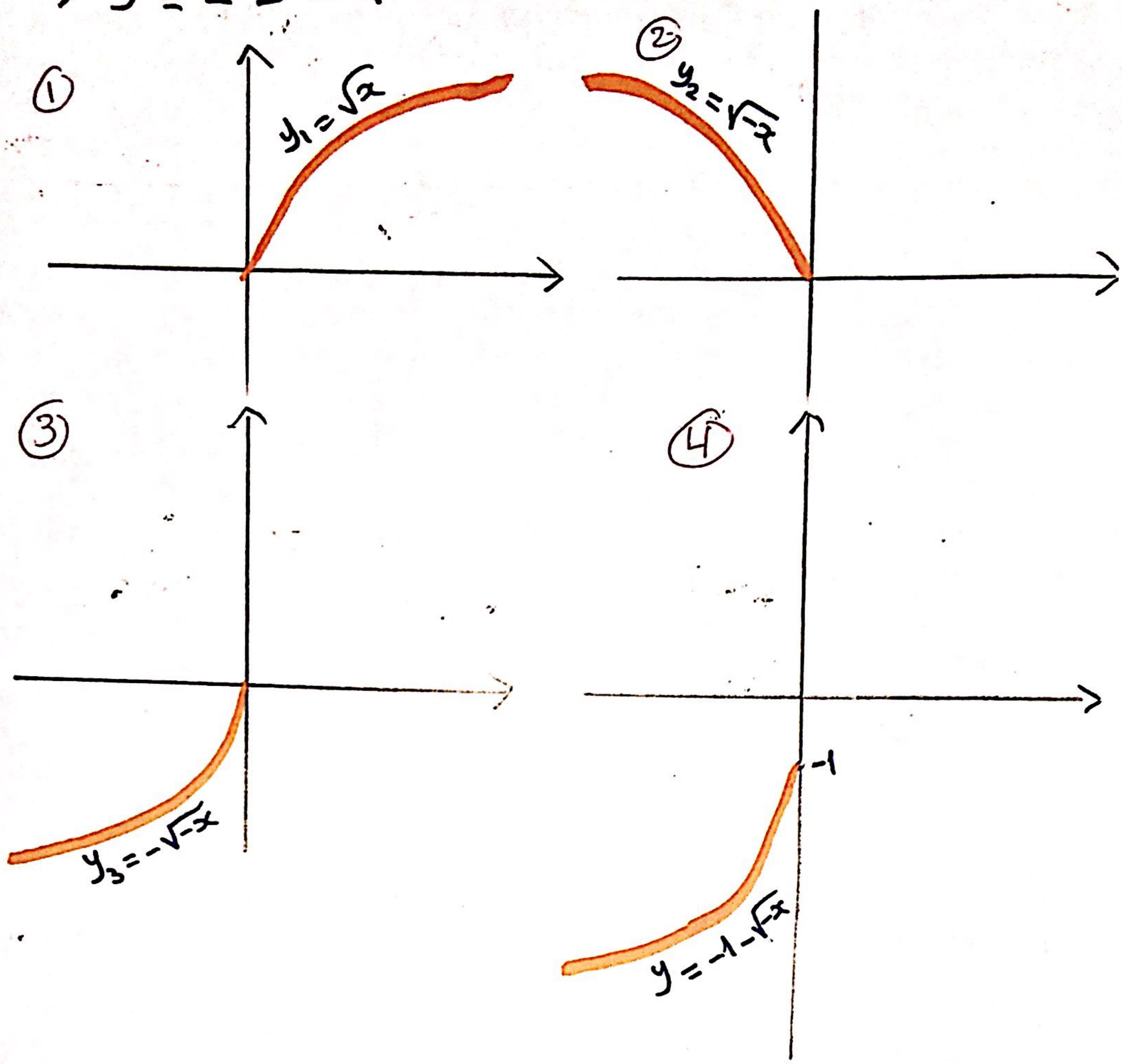
Domain of $y = 3 - \sqrt{x+2} = [-2, \infty)$

Range of $y = 3 - \sqrt{x+2} = (-\infty, 3]$

$y = 3 - \sqrt{x+2}$, Shift the graph $y_1 = \sqrt{x}$ a distance 2 units to the left and reflect the graph $y_2 = \sqrt{x+2}$ about x-axis and Shift the graph $y = -\sqrt{x+2}$ a distance 3 units upward.

Scanned by CamScanner

10) $y = -1 - \sqrt{-x}$



Domain of $y = 1 - \sqrt{-x}$ is $(-\infty, 0]$

Range of $y = -1 - \sqrt{-x}$ is $(-\infty, -1]$

$y = -1 - \sqrt{-x}$, reflect the graph of $y_1 = \sqrt{x}$ about y-axis and x-axis and Shift the graph $y_3 = -\sqrt{-x}$ a distance 1 unit downword.

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Example

Sketch and find the Domain and Range of

$f(x) = x^2 + 6x + 10$

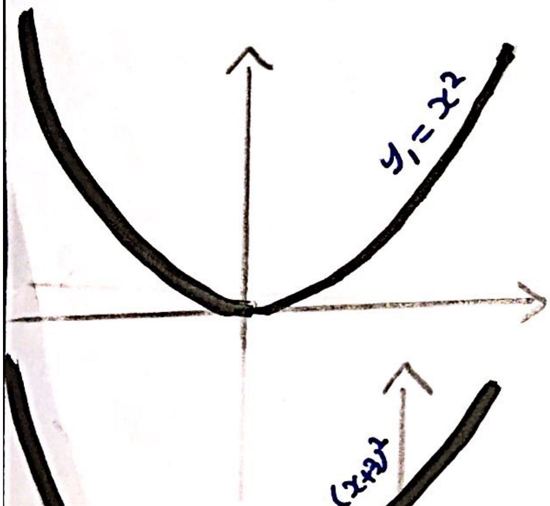
$f(x) = 5 - 4x - x^2$

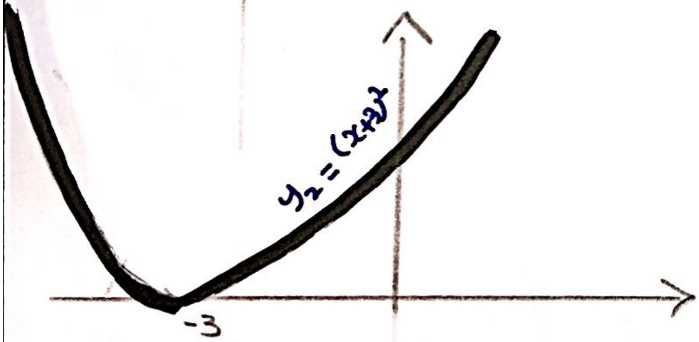
$f(x) = x^2 + 6x + 10$

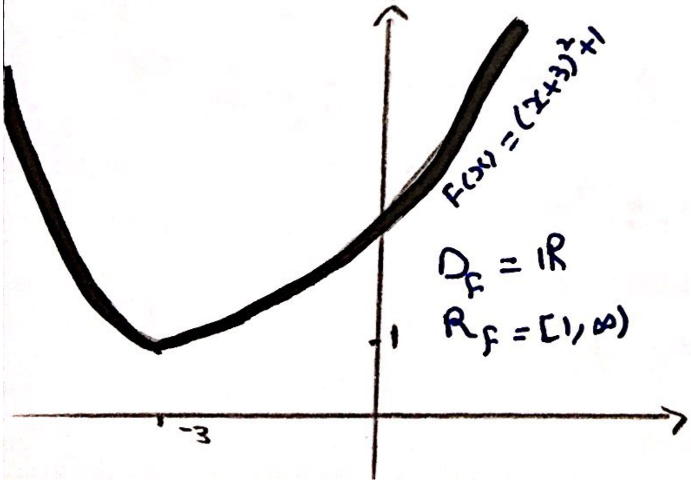
$f(x) = x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 10$

$= x^2 + 6x + 9 - 9 + 10$

$= (x+3)^2 + 1$







$D_f = \mathbb{R}$

$R_f = [1, \infty)$

$f(x) = 5 - 4x - x^2$

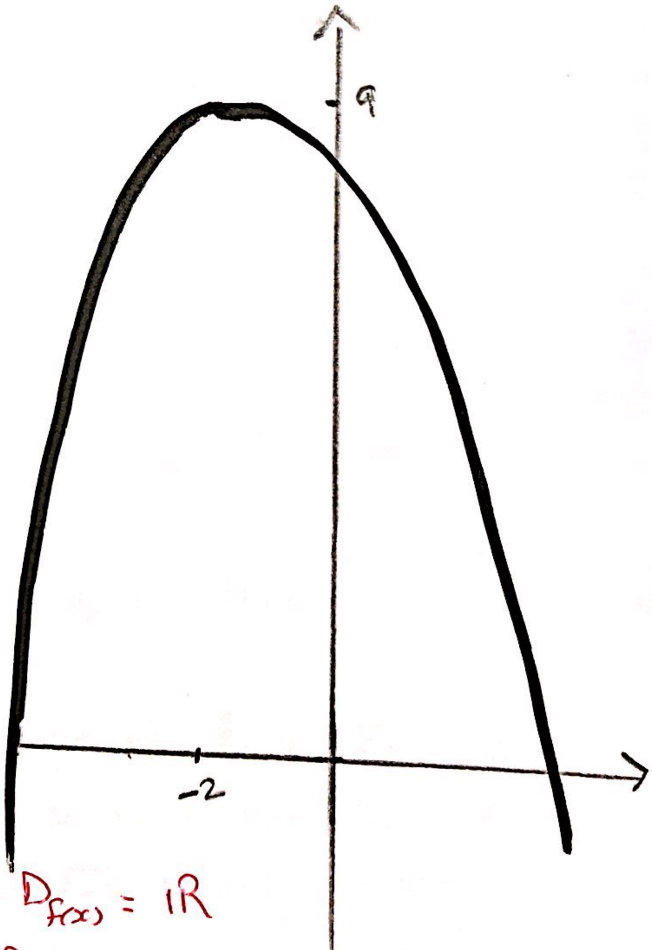
$f(x) = -\left[x^2 + 4x - 5\right]$

$= -\left[x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 5\right]$

$= -\left[x^2 + 4x + 4 - 4 - 5\right]$

$= -\left[(x+2)^2 - 9\right]$

$= 9 - (x+2)^2$



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = (-\infty, 9]$

H.W

Find the Domain and Range of the Following

① $f(x) = x^2 + 2$

② $f(x) = x^2 - 2$

③ $f(x) = (x-3)^2$

④ $f(x) = (x+3)^2$

⑤ $f(x) = -x^2$

⑥ $f(x) = 3 - x^2$

⑦ $f(x) = (x-1)^2 - 3$

⑧ $f(x) = 4 - (x+1)^2$

$f(x) = |x| + 3$

$f(x) = |x| - 3$

$f(x) = |x - 2|$

$f(x) = |x + 2|$

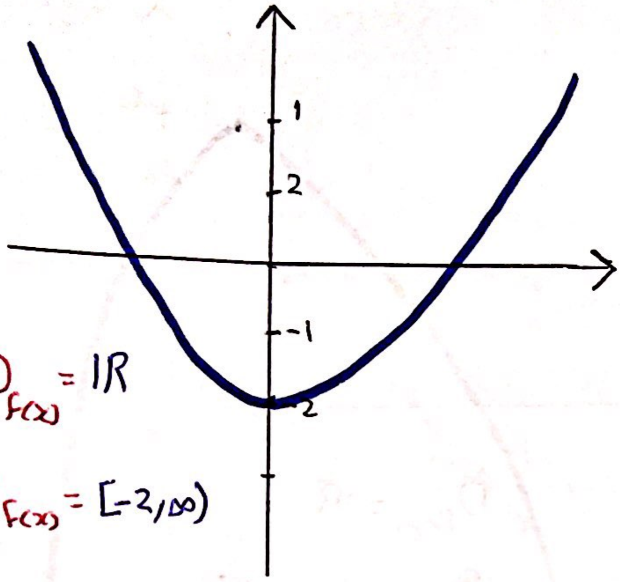
$f(x) = -|x|$

$f(x) = 5 - |x|$

$f(x) = |x - 3| + 1$

$f(x) = 2 - |x + 4|$

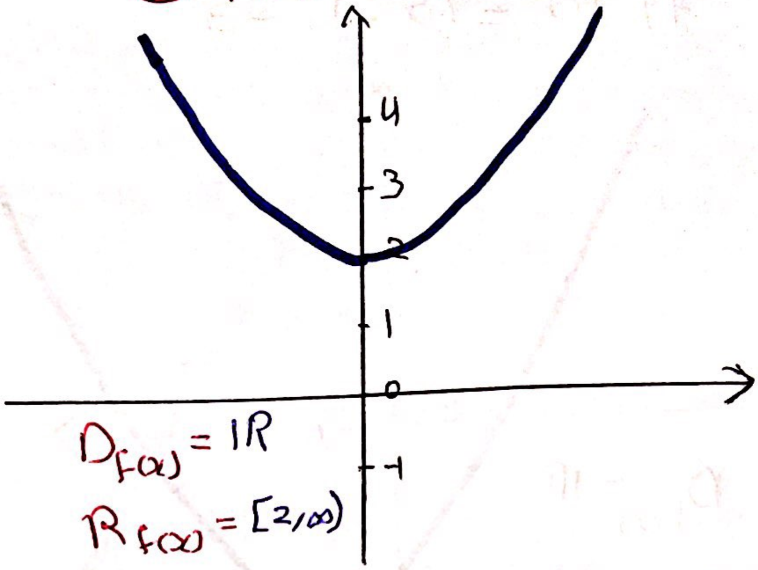
① $f(x) = x^2 - 2$



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [-2, \infty)$

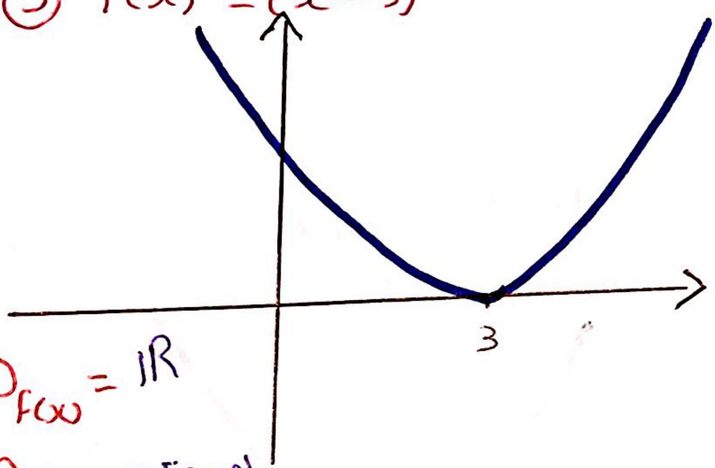
② $f(x) = x^2 + 2$



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [2, \infty)$

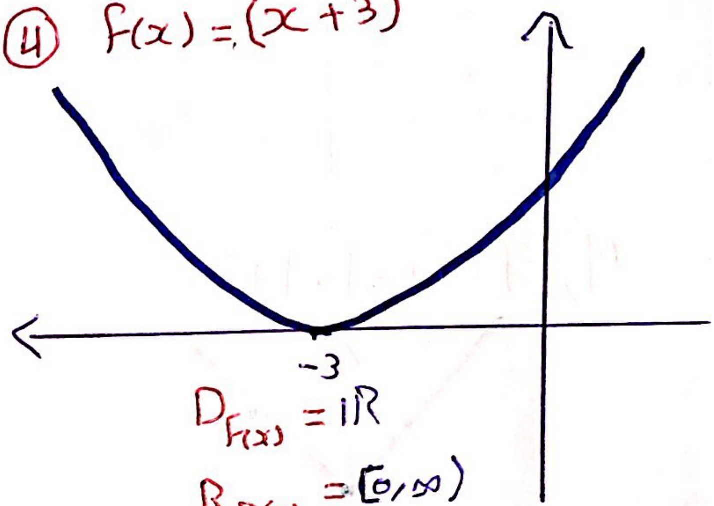
③ $f(x) = (x-3)^2$



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [0, \infty)$

④ $f(x) = (x+3)^2$



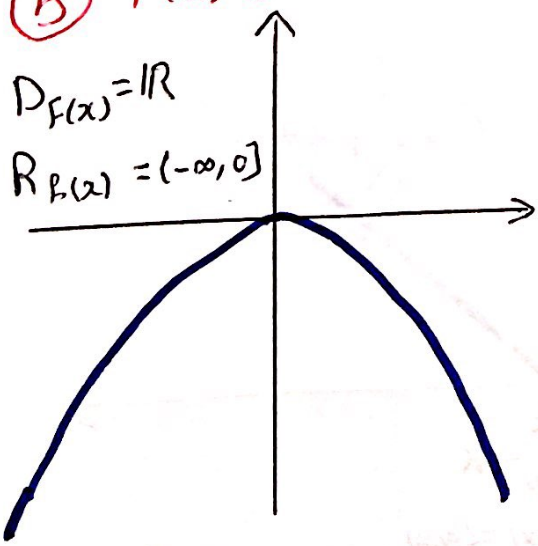
$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [0, \infty)$

⑤ $f(x) = -x^2$

$D_{f(x)} = \mathbb{R}$

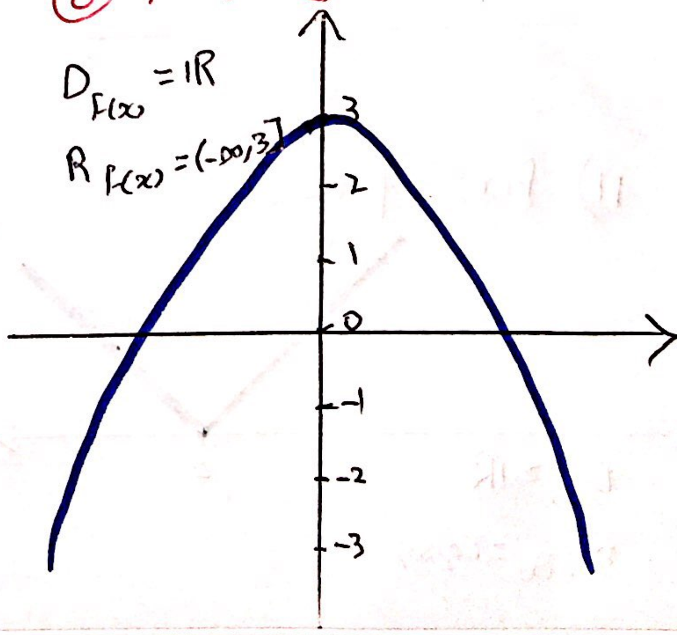
$R_{f(x)} = (-\infty, 0]$



⑥ $f(x) = 3 - x^2$

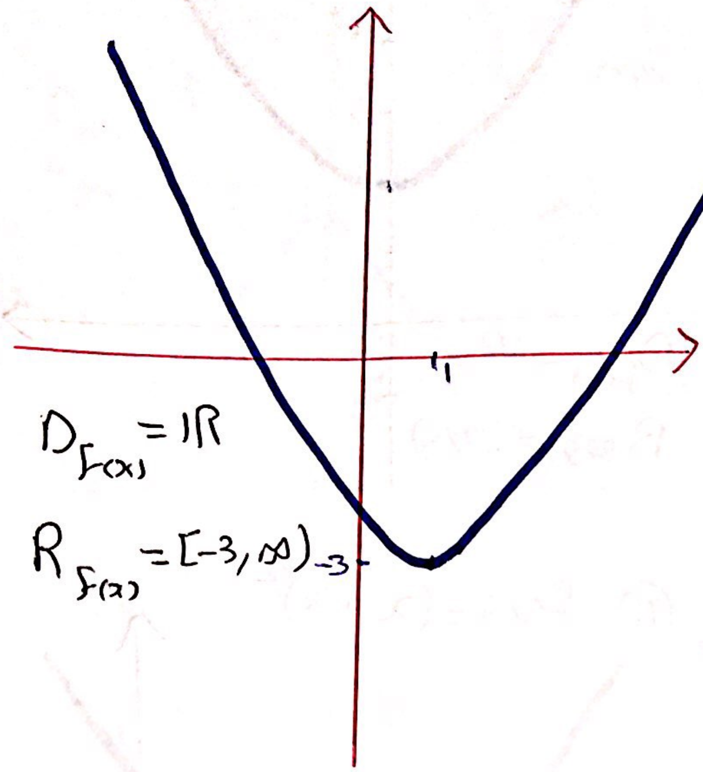
$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = (-\infty, 3]$



Scanned by CamScanner

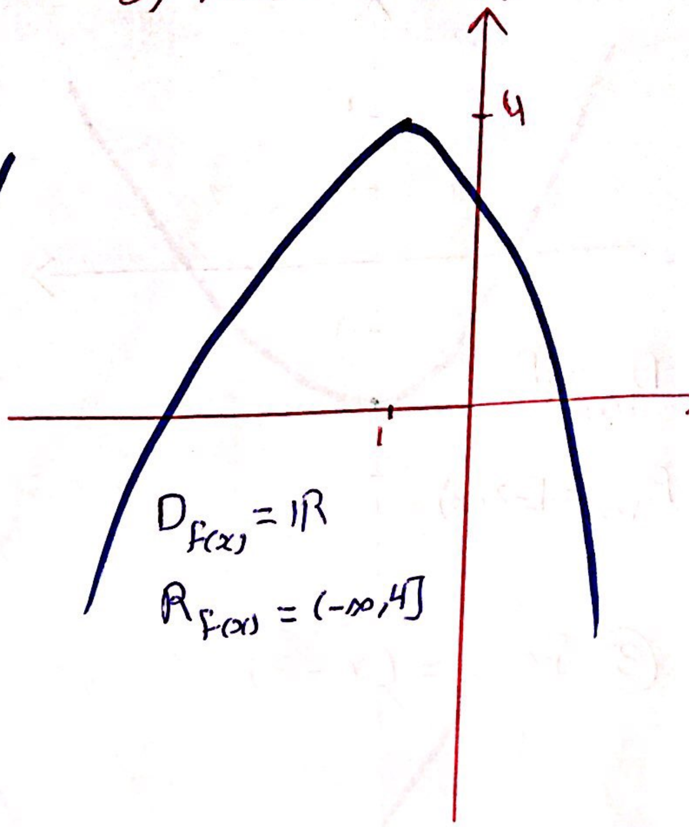
7) $f(x) = (x-1)^2 - 3$



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [-3, \infty)$

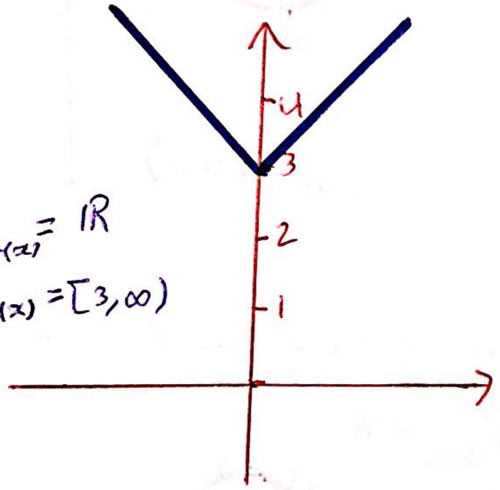
8) $f(x) = 4 - (x+1)^2$



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = (-\infty, 4]$

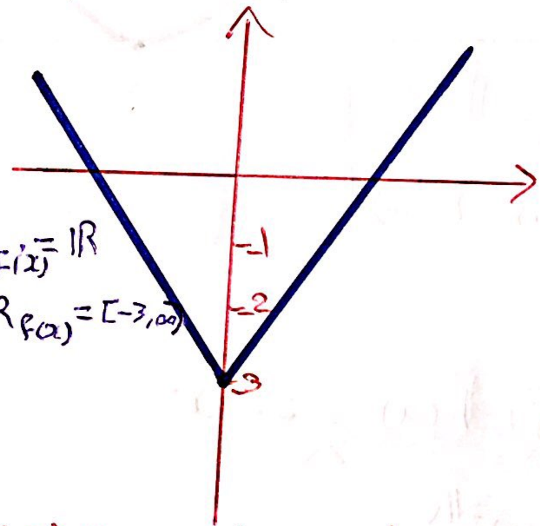
9) $f(x) = |x| + 3$



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [3, \infty)$

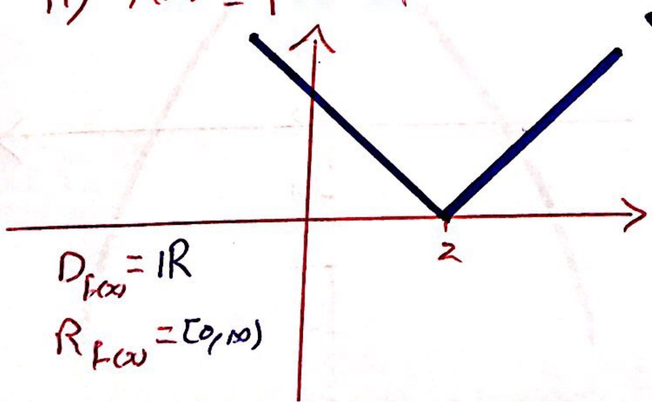
10) $f(x) = |x| - 3$



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [-3, \infty)$

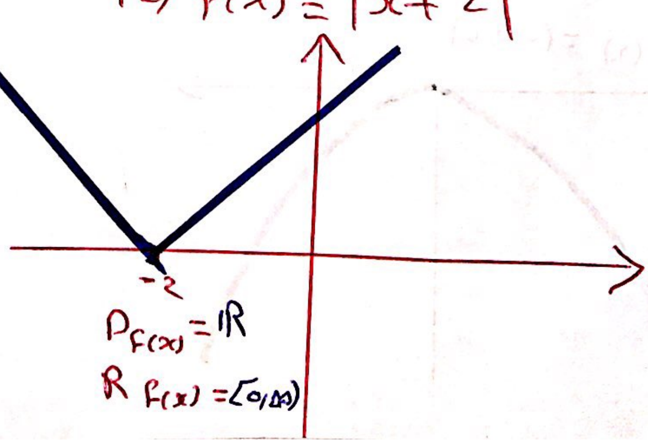
11) $f(x) = |x-2|$



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [0, \infty)$

12) $f(x) = |x+2|$



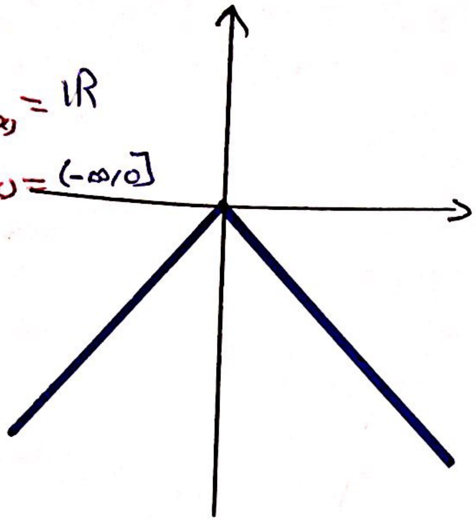
$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [0, \infty)$

13) $f(x) = -|x|$

$D_{f(x)} = \mathbb{R}$

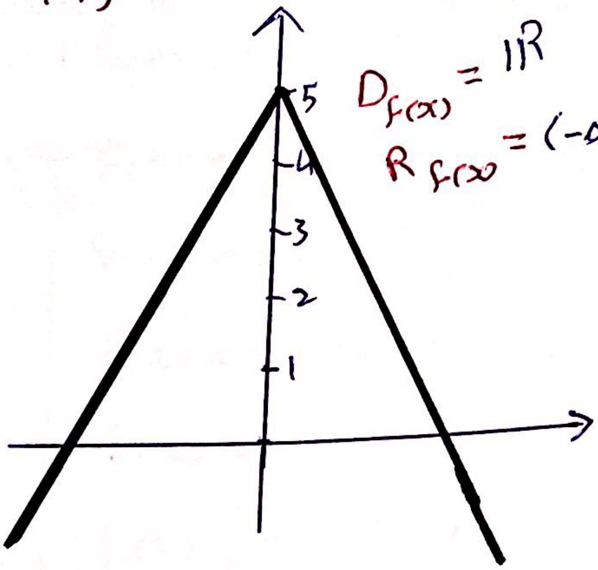
$R_{f(x)} = (-\infty, 0]$



14) $f(x) = 5 - |x|$

$D_{f(x)} = \mathbb{R}$

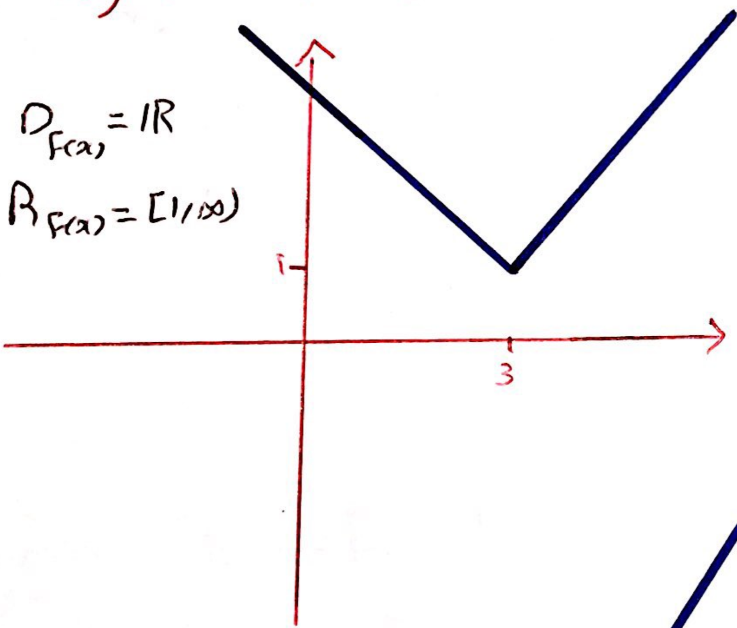
$R_{f(x)} = (-\infty, 5]$



15) $f(x) = |x-3| + 1$

$D_{f(x)} = \mathbb{R}$

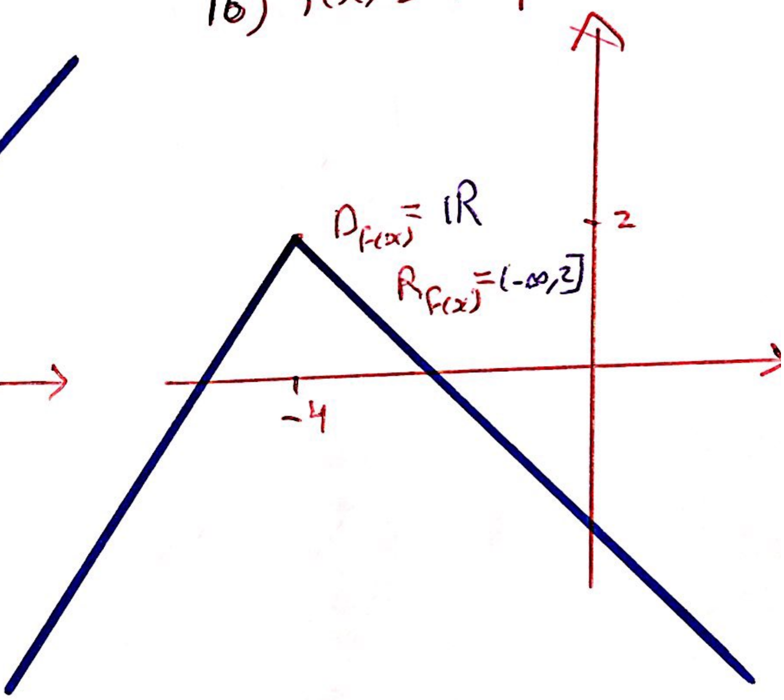
$R_{f(x)} = [1, \infty)$



16) $f(x) = 2 - |x+4|$

$D_{f(x)} = \mathbb{R}$

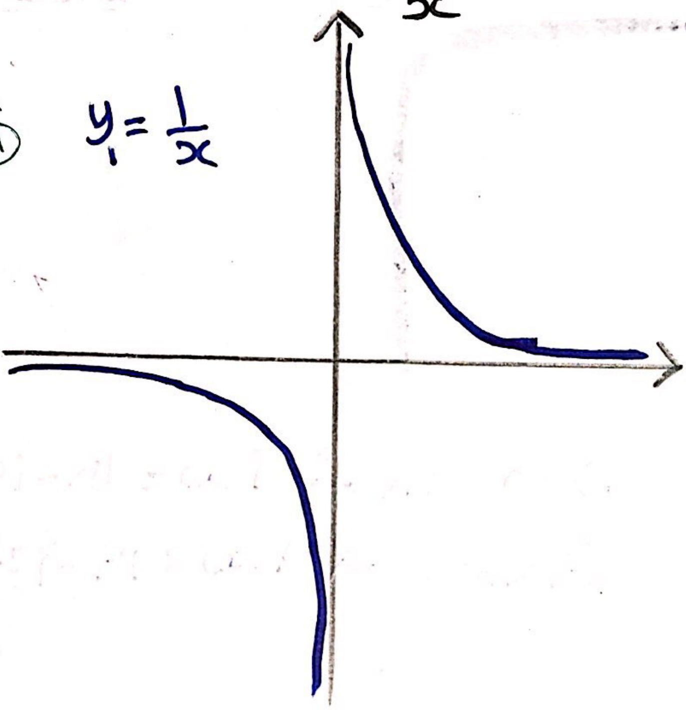
$R_{f(x)} = (-\infty, 2]$



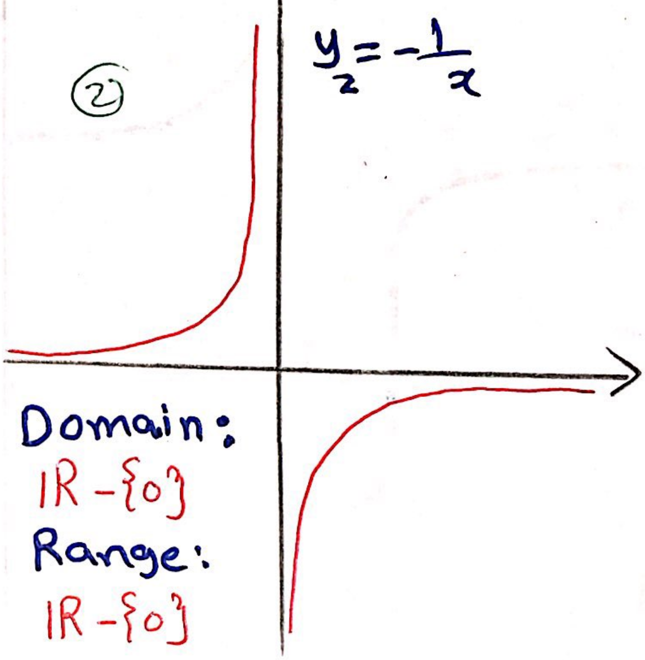
Example (3): Sketch the graph and find the Domain and range.

1) $f(x) = -\frac{1}{x}$

① $y_1 = \frac{1}{x}$



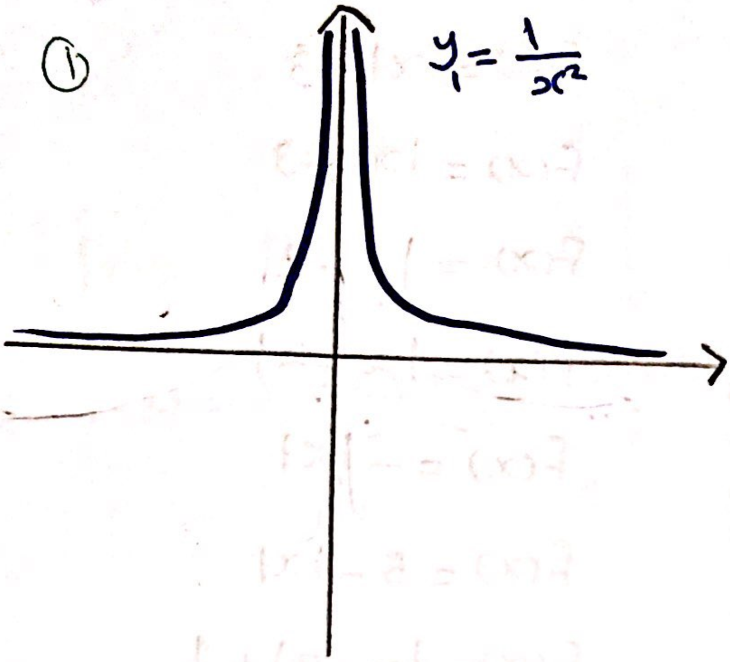
② $y_2 = -\frac{1}{x}$

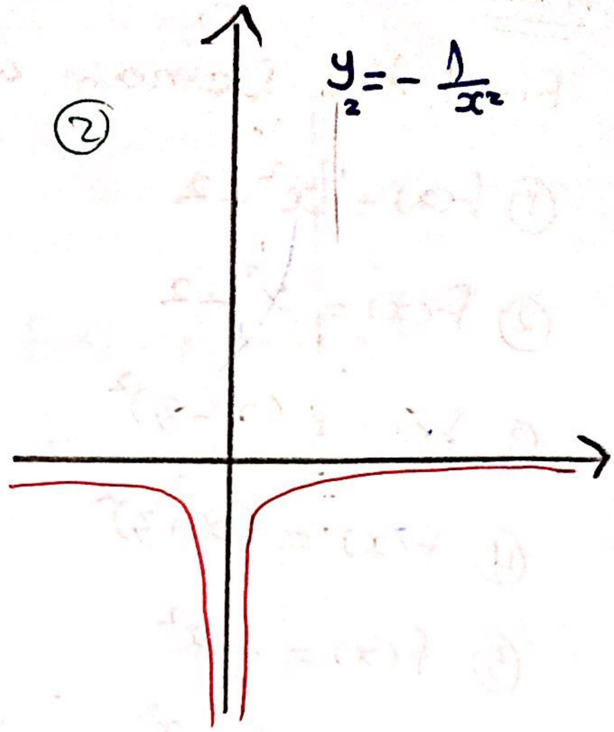


Domain:
$\mathbb{R} - \{0\}$

Range:
$\mathbb{R} - \{0\}$

② $f(x) = -\frac{1}{x^2}$

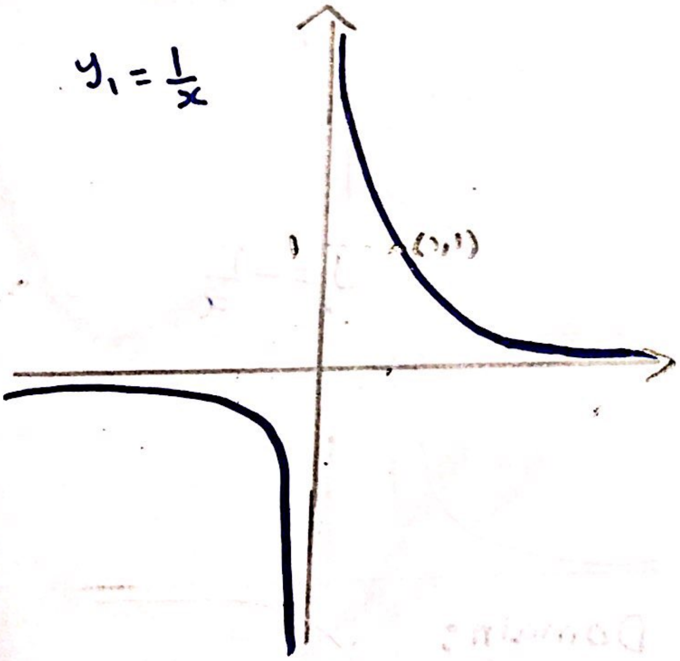


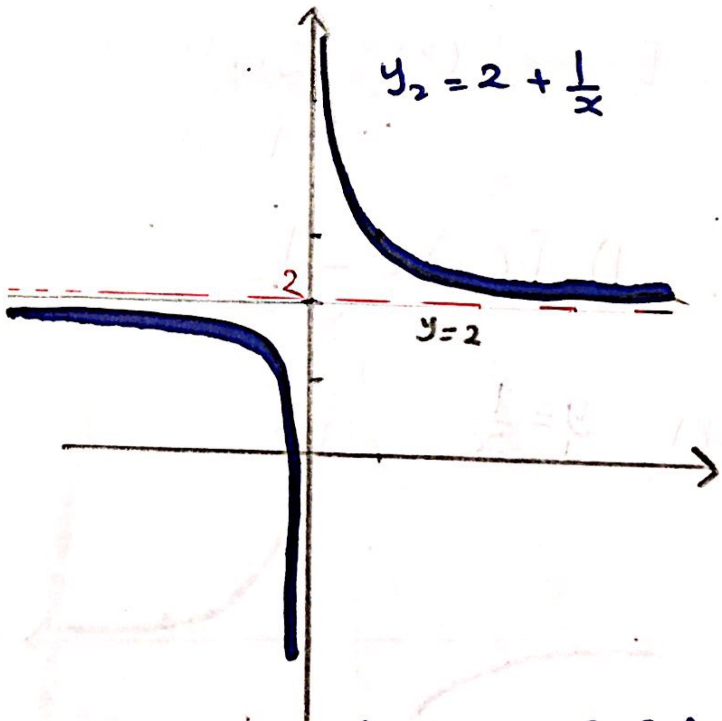


Domain: $\mathbb{R} - \{0\}$

Range: $(-\infty, 0)$

③ $f(x) = 2 + \frac{1}{x}$

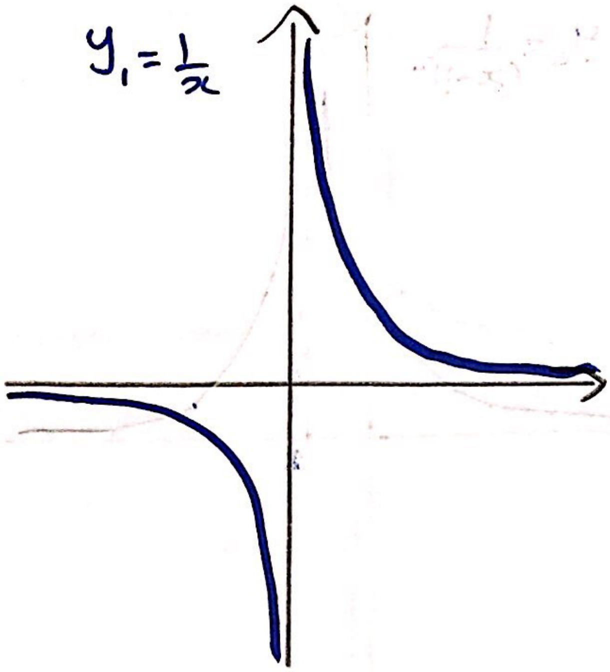


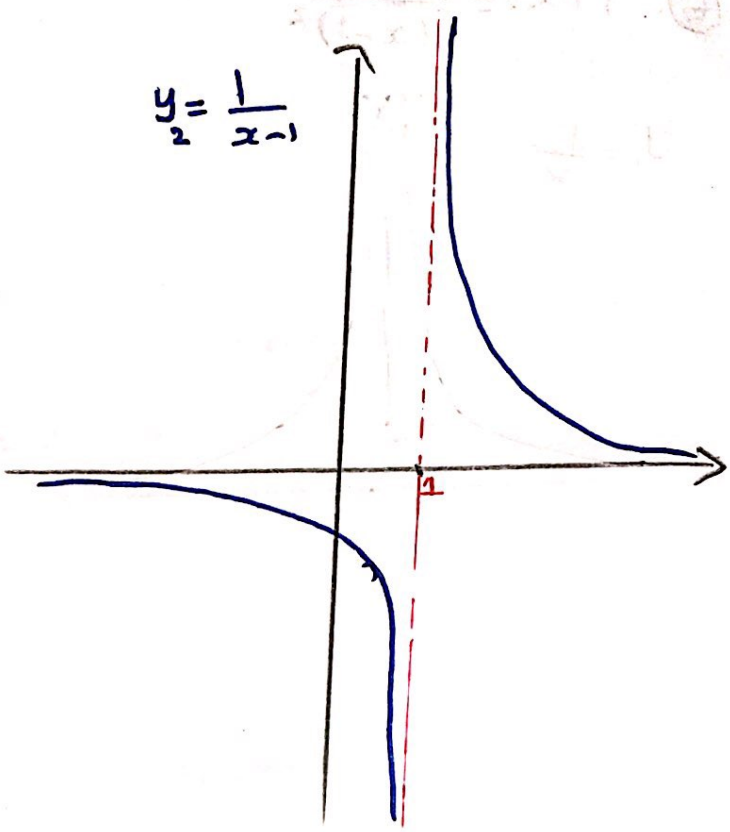


Domain of $f(x) = \mathbb{R} - \{0\}$

Range of $f(x) = \mathbb{R} - \{2\}$

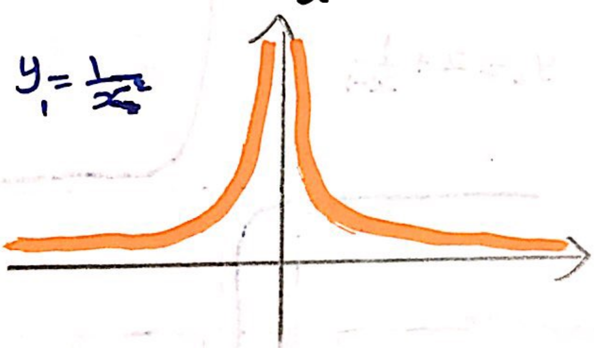
4) $f(x) = \frac{1}{x-1}$

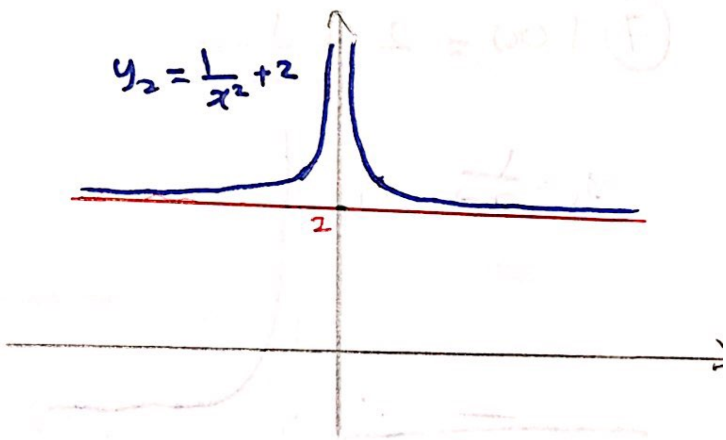




Domain: $\mathbb{R} - \{1\}$
Range: $\mathbb{R} - \{0\}$

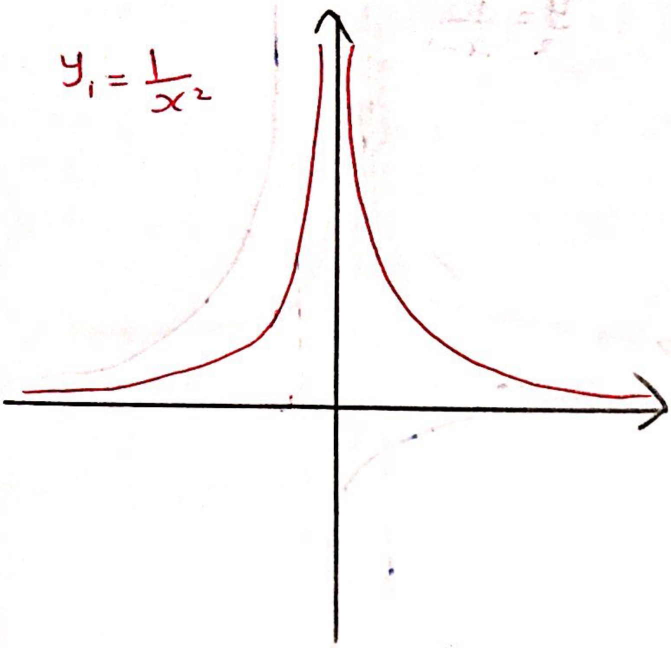
5) $f(x) = \frac{1}{x^2} + 2$

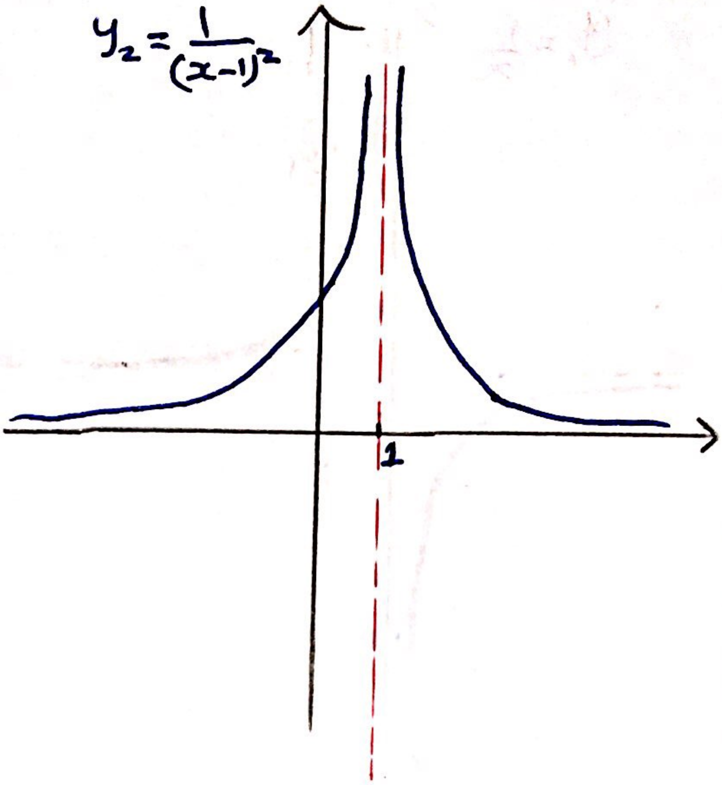




Domain: $\mathbb{R} - \{0\}$
Range: $(2, \infty)$

6 $f(x) = \dfrac{1}{(x-1)^2}$

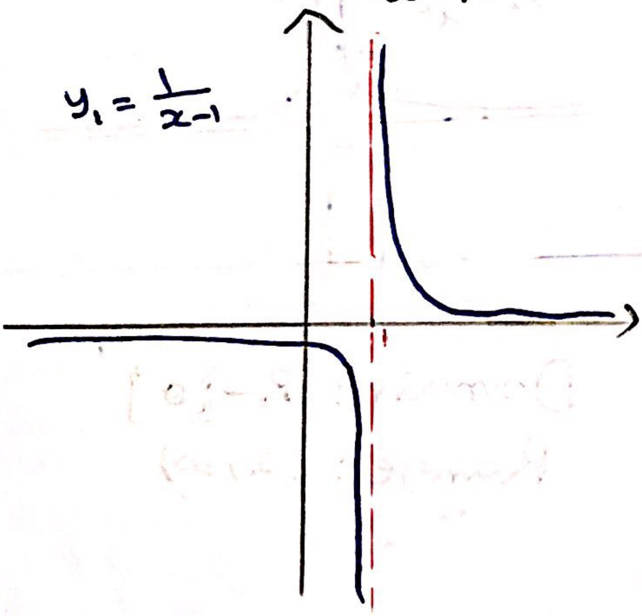


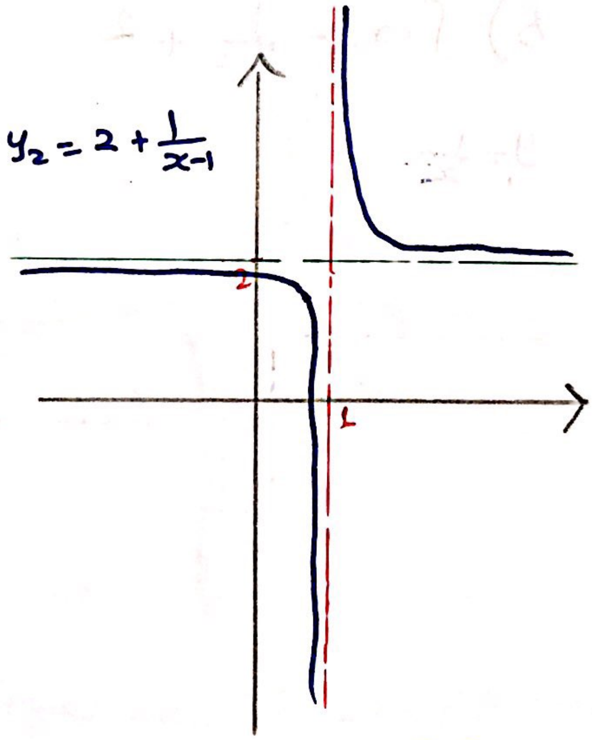


Domain : $\mathbb{R} - \{1\}$

Range : $(0, \infty)$

7 $f(x) = 2 + \dfrac{1}{x-1}$

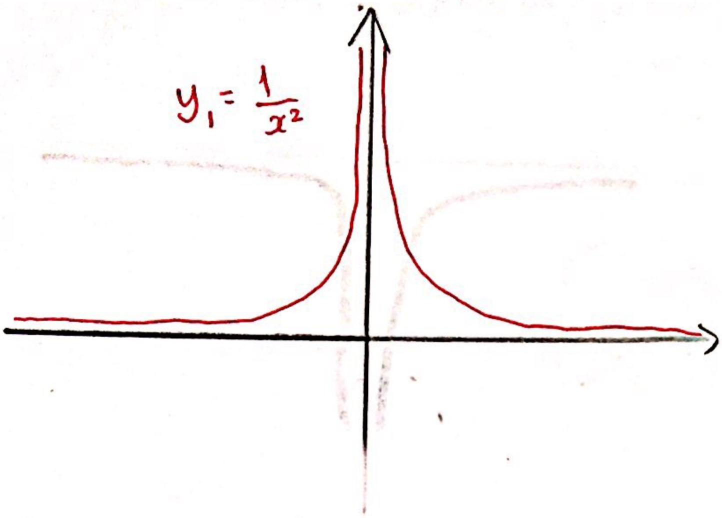


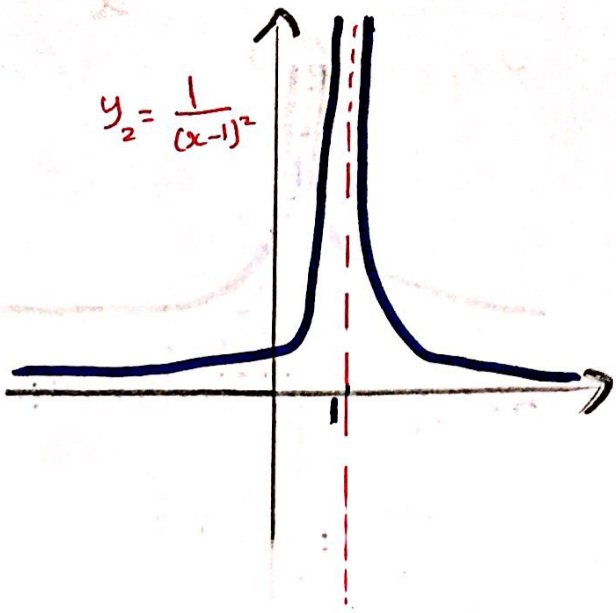


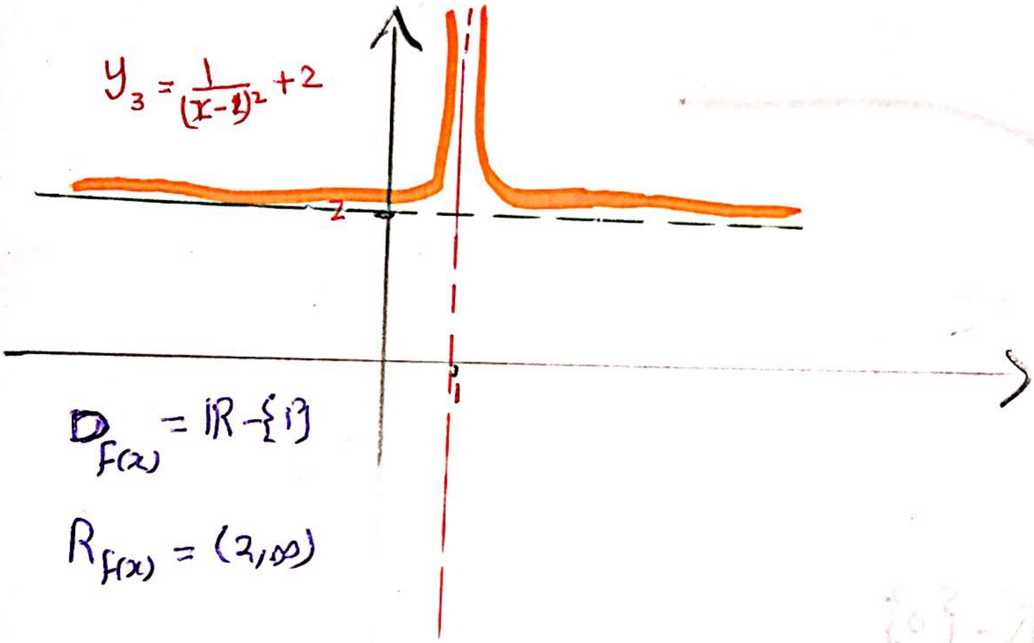
Domain : $\mathbb{R} - \{1\}$

Range : $\mathbb{R} - \{2\}$

8 $f(x) = \frac{1}{(x-1)^2} + 2$ H.W



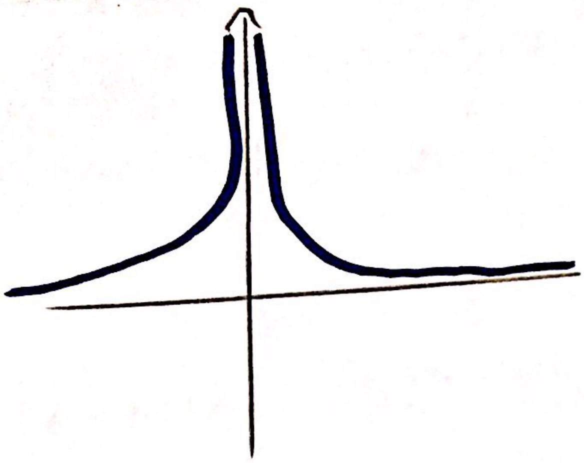


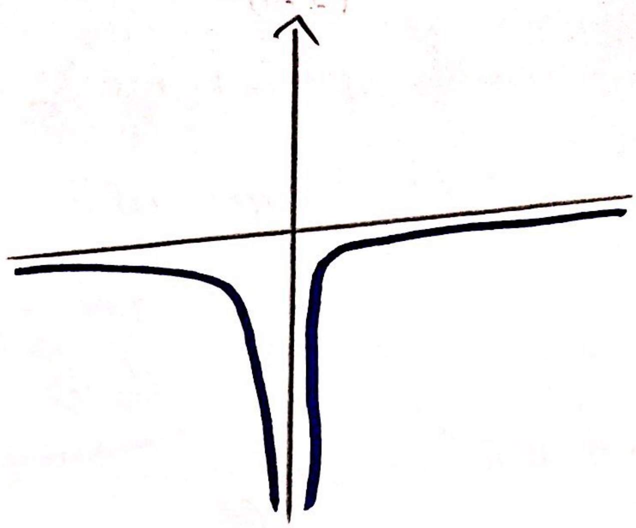


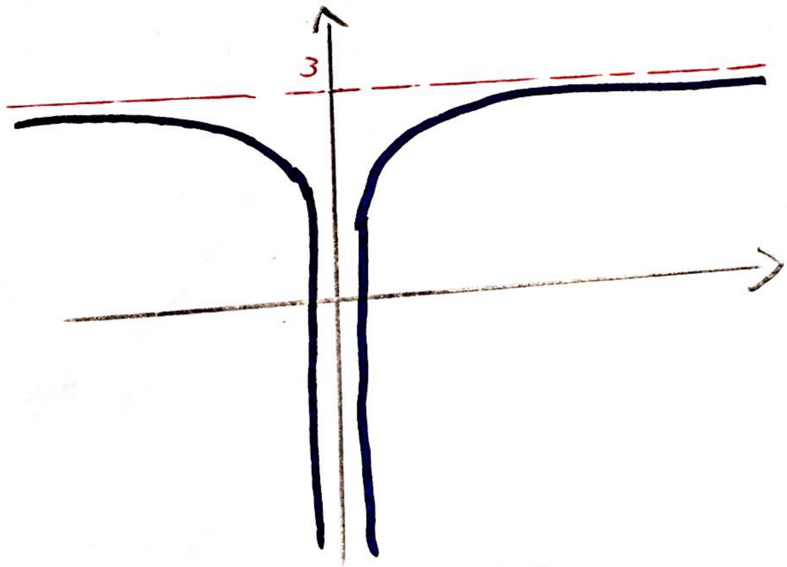
$D_{f(x)} = \mathbb{R} - \{1\}$

$R_{f(x)} = (2, \infty)$

⑨ $f(x) = 3 - \frac{1}{x^2}$



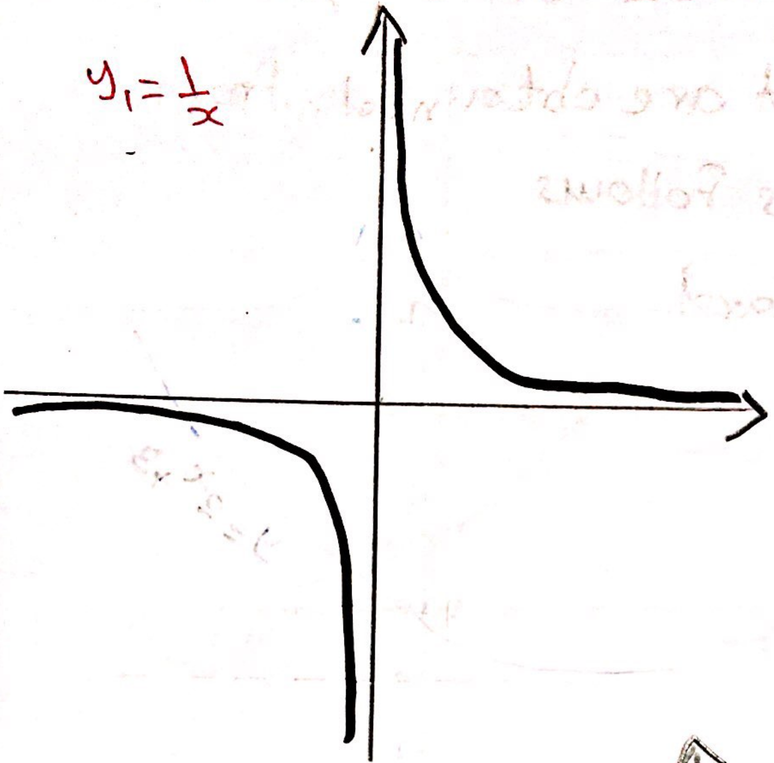


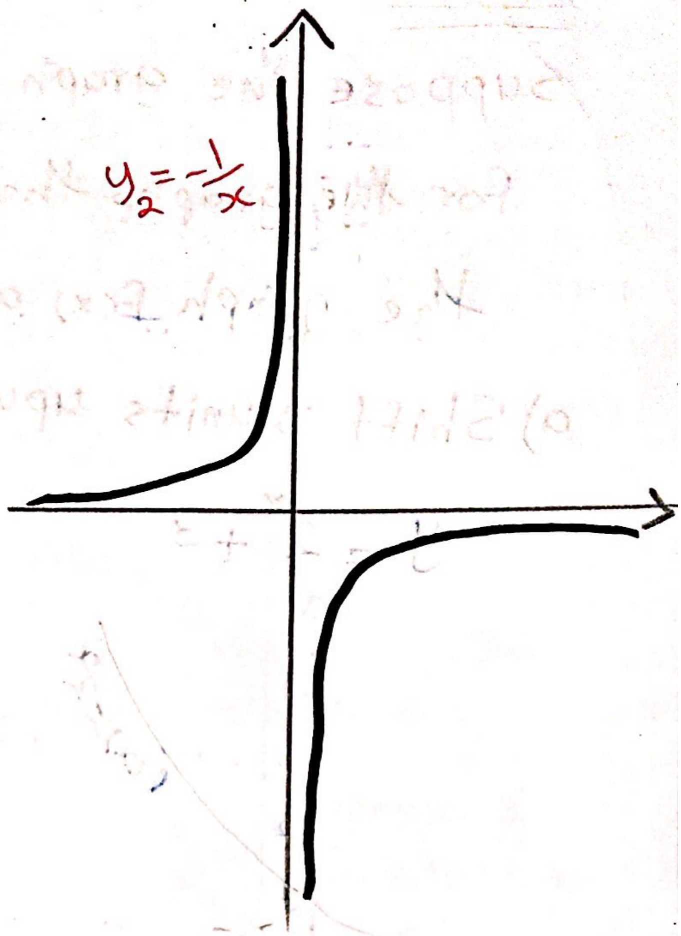


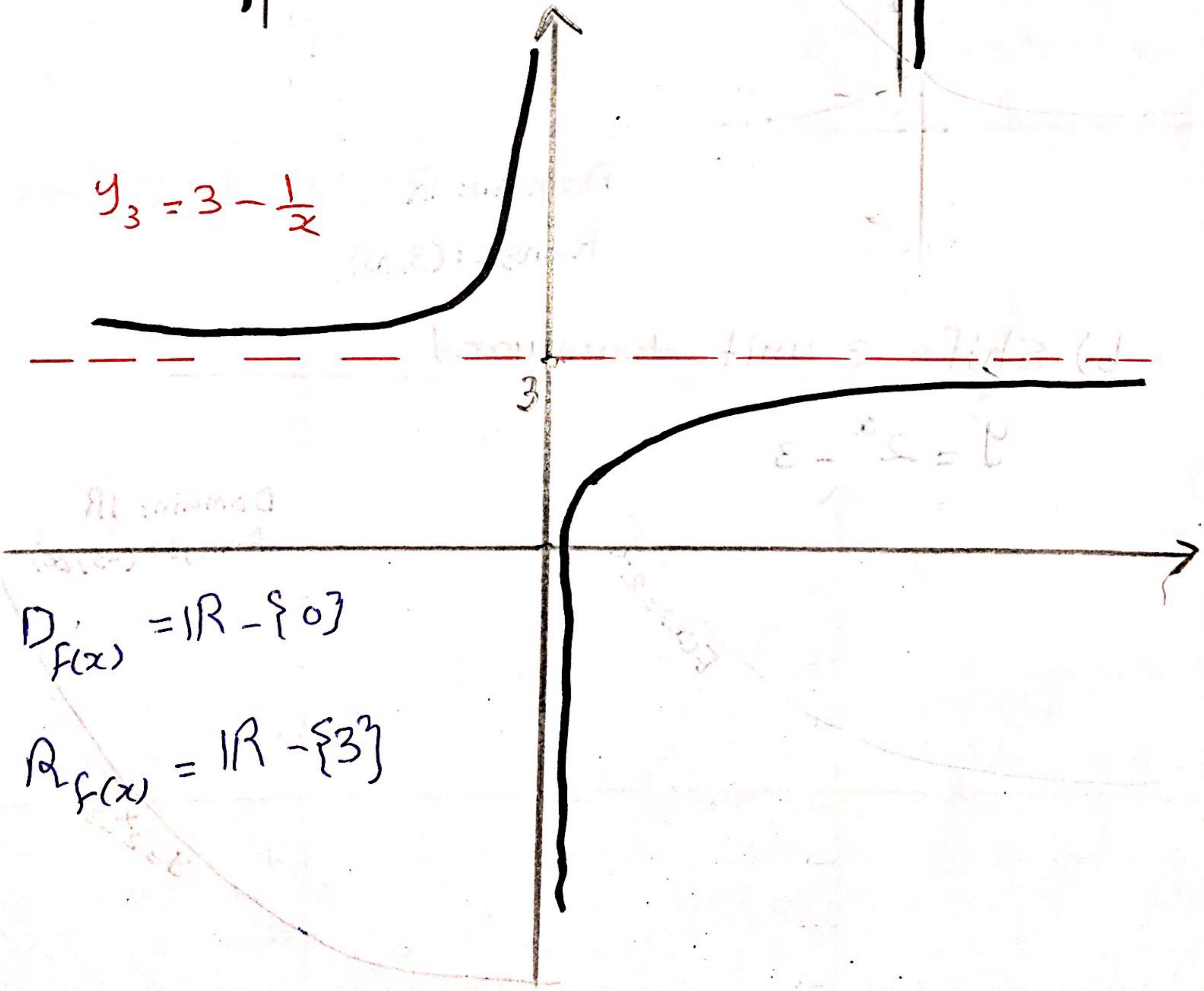
Domain: $\mathbb{R} - \{0\}$

Range: $(-\infty, 3)$

(10) $f(x) = 3 - \frac{1}{x}$ H.W







$D_{f(x)} = \mathbb{R} - \{0\}$

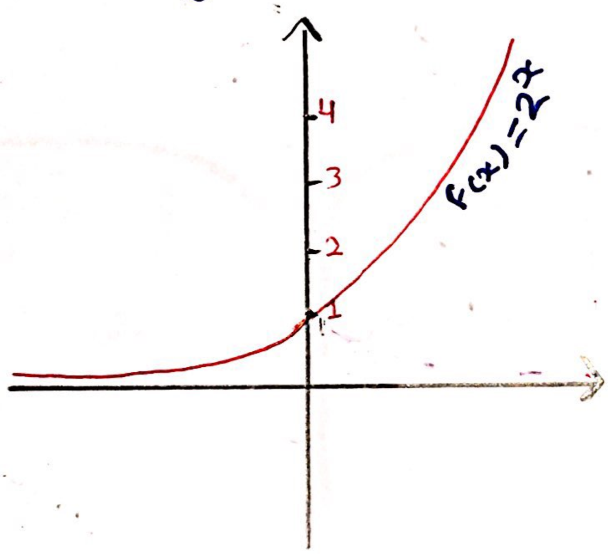
$R_{f(x)} = \mathbb{R} - \{3\}$

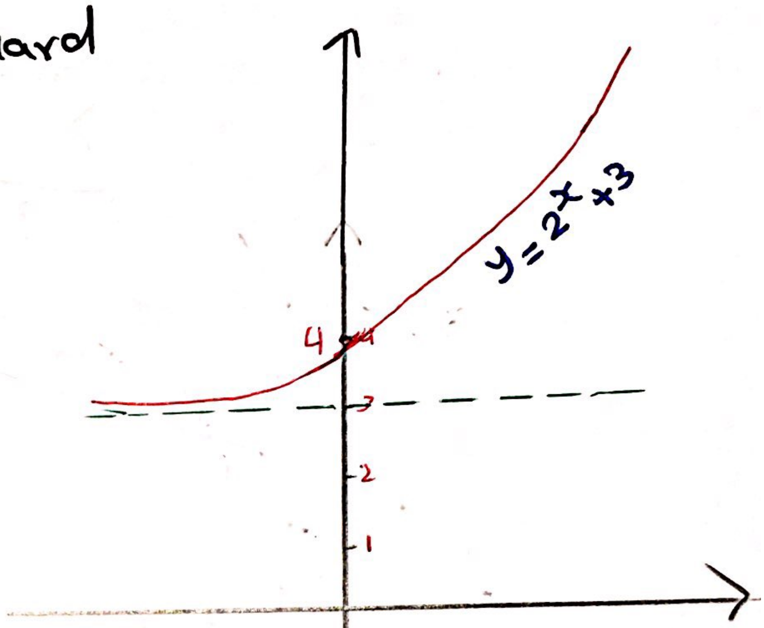
Example

Suppose the graph $f(x) = 2^x$ write equation for the graphs that are obtained from the graph $f(x)$ as Follows

a) Shift 3 units upward

$$y = 2^x + 3$$



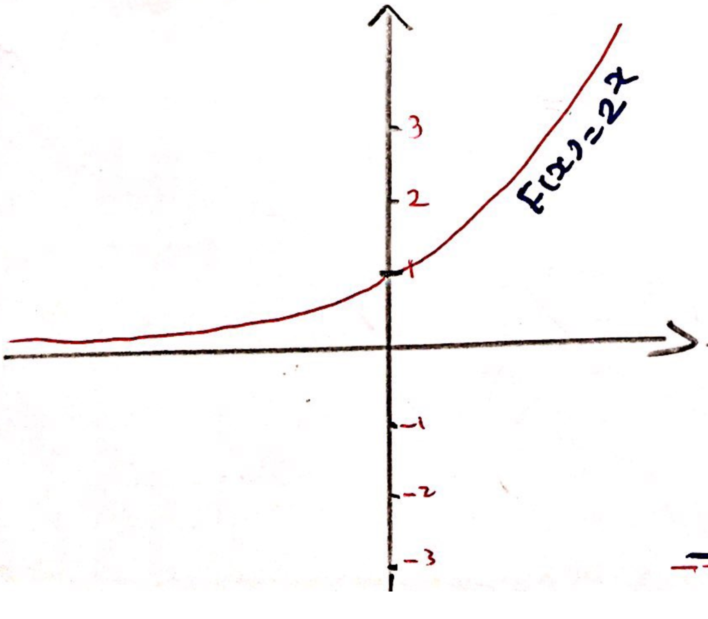


Domain: IR

Range: $(3, \infty)$

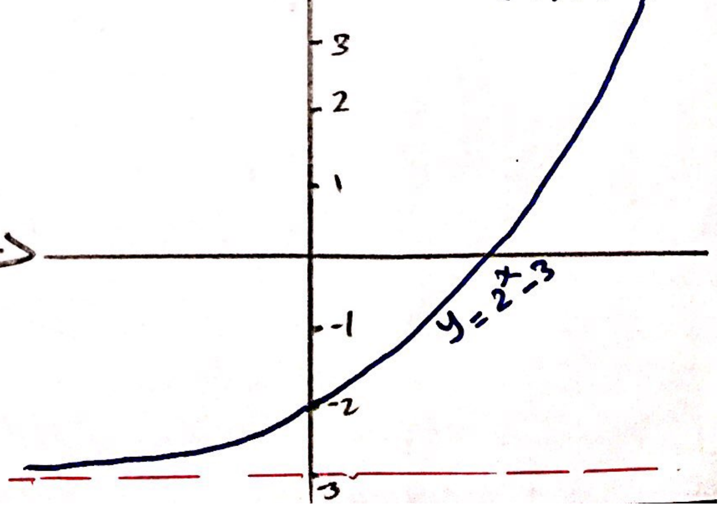
b) Shift 3 unit downward

$$y = 2^x - 3$$



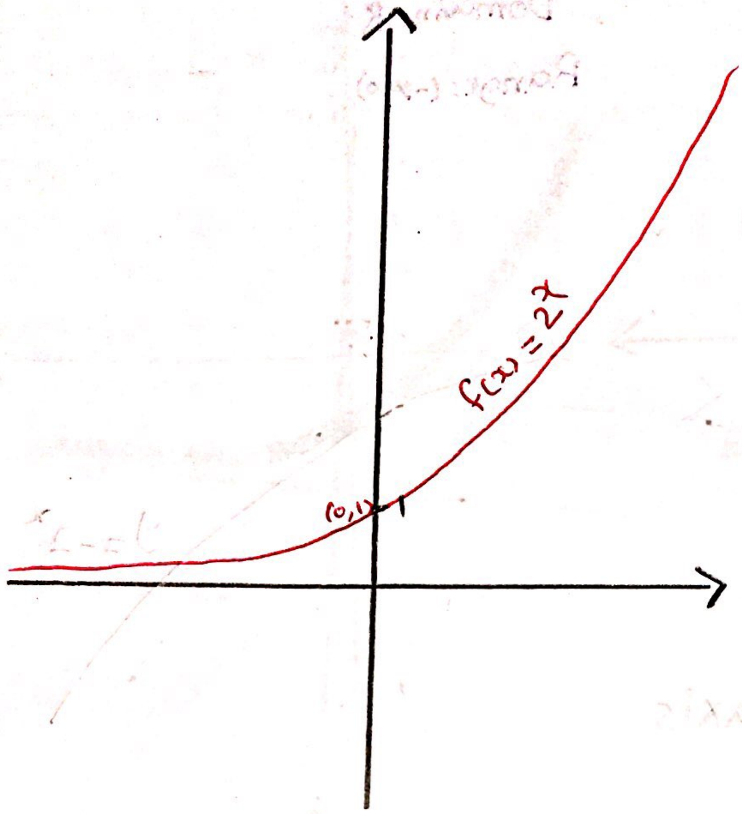
Domain: IR

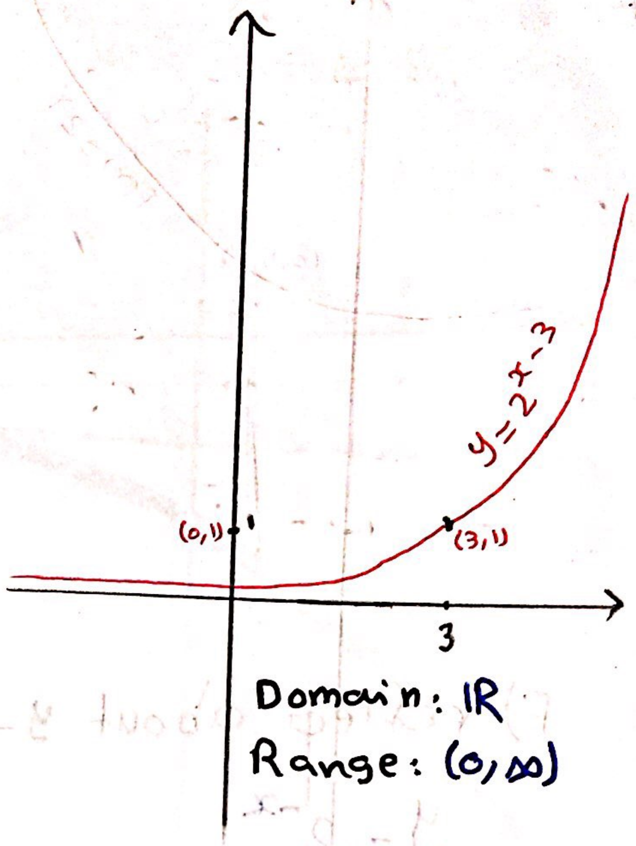
Range: $(-3, \infty)$



c) Shift 3 units to the right

$y = 2^{x-3}$



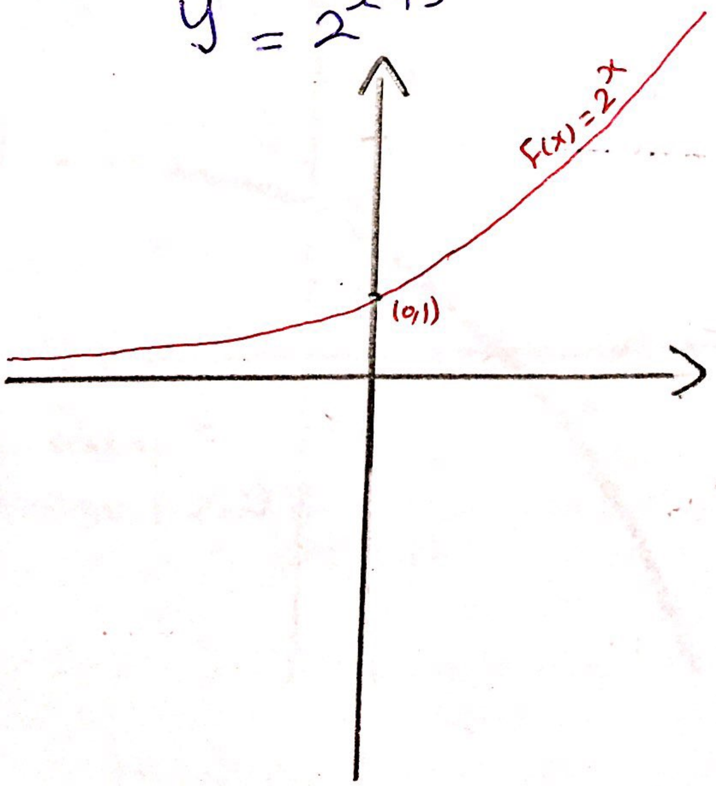


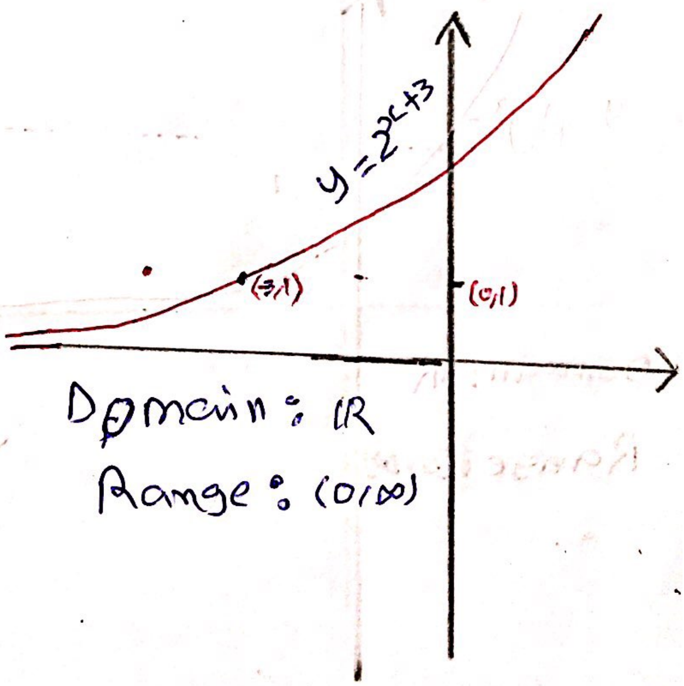
Domain: $\mathbb{R}$

Range: $(0, \infty)$

D) Shift 3 unit to the left. H.W

$y = 2^{x+3}$



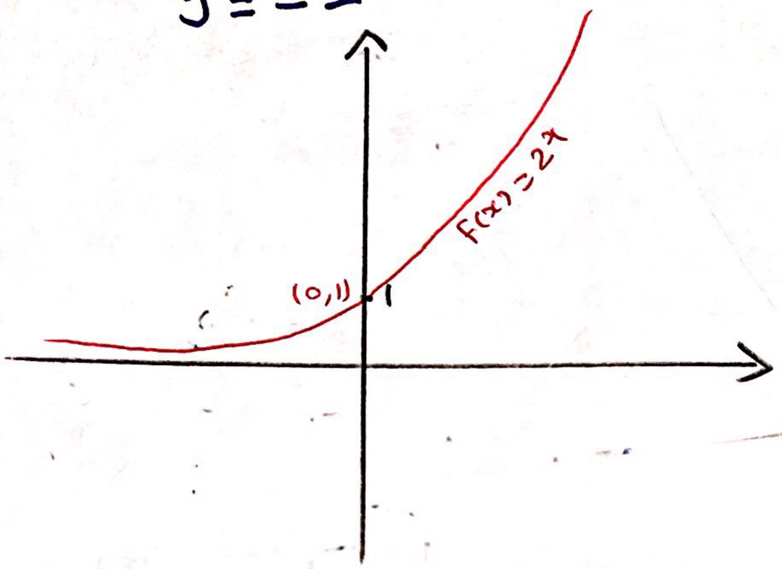


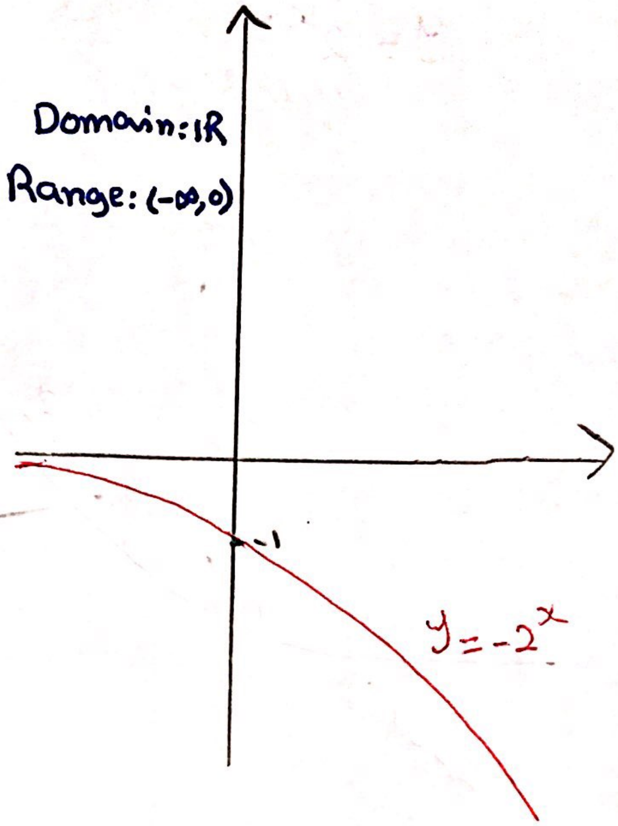
Domain: $\mathbb{R}$

Range: $(0, \infty)$

e) reflect about x-axis

$y = -2^x$

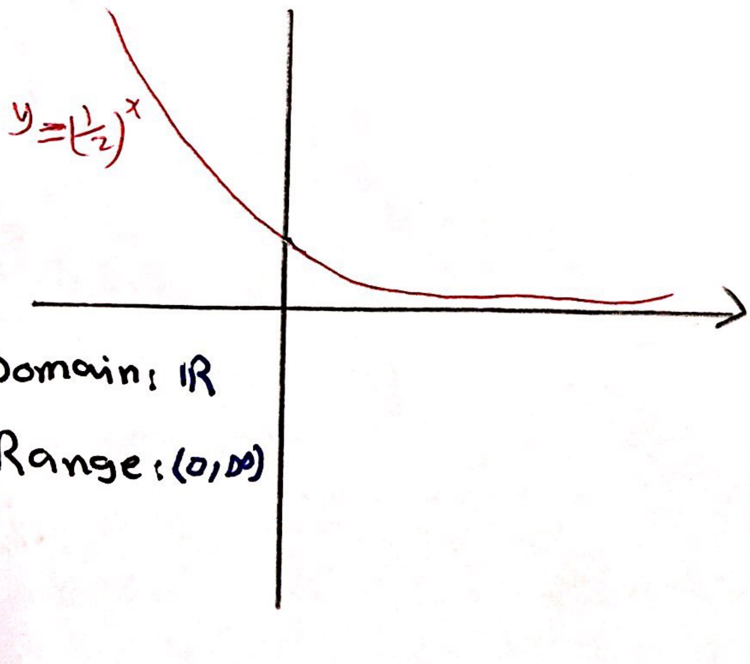




F) reflect about y-axis

$y = 2^{-x}$

$= \left(\frac{1}{2}\right)^x$

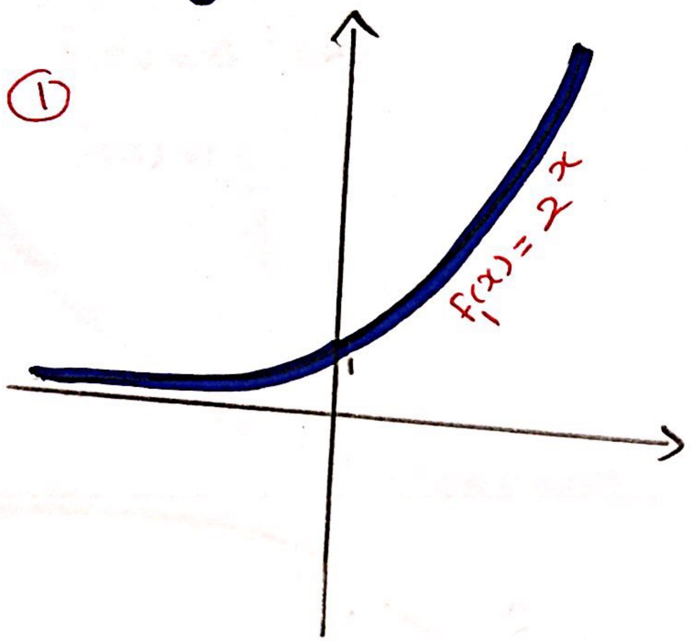


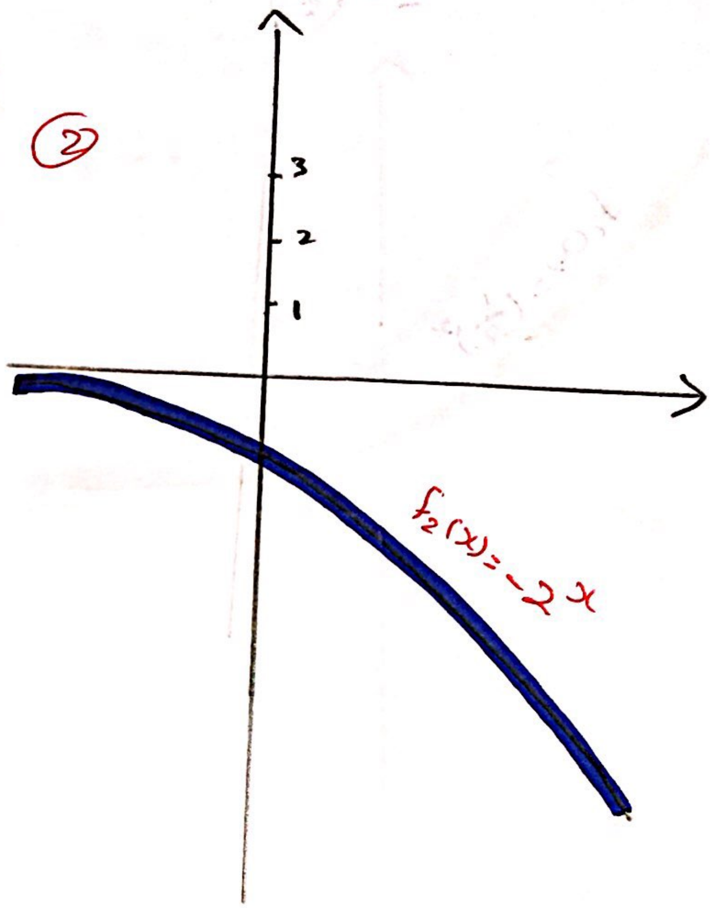
Domain: $\mathbb{R}$

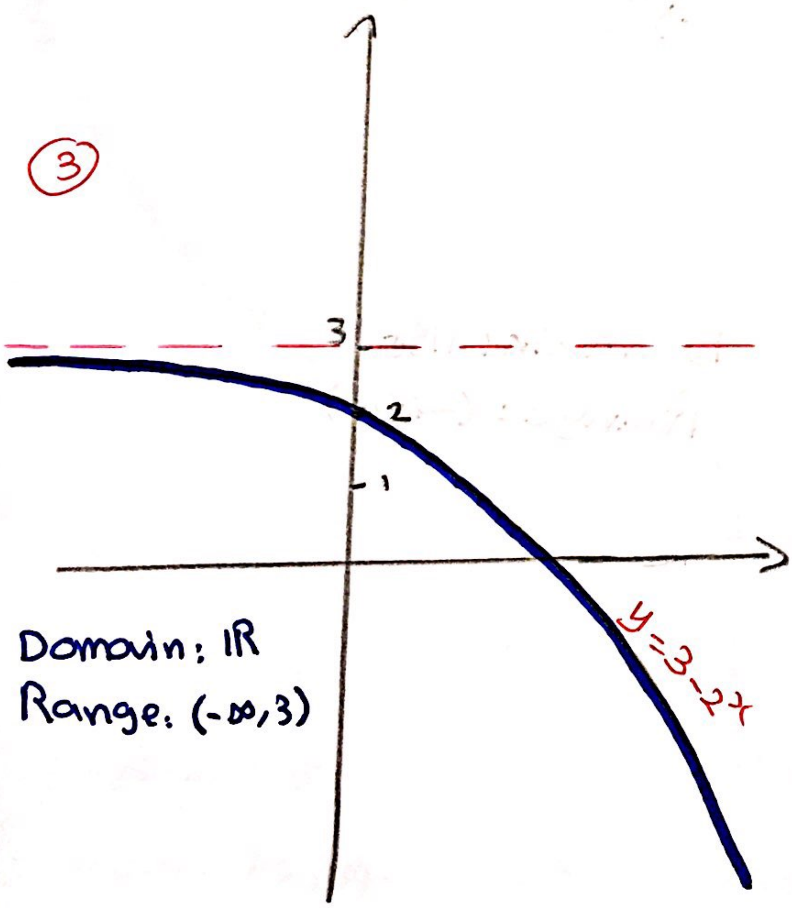
Range: $(0, \infty)$

9) reflect about x-axis and Shift 3 units upward.

$y = 3 - 2^x$





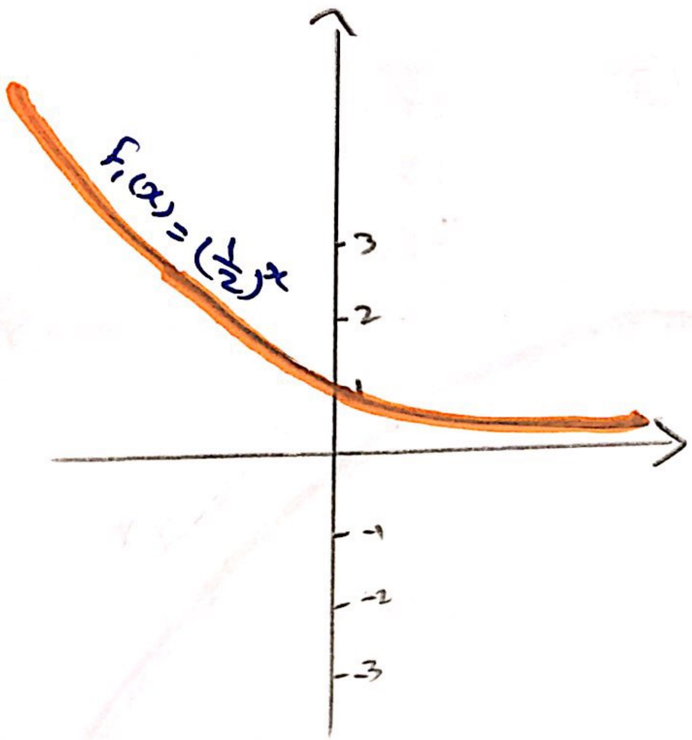


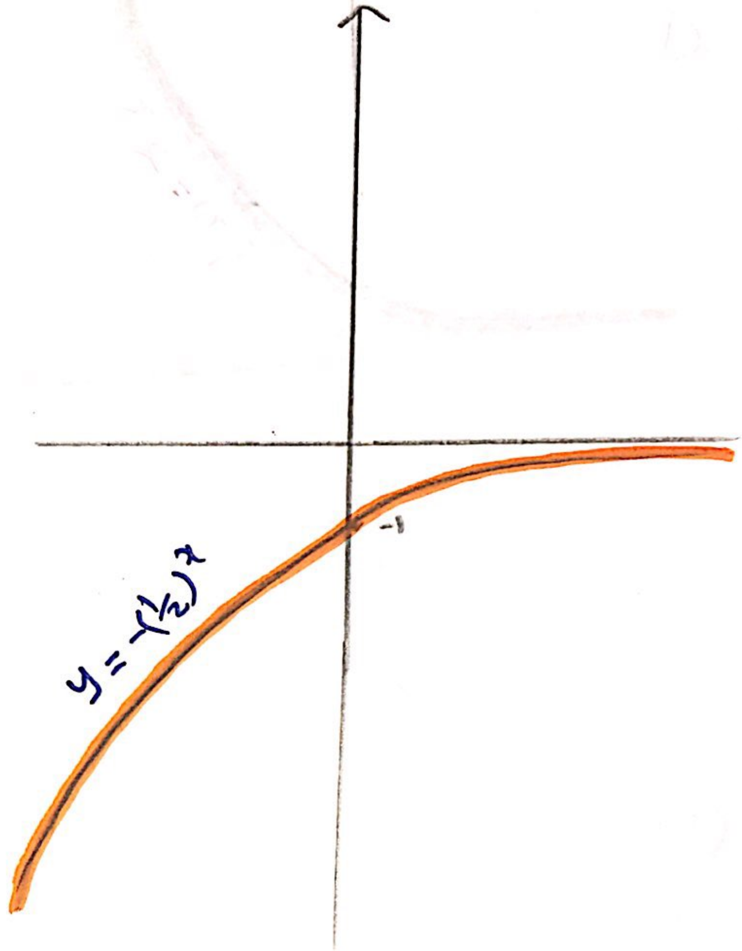
Domain: $\mathbb{R}$
Range: $(-\infty, 3)$

h) reflect about x-axis and y-axis

$$y = -2^{-x}$$

$$= -\left(\frac{1}{2}\right)^x$$





Domain: $\mathbb{R}$

Range: $(-\infty, 0)$

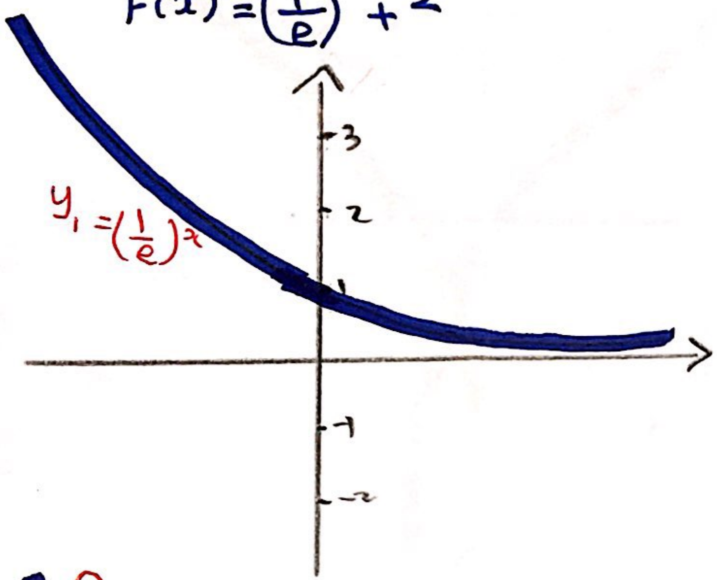
Example find the Domain and Range of

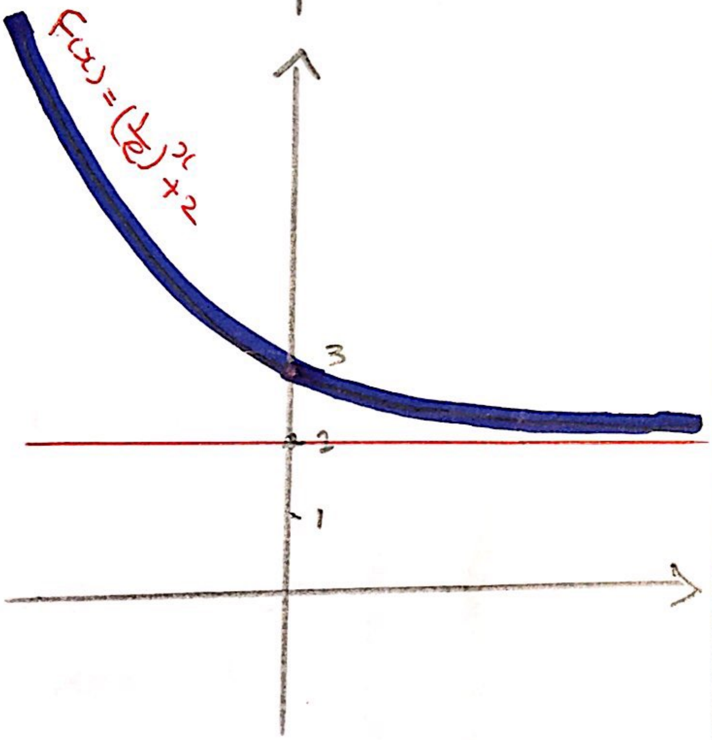
① $f(x) = e^{-x} + 2$

② $f(x) = \left(\frac{1}{\pi}\right)^{-x} - 2$

$f(x) = e^{-x} + 2$

$f(x) = \left(\frac{1}{e}\right)^{x} + 2$



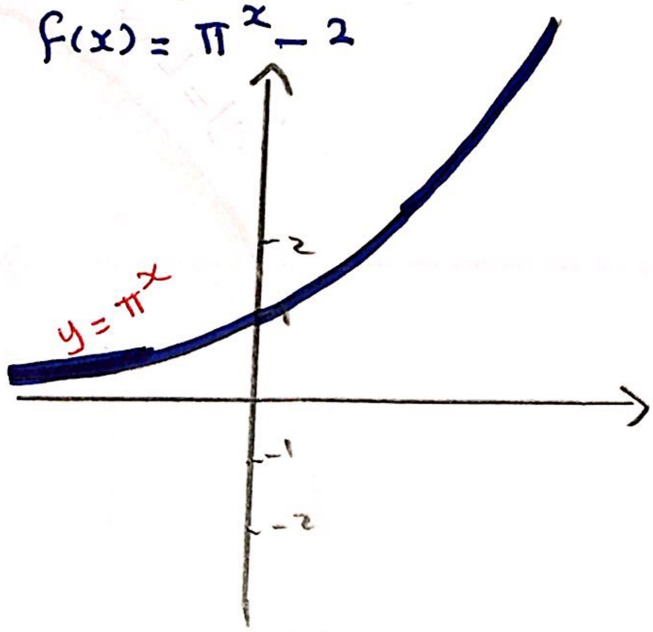


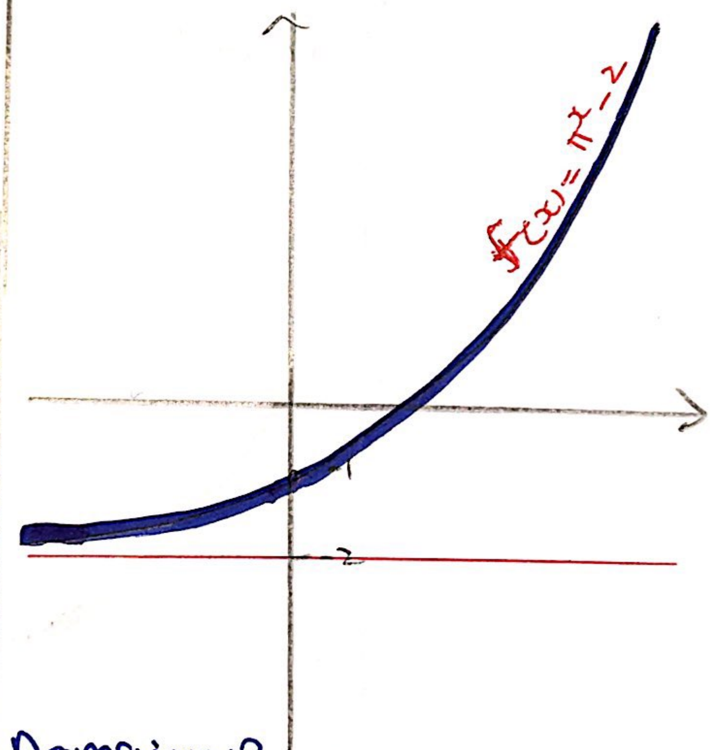
Domain: $\mathbb{R}$

Range: $(2, \infty)$

$f(x) = \left(\frac{1}{\pi}\right)^{-x} - 2$

$f(x) = \pi^{x} - 2$





Domain: $\mathbb{R}$

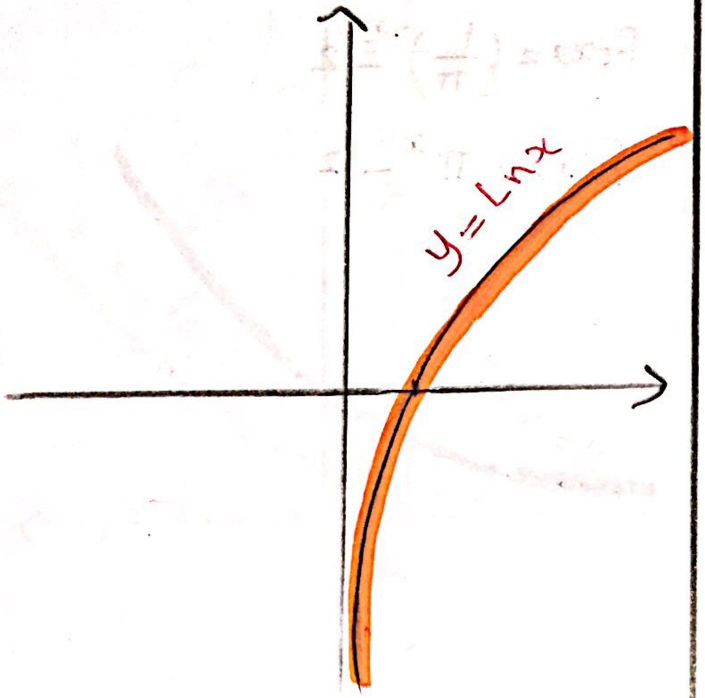
Range: $(-2, \infty)$

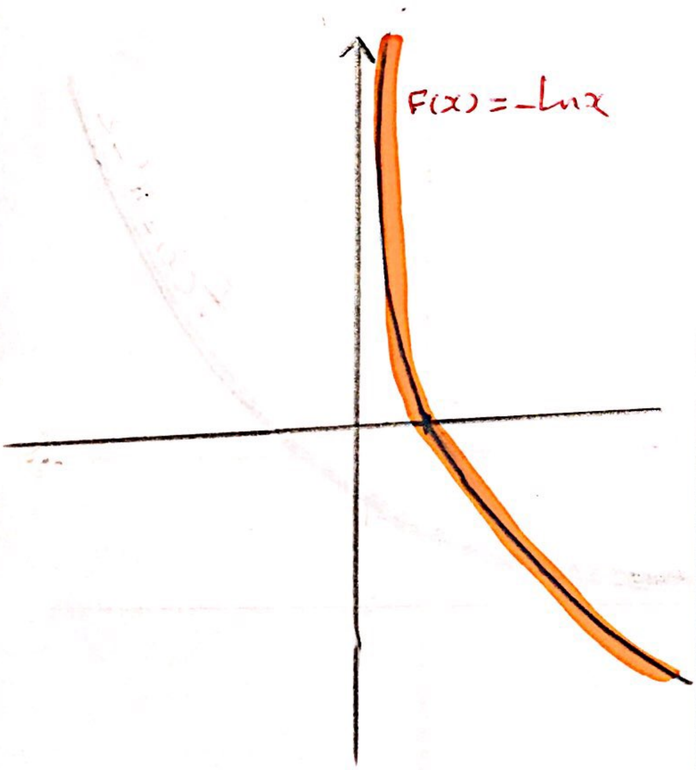
Scanned by CamScanner

③ $f(x) = -\ln x$

④ $f(x) = \ln(-x)$

$f(x) = -\ln x$

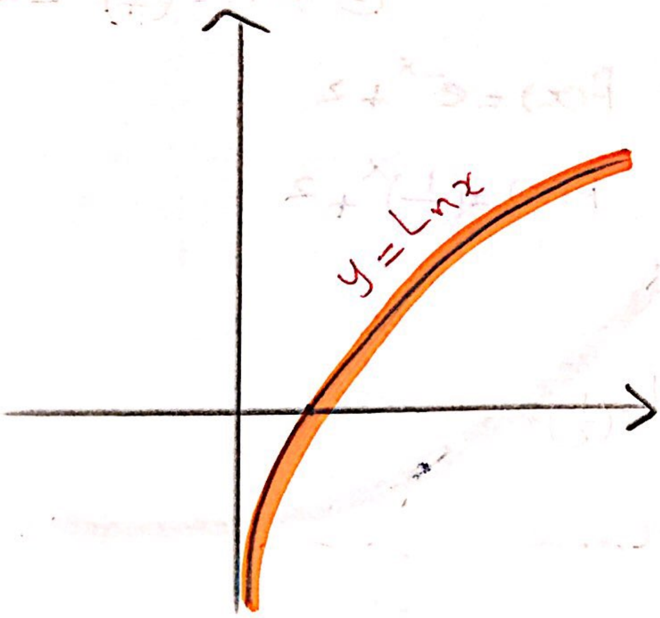


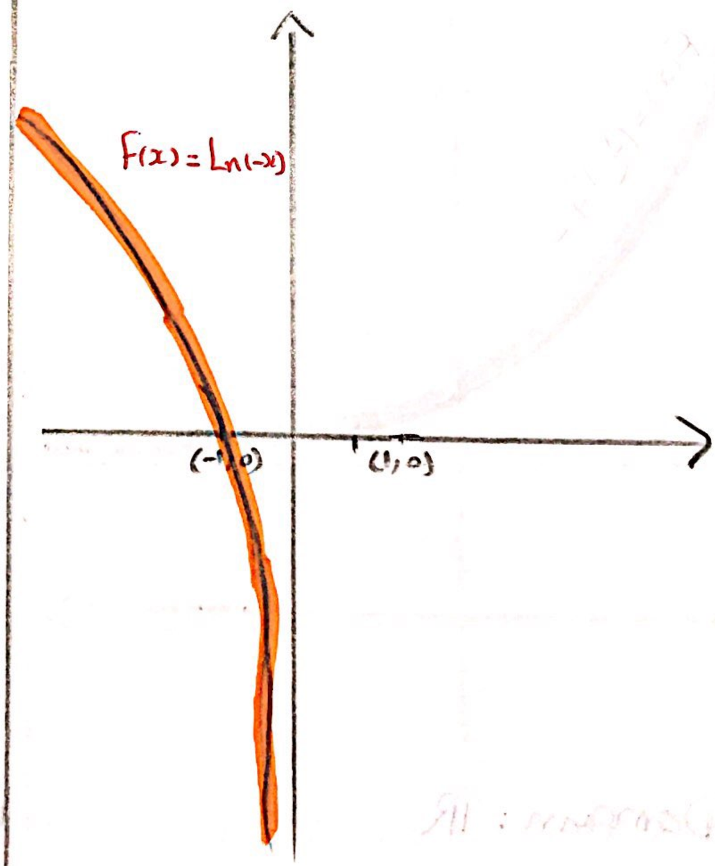


Domain: $(0, \infty)$

Range: $\mathbb{R}$

$f(x) = \ln(-x)$





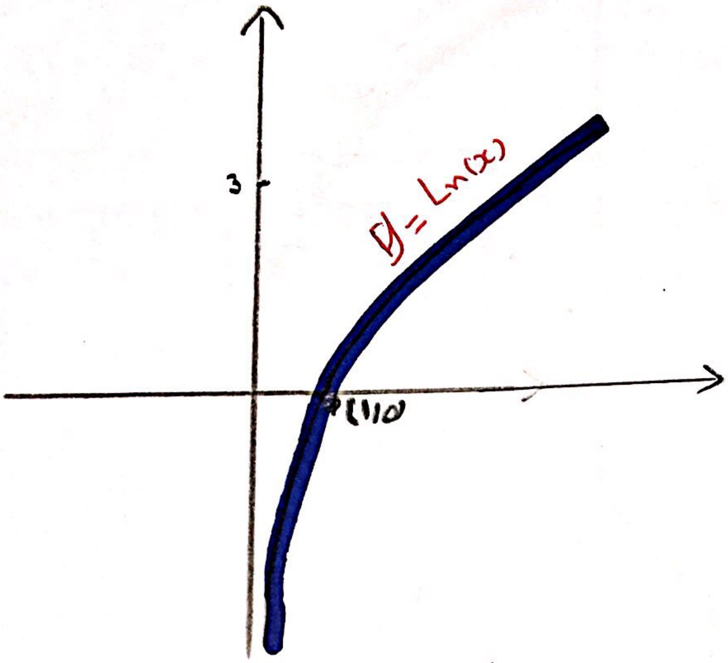
Domain: $(-\infty, 0)$

Range: $\mathbb{R}$

⑤ $f(x) = \ln(x) + 3$

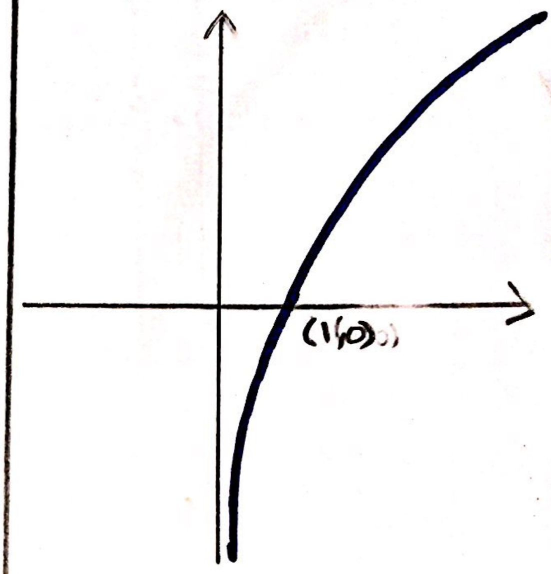
⑥ $f(x) = \ln(x) - 3$

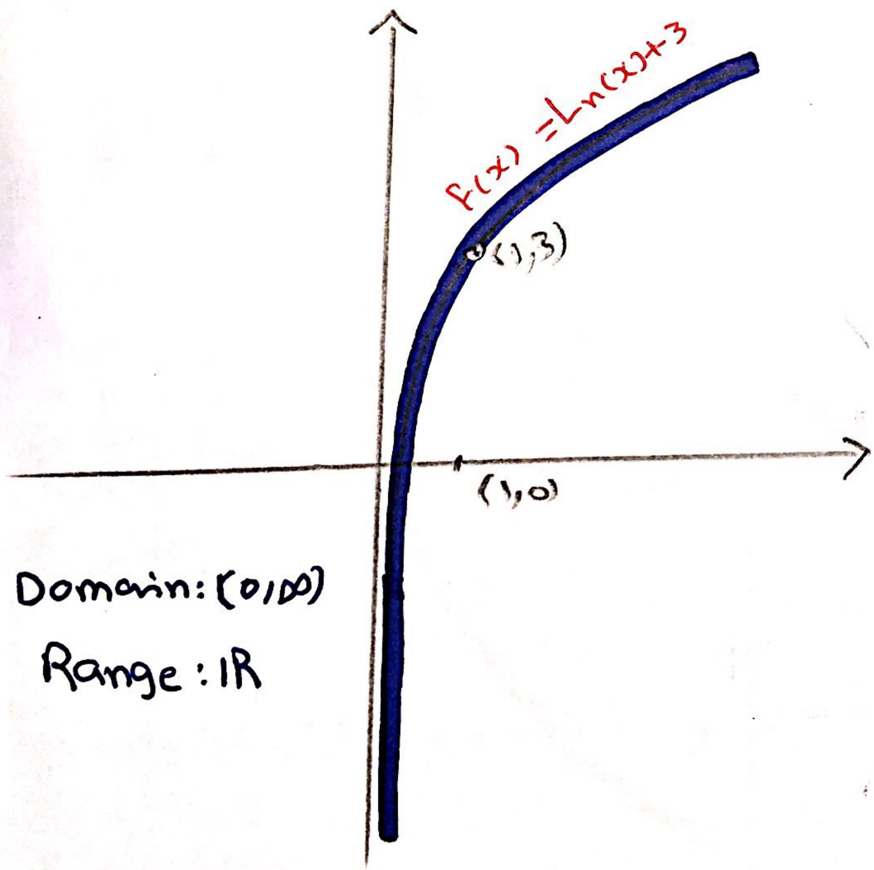
$f(x) = \ln(x) + 3$



$f(x) = \ln(x) - 3$

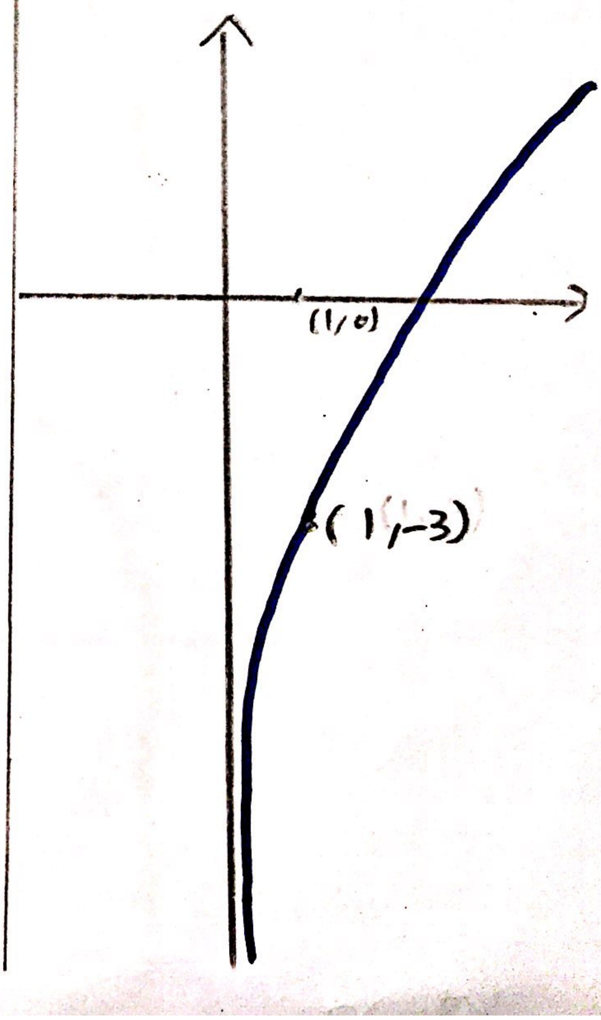
H.W





Domain: $(0, \infty)$

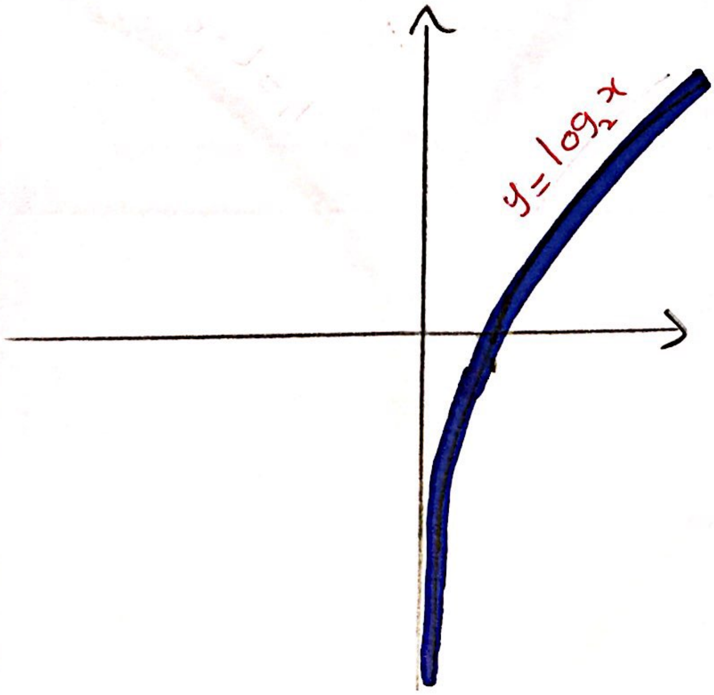
Range: $\mathbb{R}$

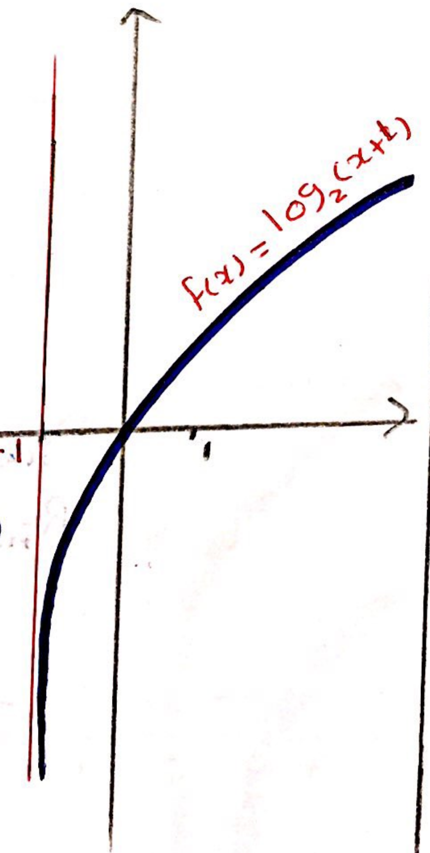


⑦ $f(x) = \log_2(x-1)$

⑧ $f(x) = \log_2(x+1)$

$f(x) = \log_2 (x+1)$

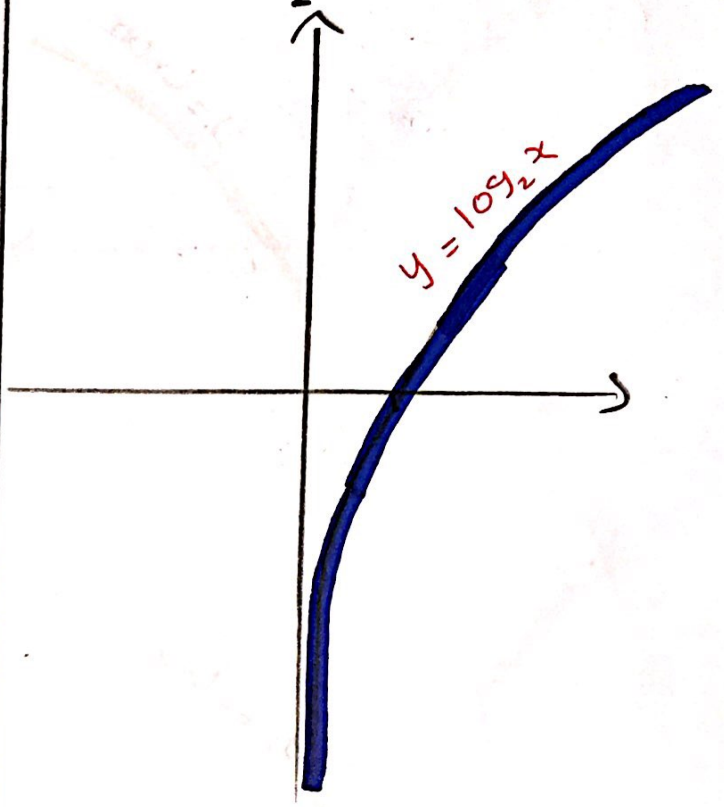


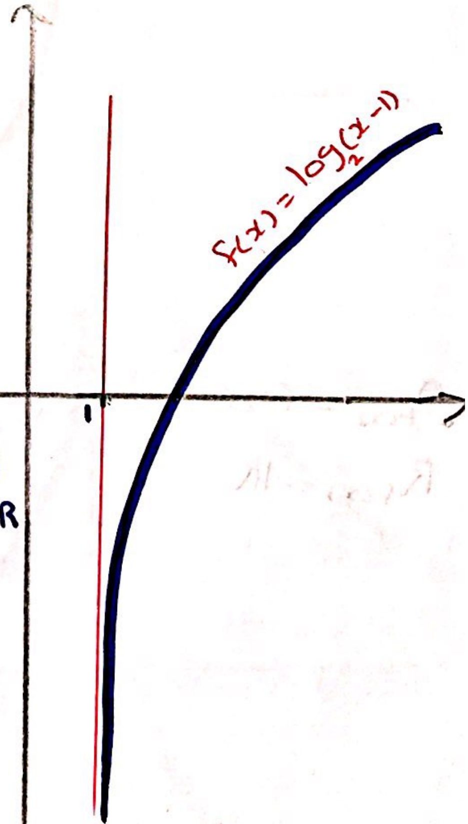


Domain: $(-1, \infty)$

Range: $\mathbb{R}$

$f(x) = \log_2(x-1)$





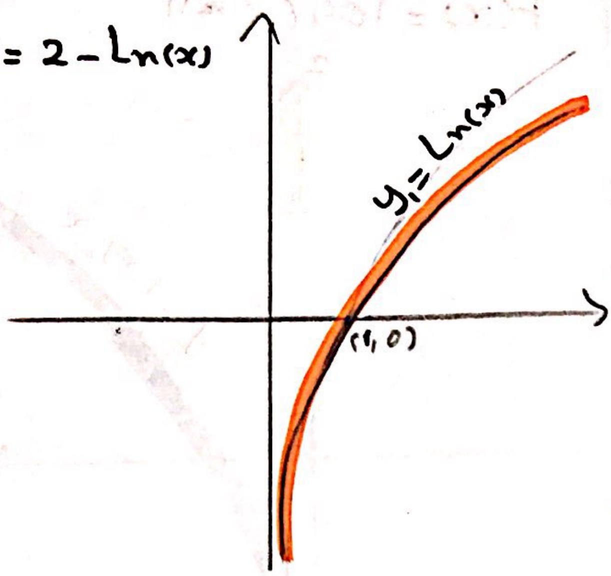
Domain: $(1, \infty)$

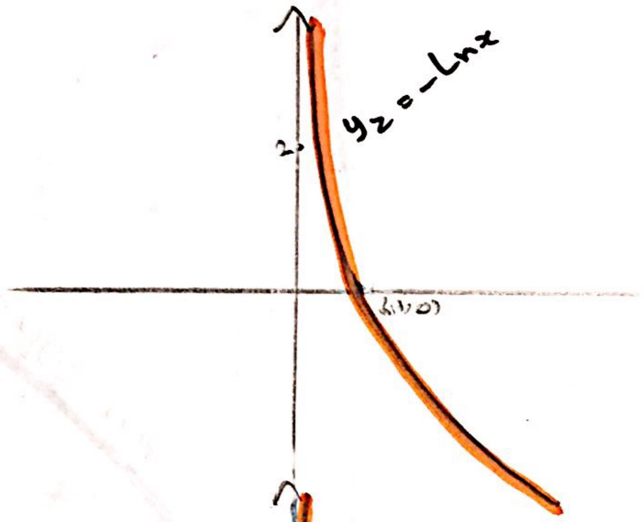
Range: $\mathbb{R}$

⑨ $f(x) = 2 - \ln(x)$

⑩ $f(x) = 2 + \ln(-x)$

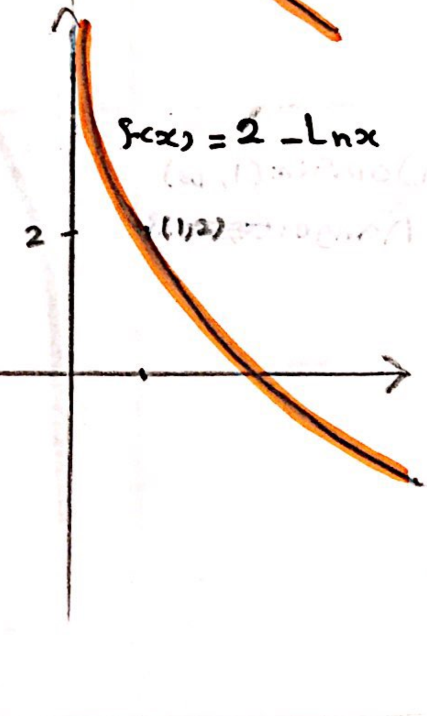
$f(x) = 2 - \ln(x)$



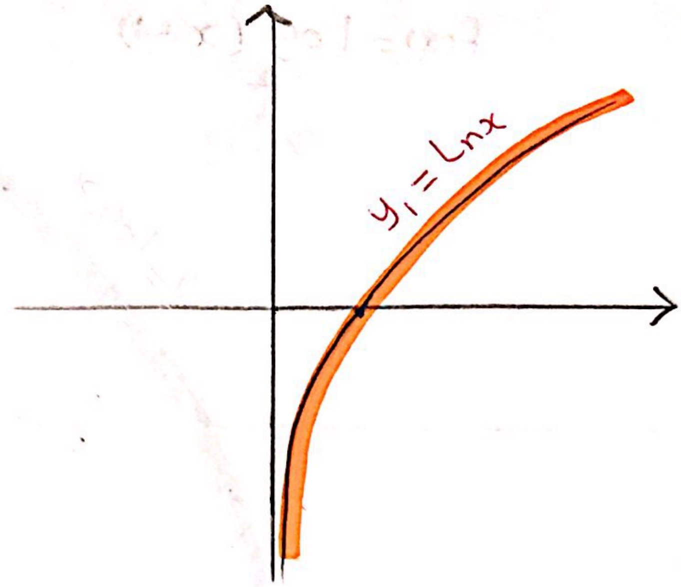


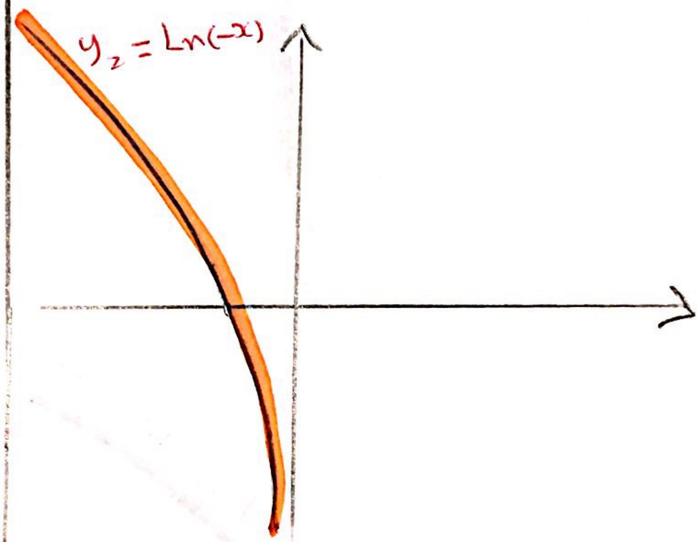
$D_{f(x)} = (0, \infty)$

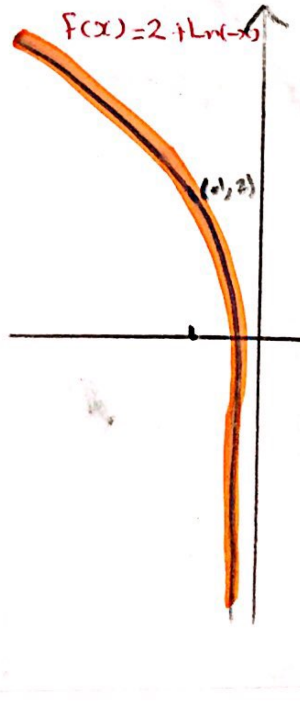
$R_{f(x)} = \mathbb{R}$



$f(x) = 2 + \ln(-x)$



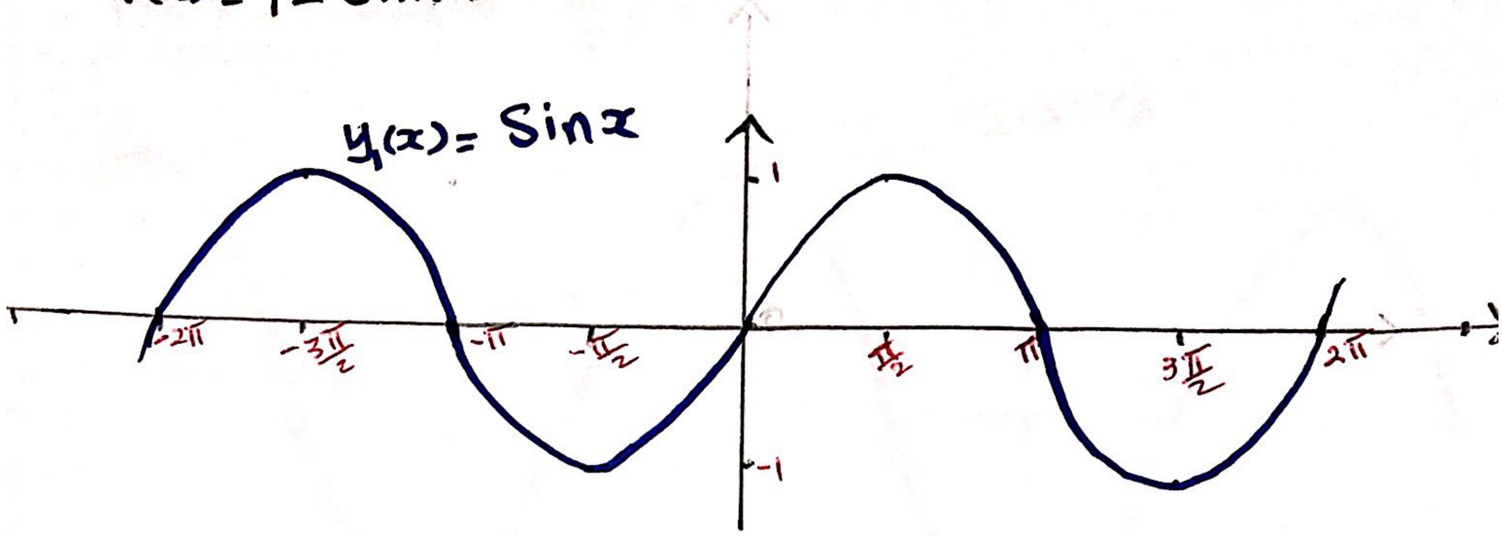


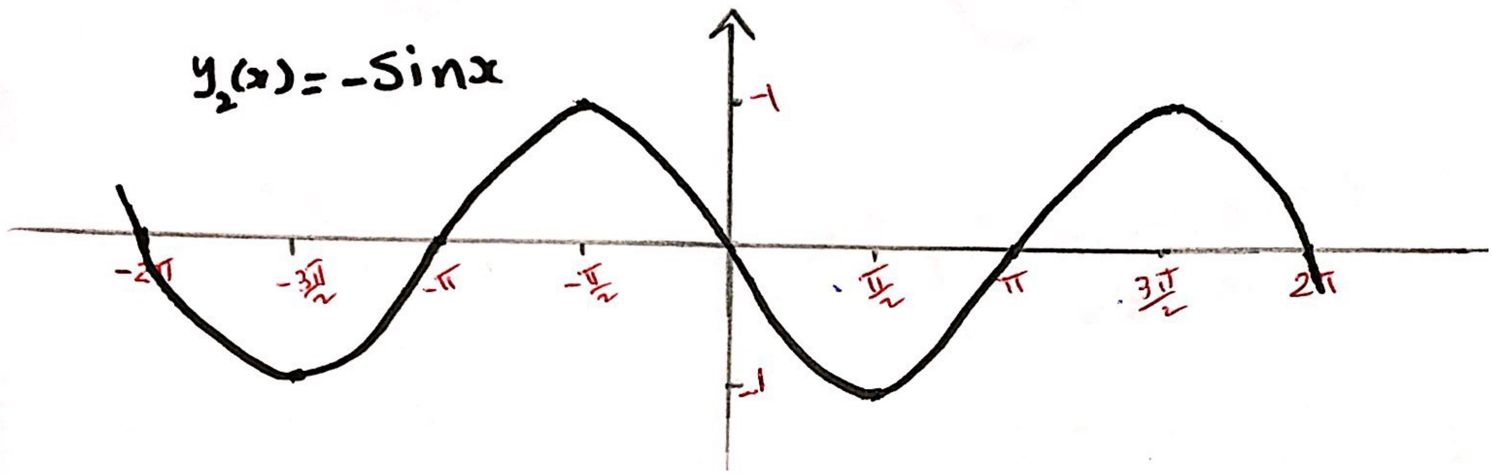


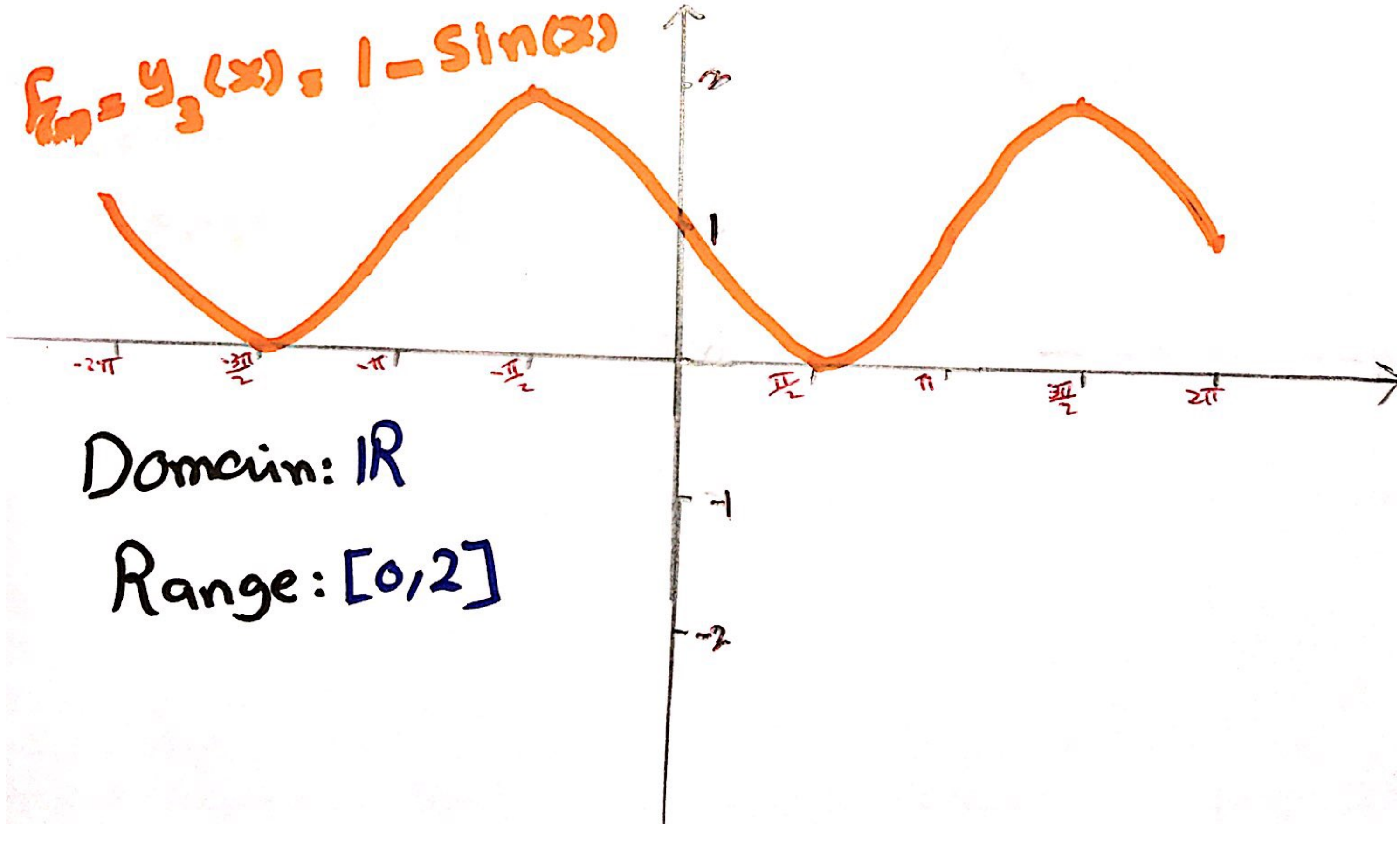
$D_{f(x)} = (-\infty, 0)$

$R_{f(x)} = \mathbb{R}$

$f(x) = 1 - \sin(x)$







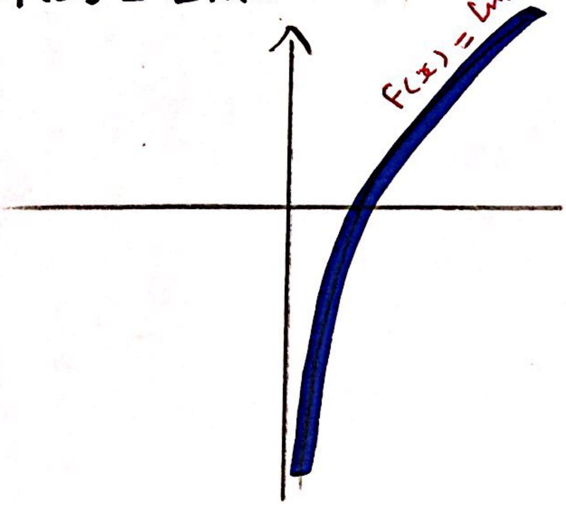
Domain: $\mathbb{R}$

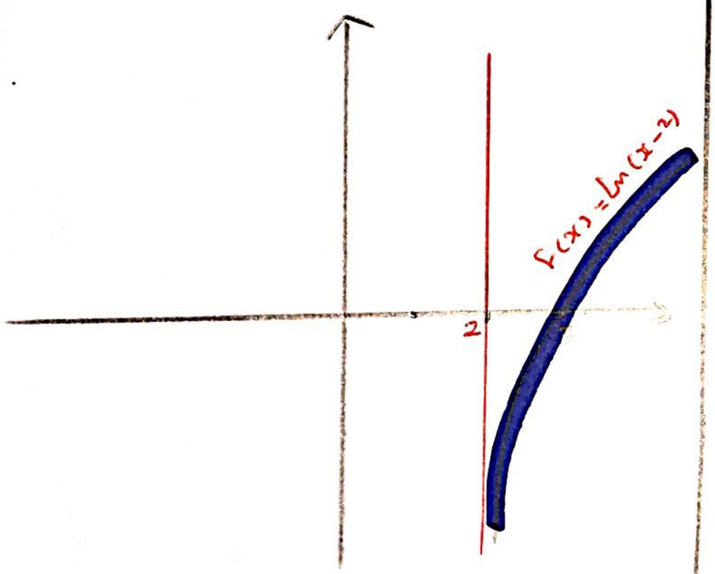
Range: $[0,2]$

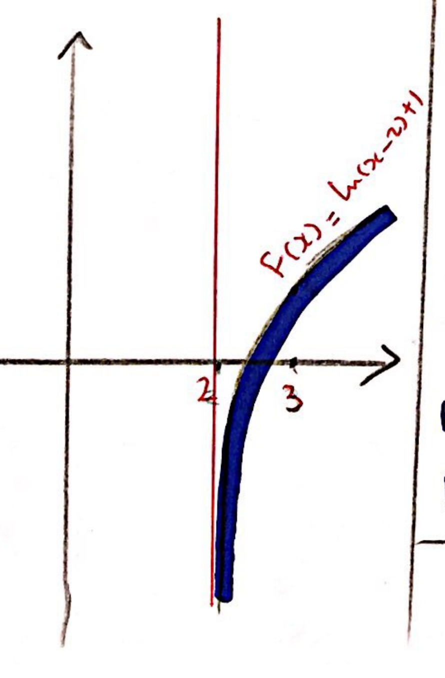
(11) $f(x) = \ln(x-2) + 1$

(12) $f(x) = 1 - \ln(x-2)$

$f(x) = \ln(x-2)+1$



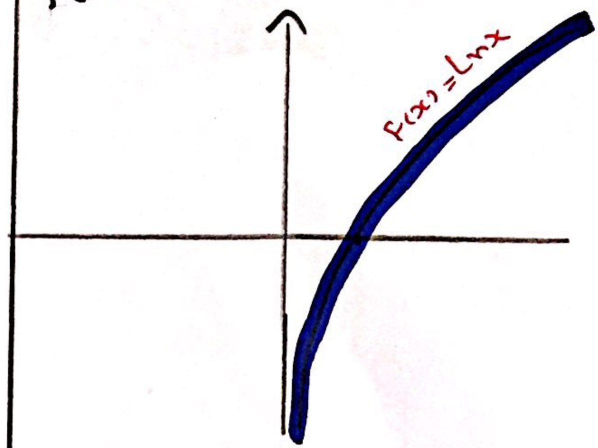


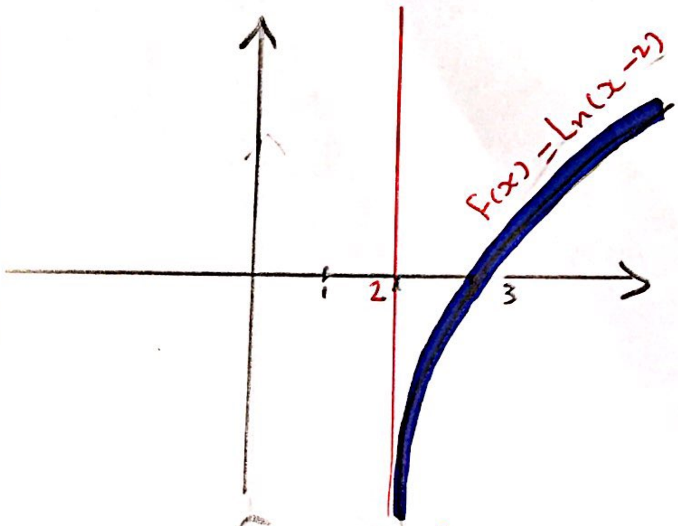


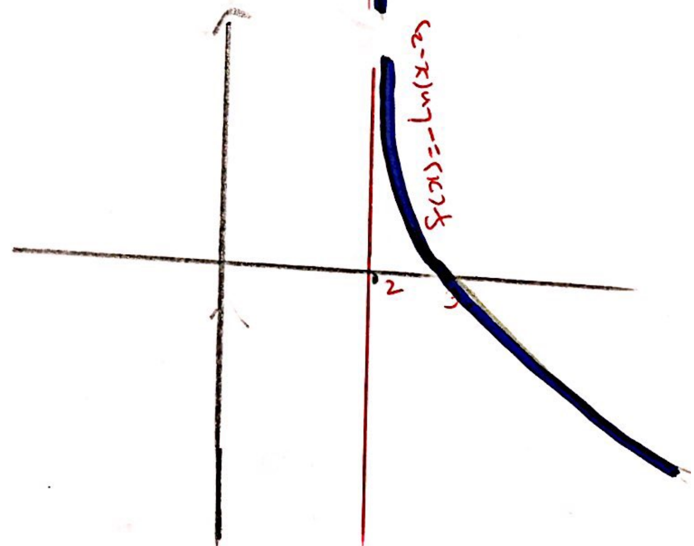
$D_{f(x)} = (2, \infty)$

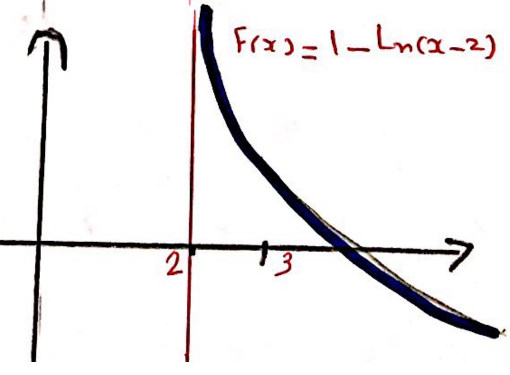
$R_{f(x)} = \mathbb{R}$

$f(x) = 1 - \ln(x-2)$





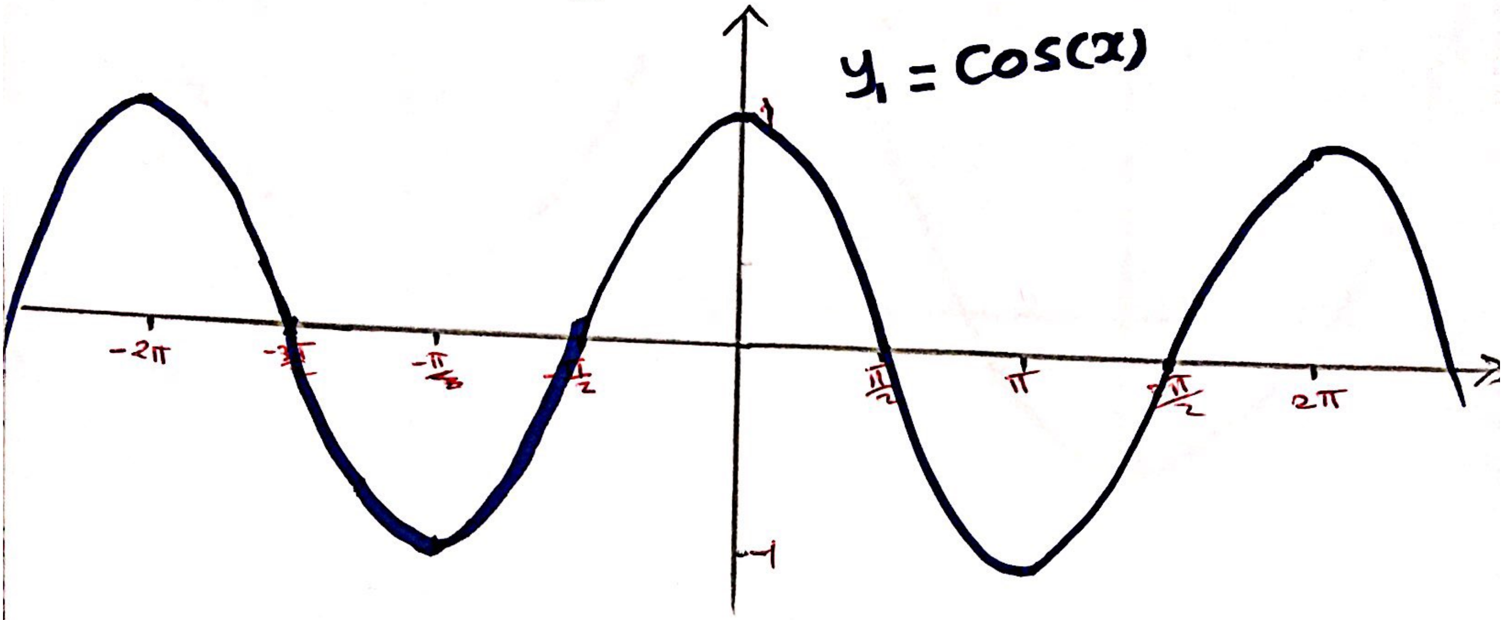


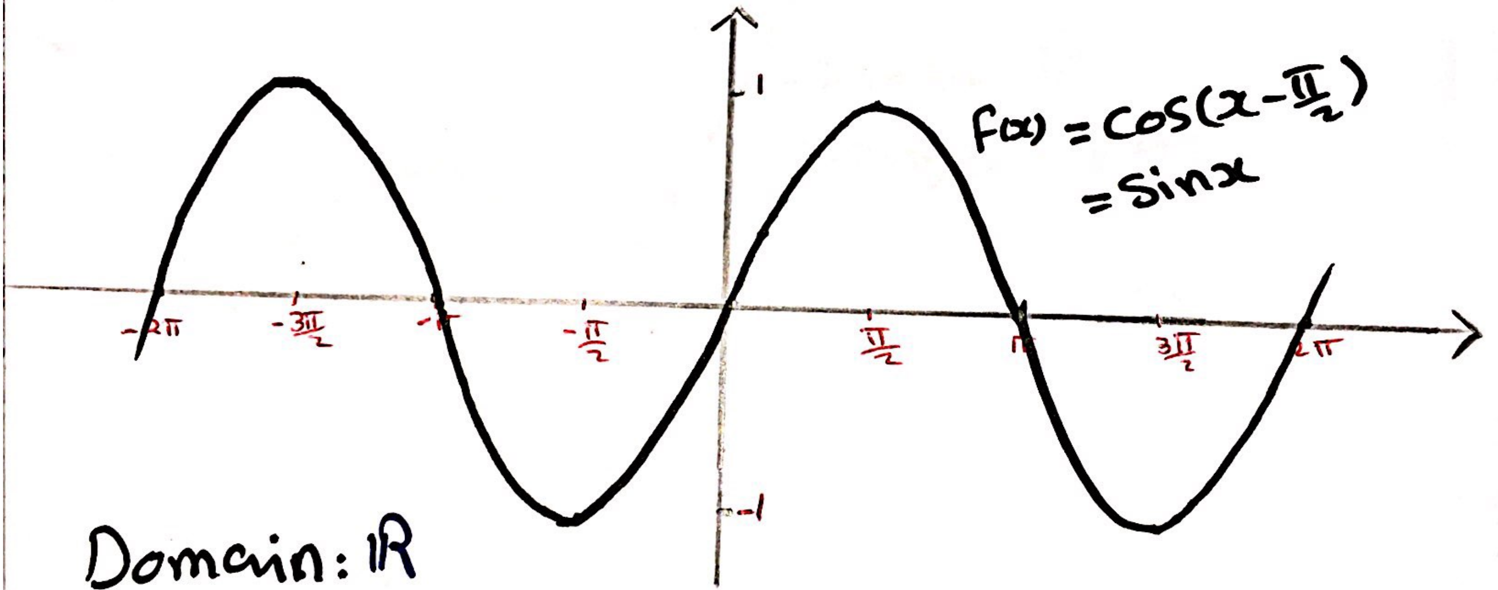


$D_{f(x)} = (2, \infty)$

$R_{f(x)} = \mathbb{R}$

$f(x) = \cos(x - \frac{\pi}{2})$

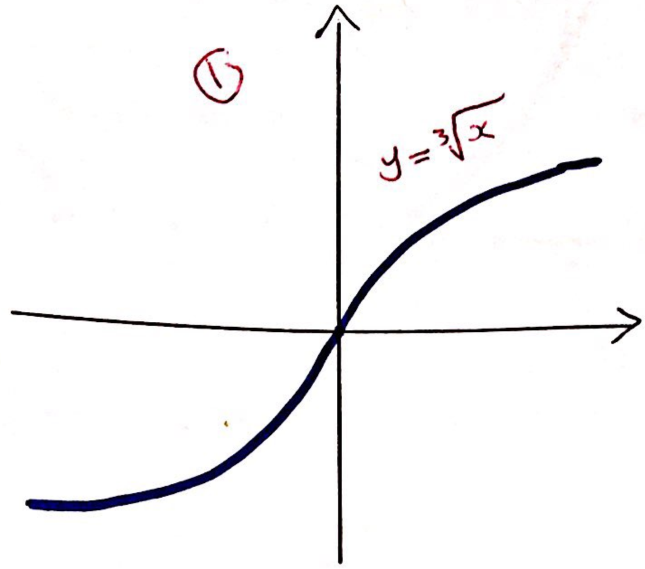


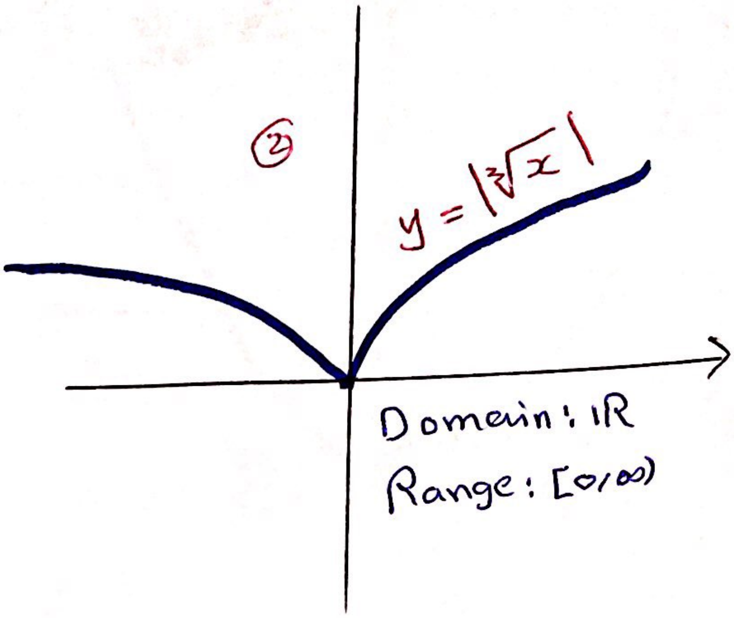


Domain: $\mathbb{R}$

Range: $[-1, 1]$

$f(x) = |\sqrt[3]{x}|$

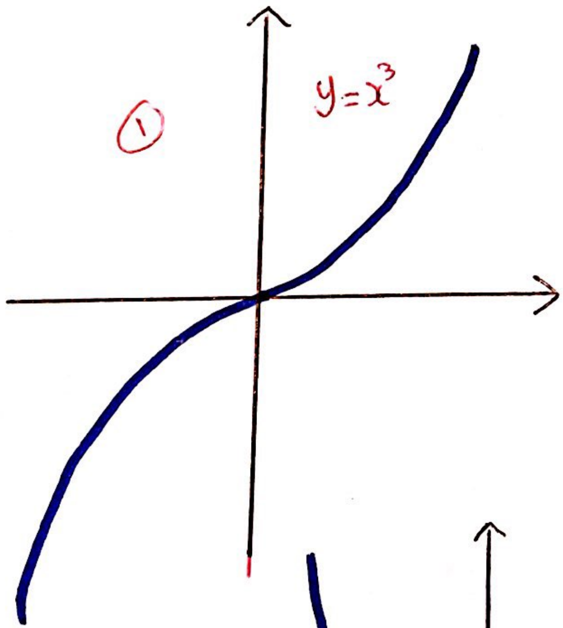


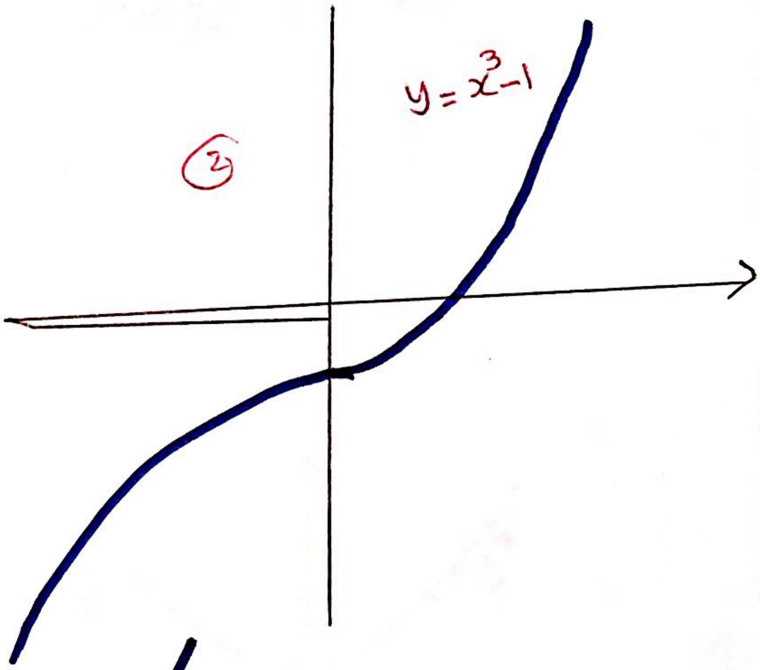


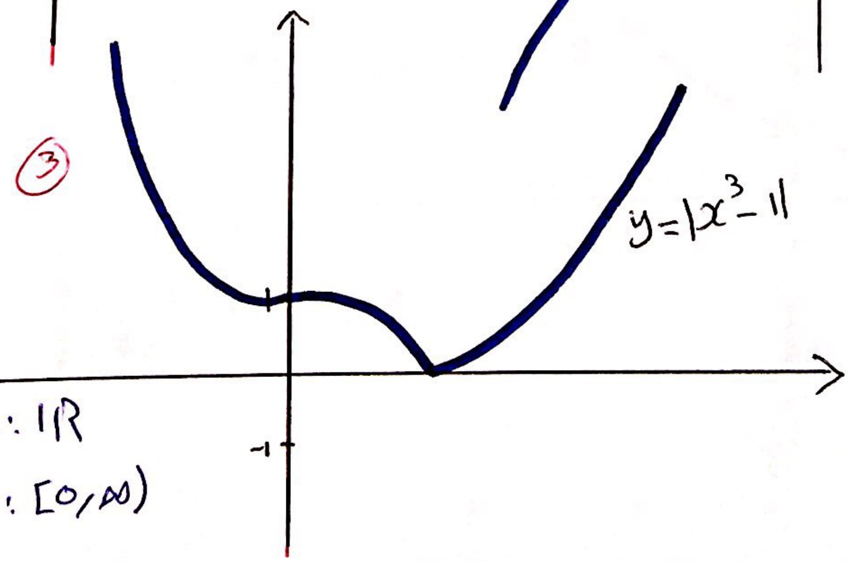
Domain: $\mathbb{R}$

Range: $[0, \infty)$

$f(x) = |x^3 - 1|$



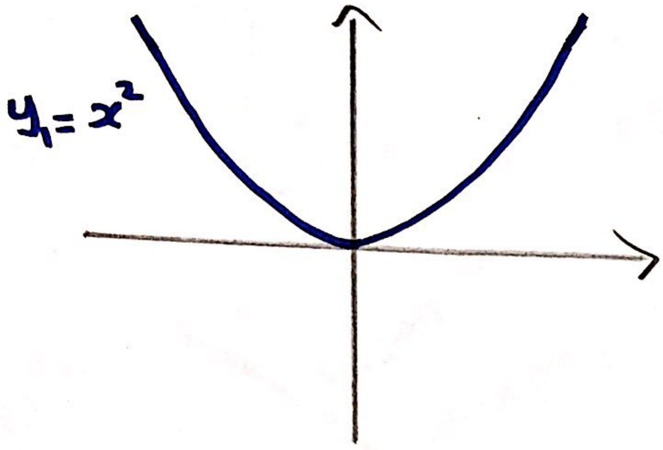


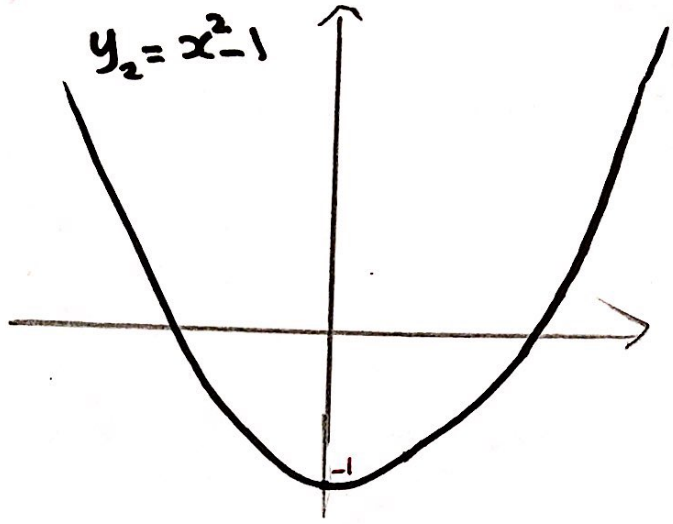


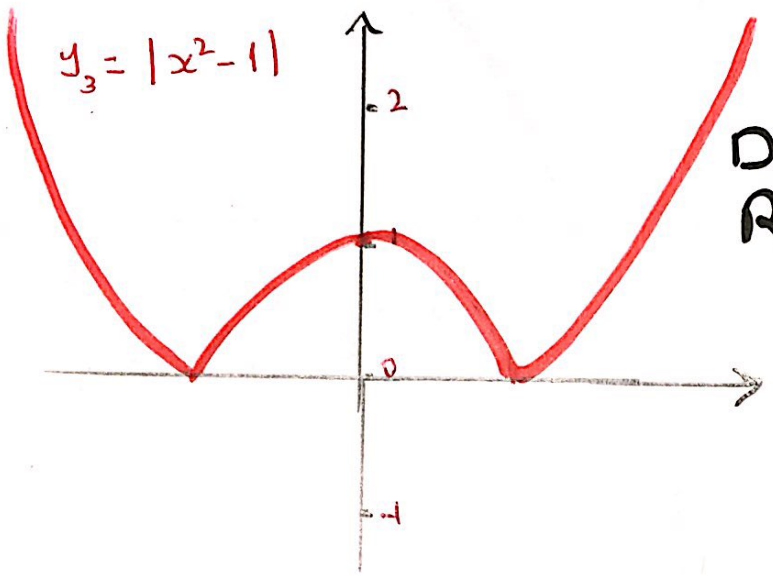
Domain: $\mathbb{R}$

Range: $[0, \infty)$

$$f(x) = |x^2 - 1|$$







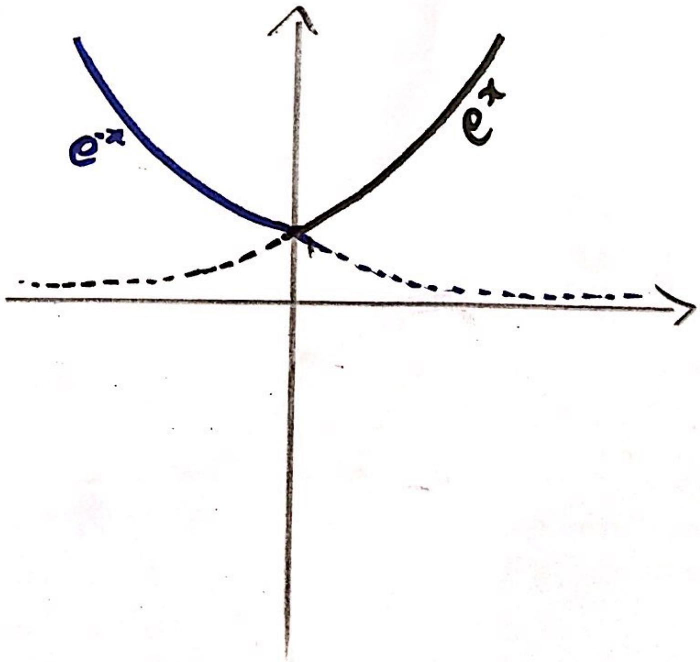
Domain: $\mathbb{R}$

Range: $[0, \infty)$

$$f(x) = e^{|x|}$$

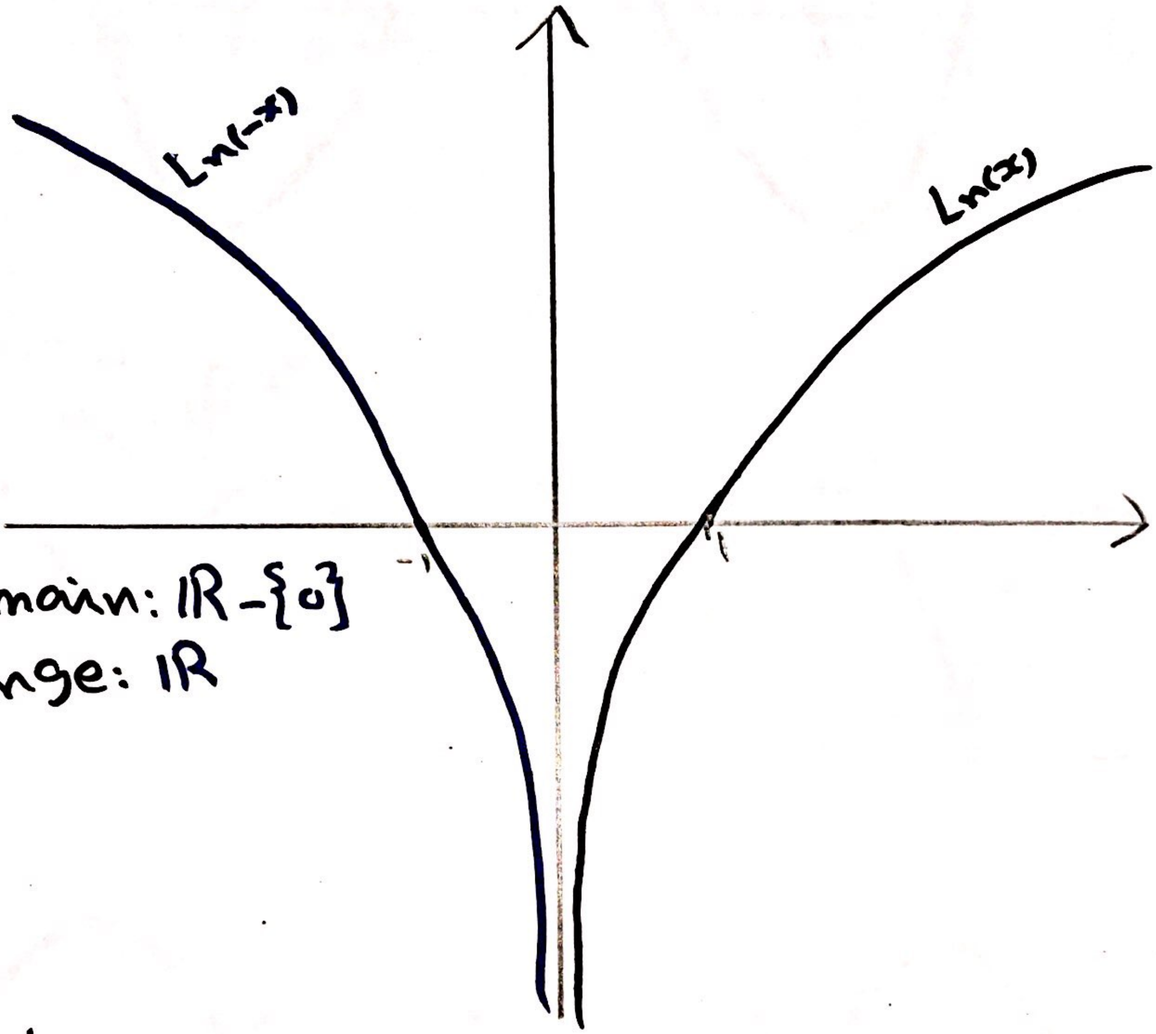
$$f(x) = \begin{cases} e^{x} & \text{if } x \geq 0 \\ e^{-x} & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} e^{x} & \text{if } x \geq 0 \\ \left(\frac{1}{e}\right)^{x} & \text{if } x < 0 \end{cases}$$



$f(x) = \ln|x|$

$$= \begin{cases} \ln x & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0 \end{cases}$$

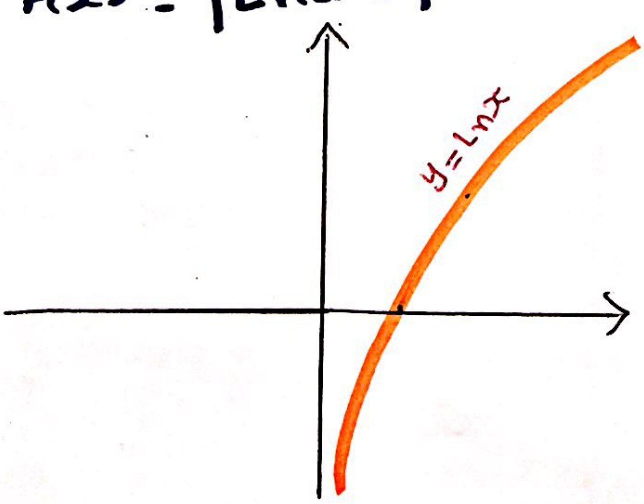


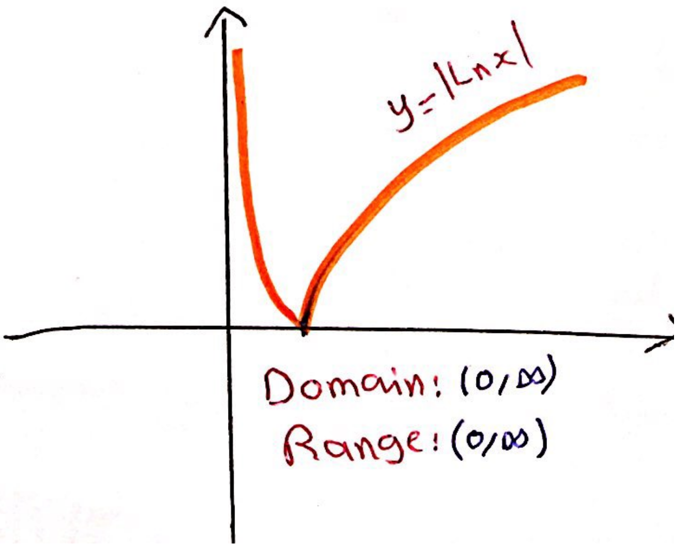
Domain: $\mathbb{R} - \{0\}$

Range: $\mathbb{R}$

H.W

$f(x) = |\ln(x)|$

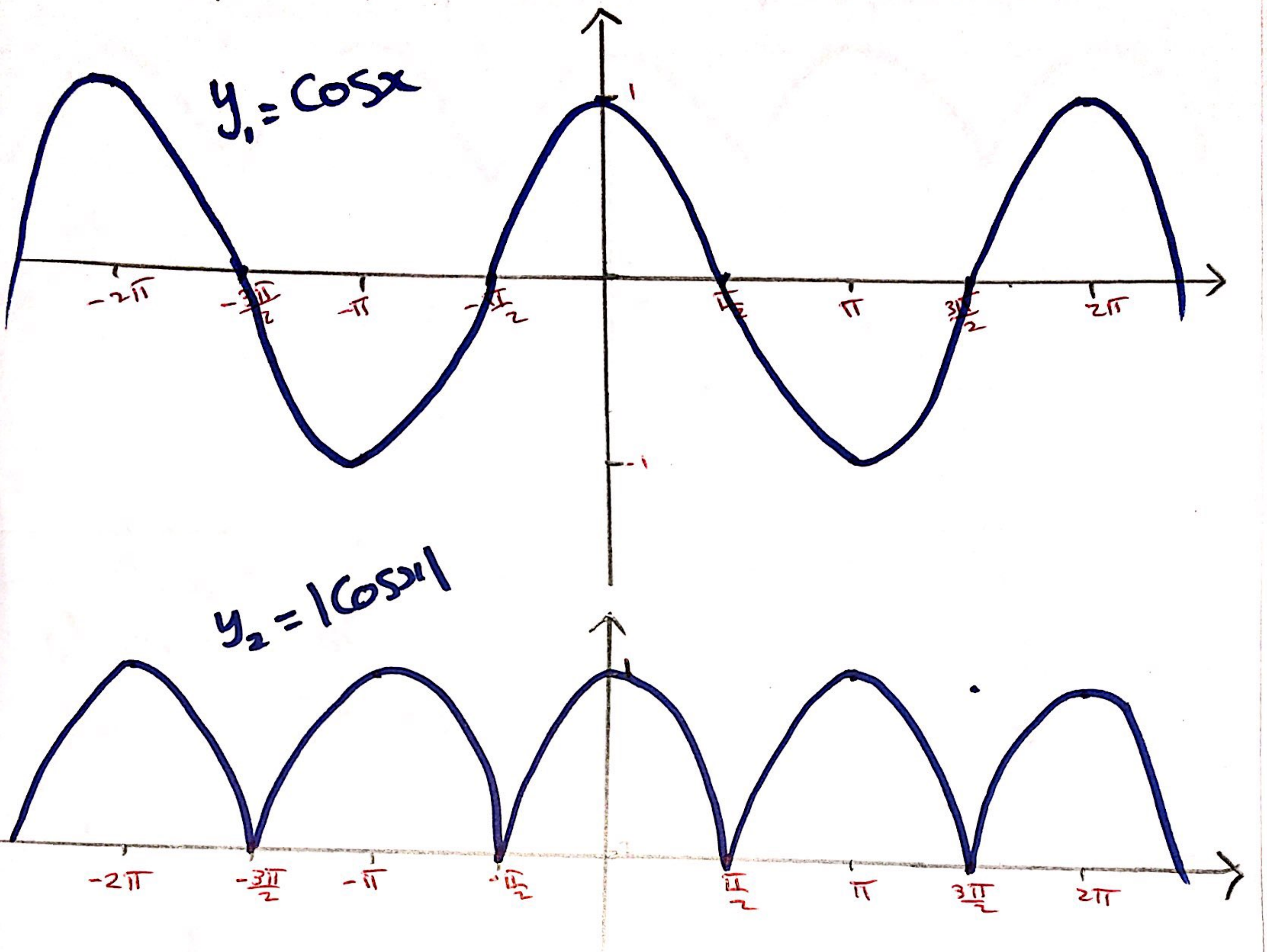




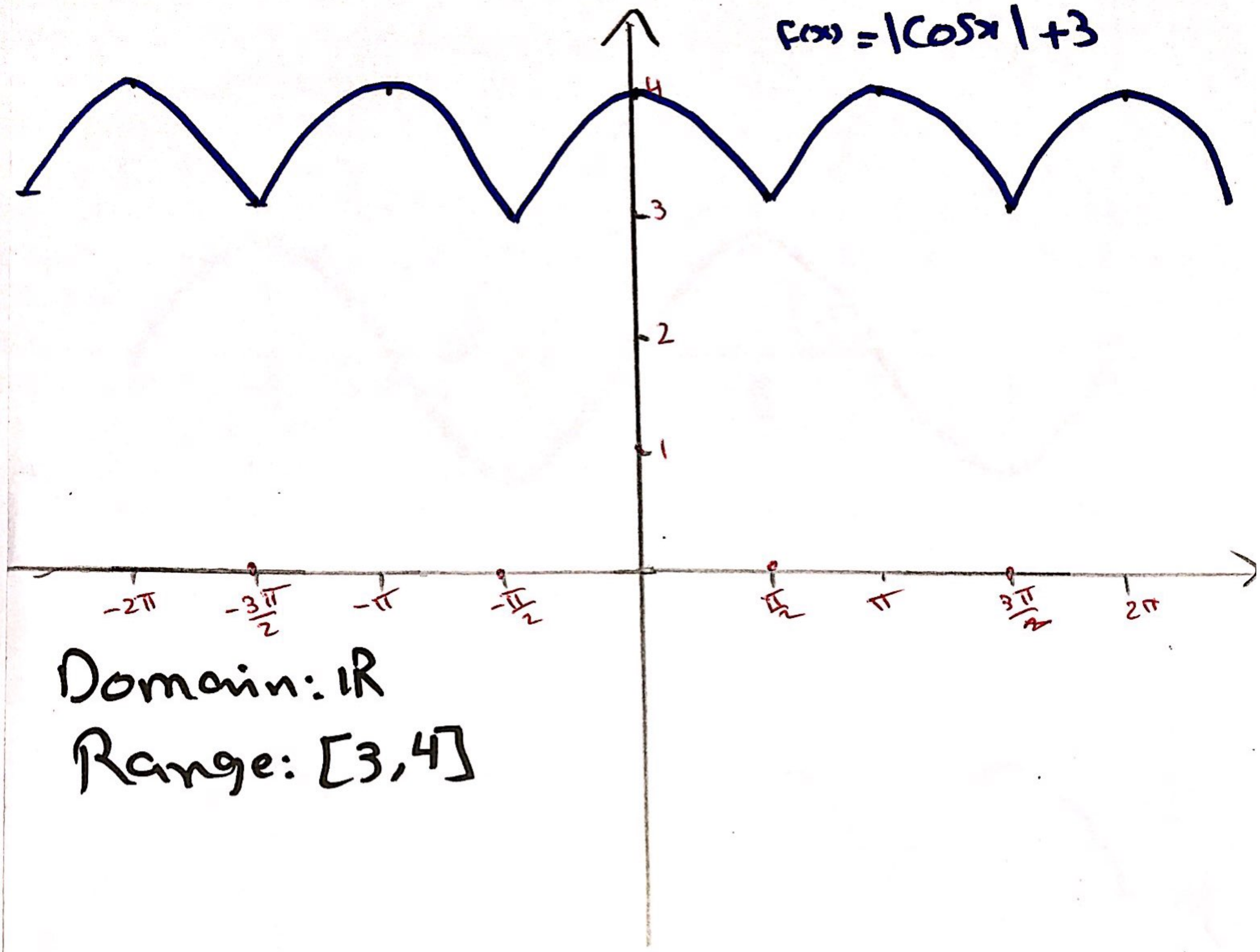
Domain: $(0, \infty)$

Range: $(0, \infty)$

$F(x) = |\cos x| + 3$



$y_1 = \cos x$
1
-1
-2π
$-\frac{3\pi}{2}$
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$y_2 = |\cos x|$
-2π
$-\frac{3\pi}{2}$
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π



Domain: $\mathbb{R}$

Range: $[3, 4]$

H.W

$f(x) = |\sin x| + 2$

$f(x) = \sin(x - \frac{\pi}{2})$

$f(x) = 3 + \cos x$

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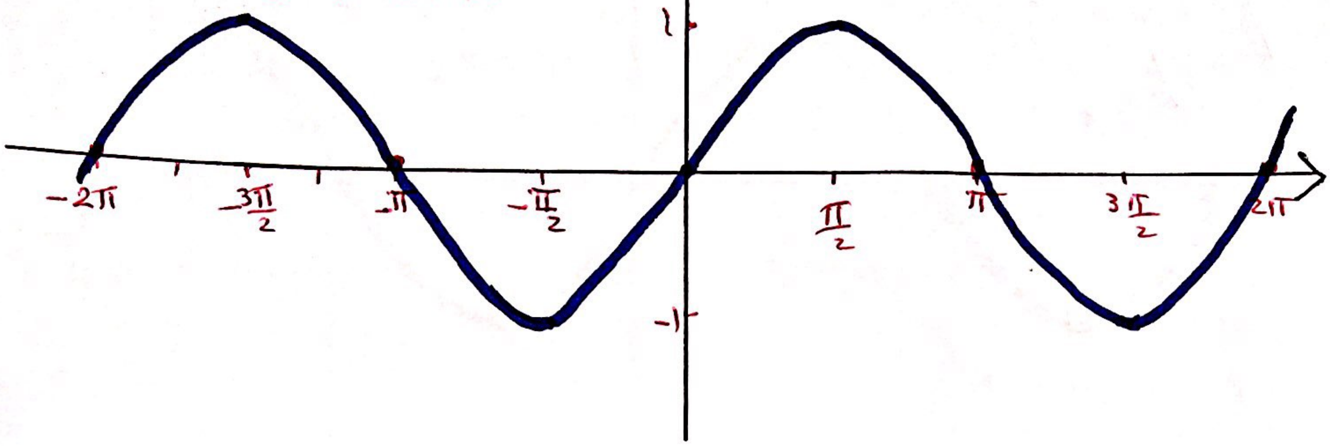
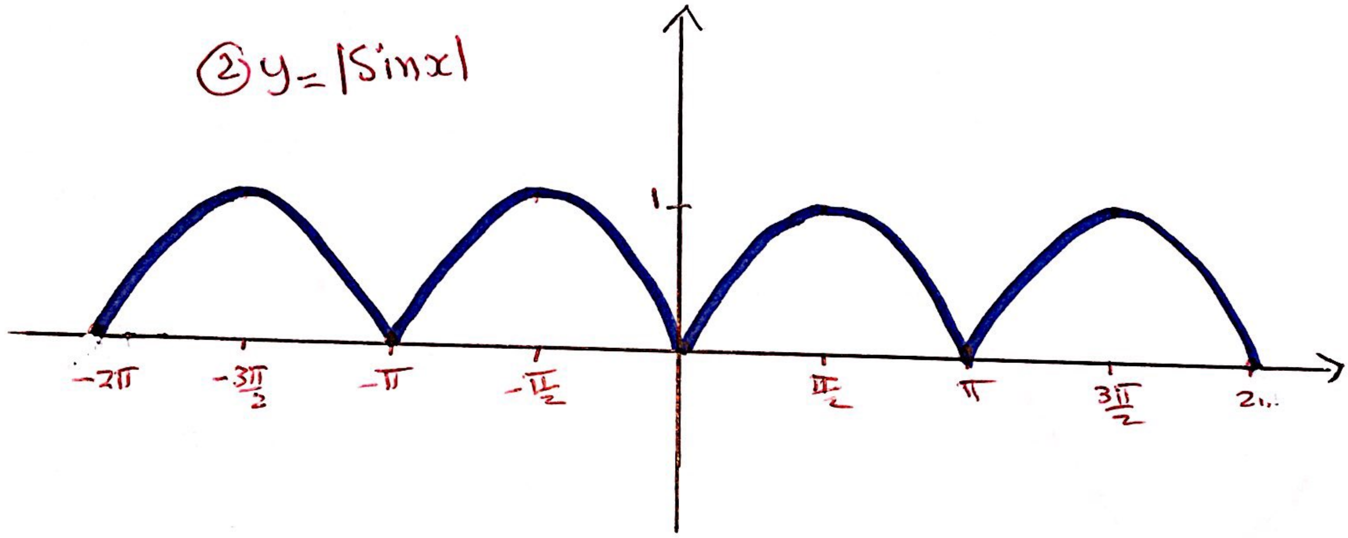
$f(x) = |\sin x| + 2$

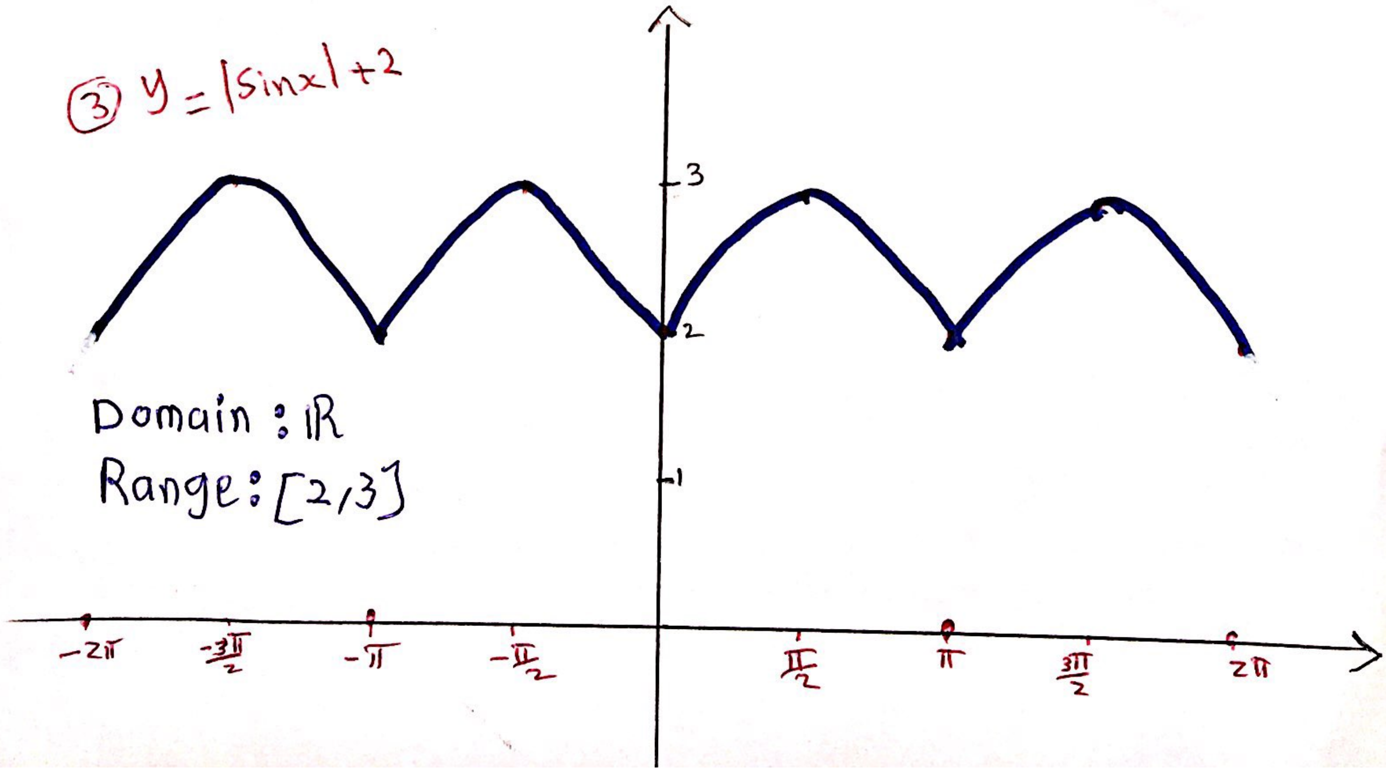
① $y = \sin x$



② $y = |\sin x|$



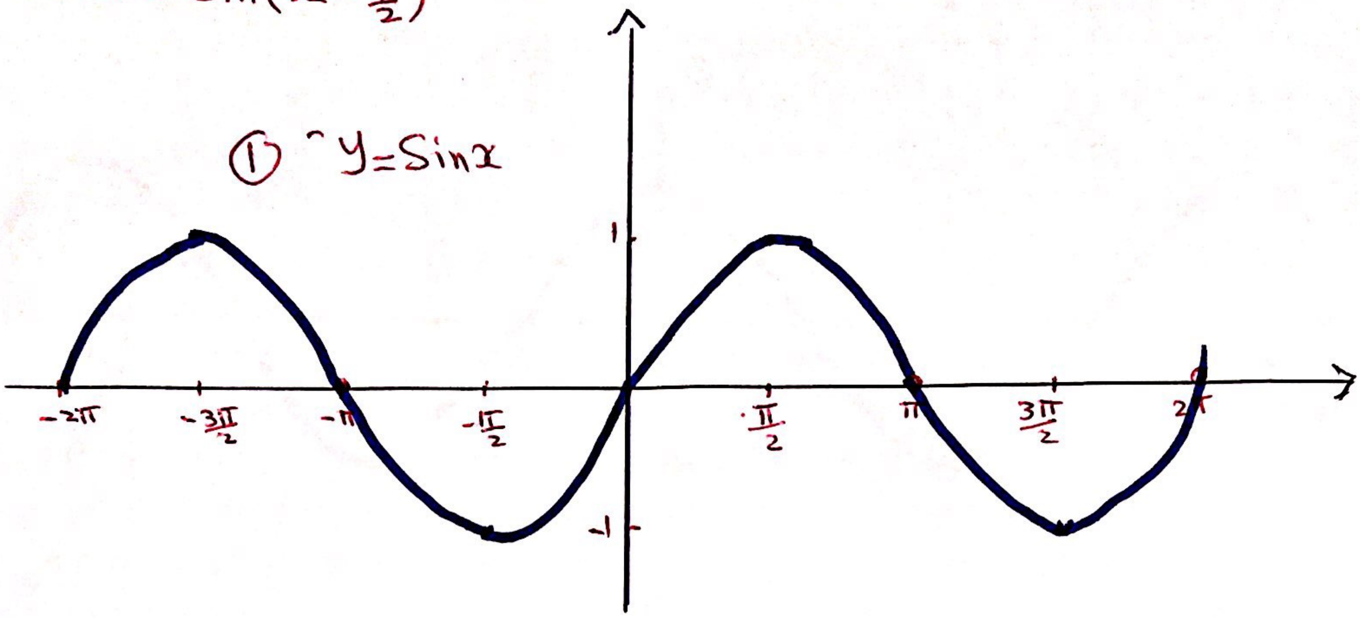
③ $y = |\sin x| + 2$

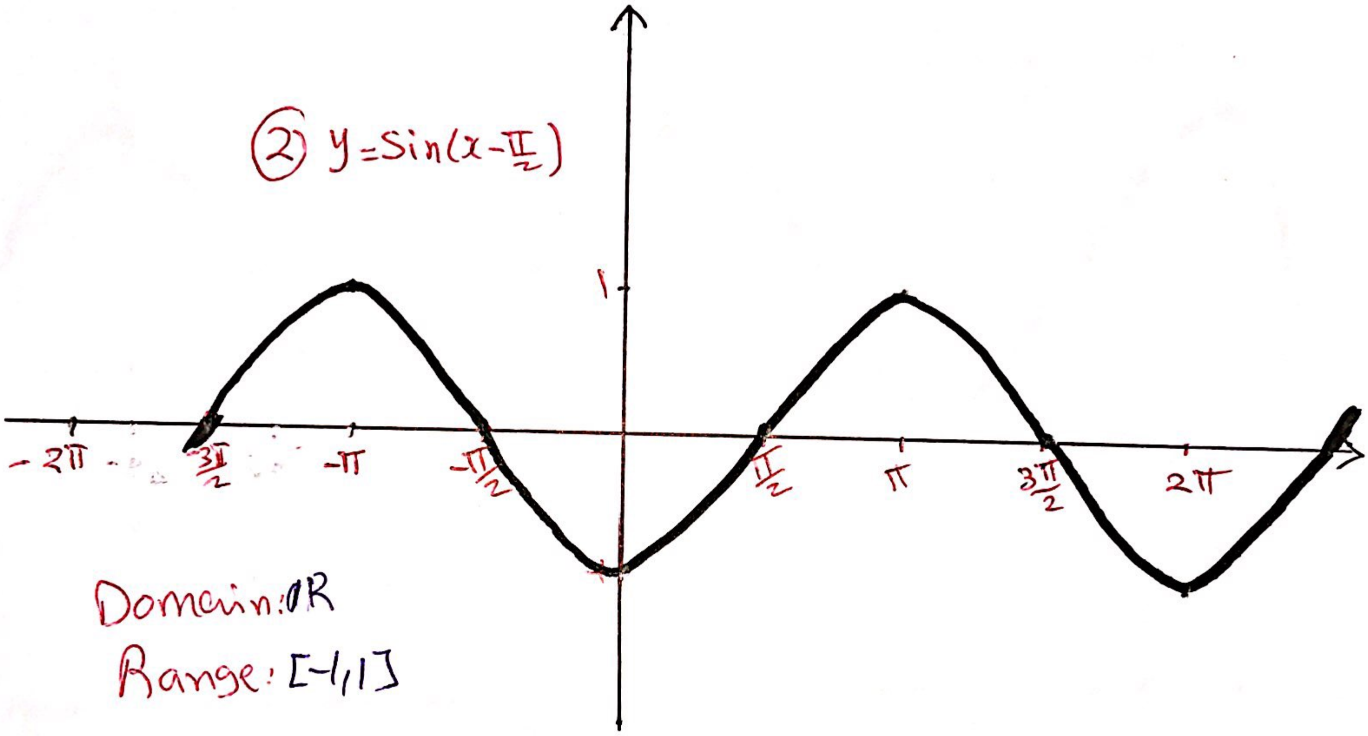


Domain : $\mathbb{R}$

Range : $[2, 3]$

$f(x) = \sin(x - \frac{\pi}{2})$

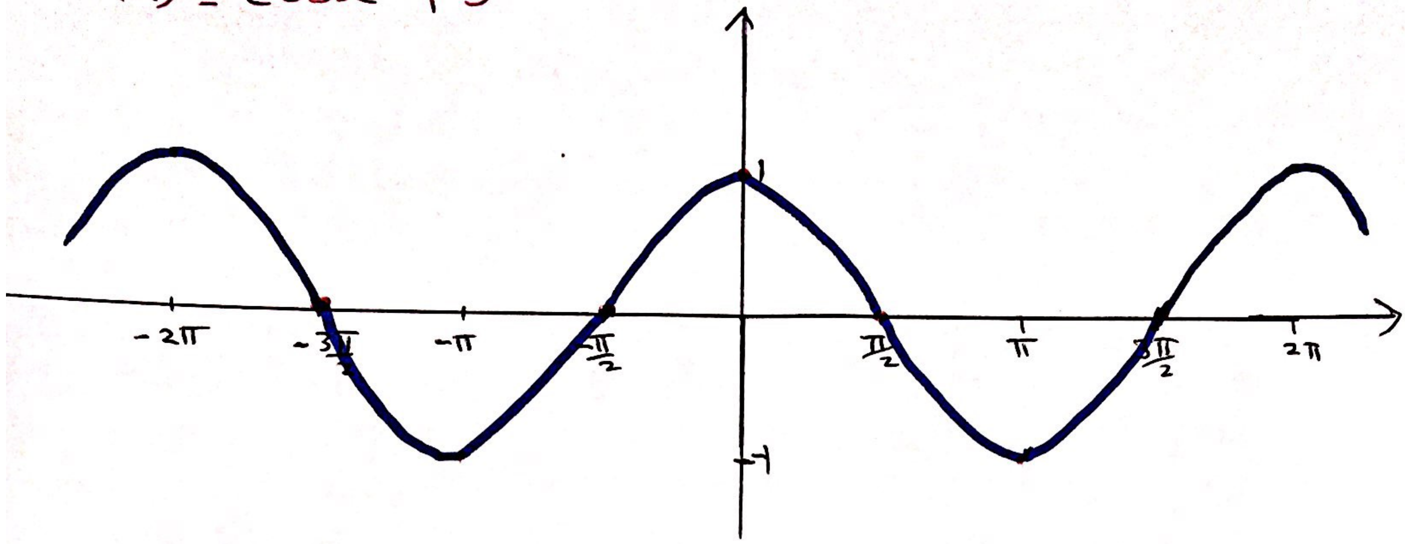


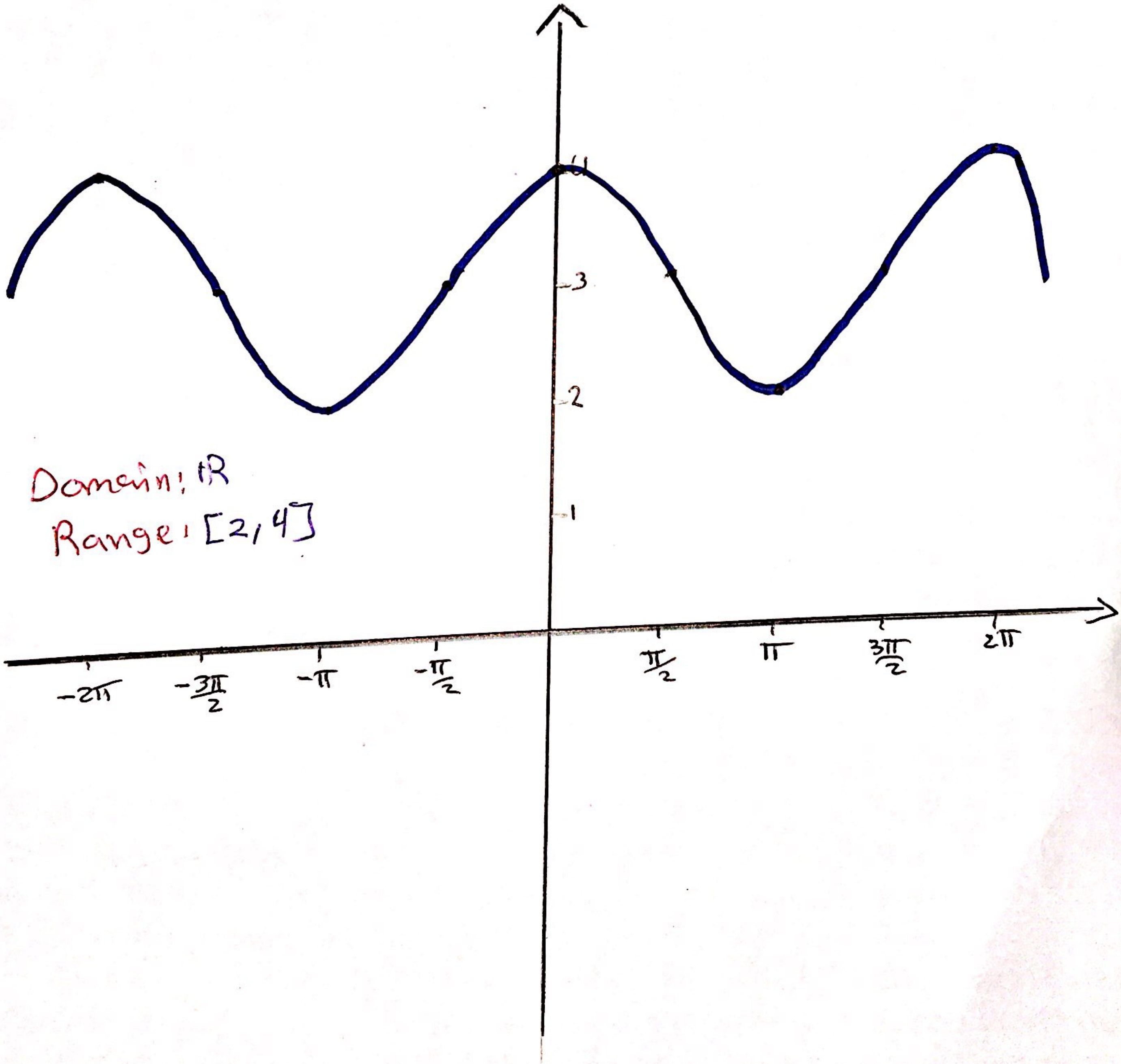


Domain: $\mathbb{R}$

Range: $[-1, 1]$

$f(x) = \cos x + 3$



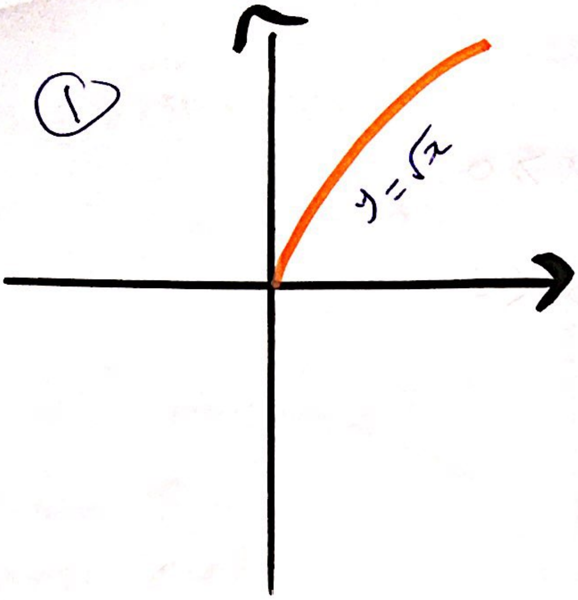


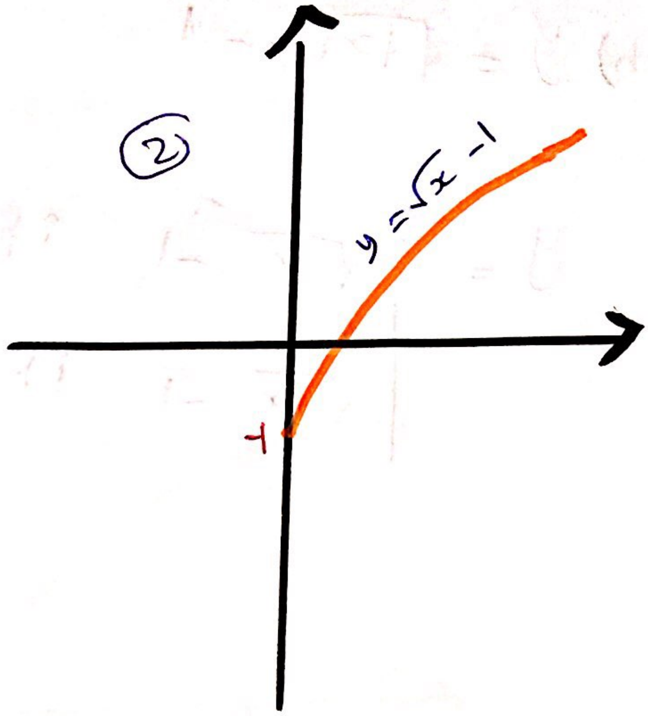
Domain: $\mathbb{R}$

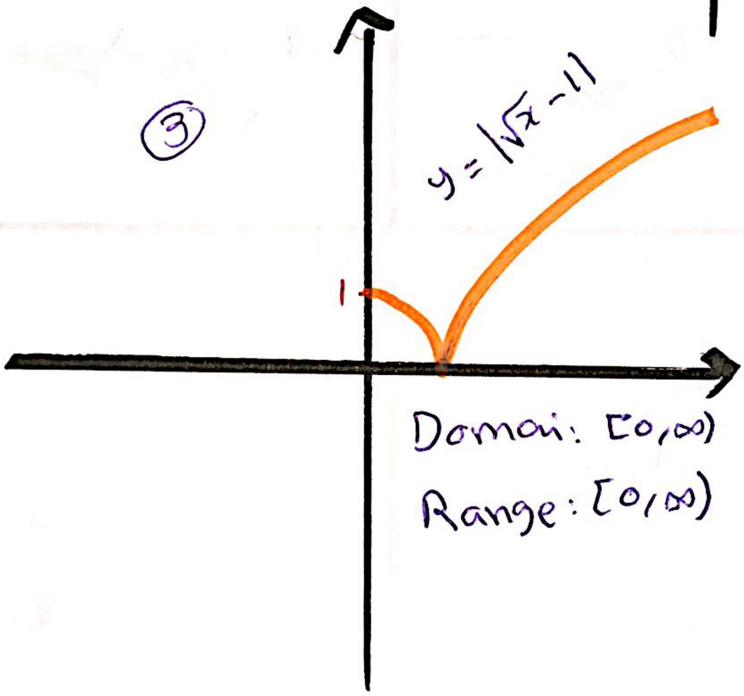
Range: $[2, 4]$

find the Domain and Range of the following:

a) $y = |\sqrt{x} - 1|$

① 

② 

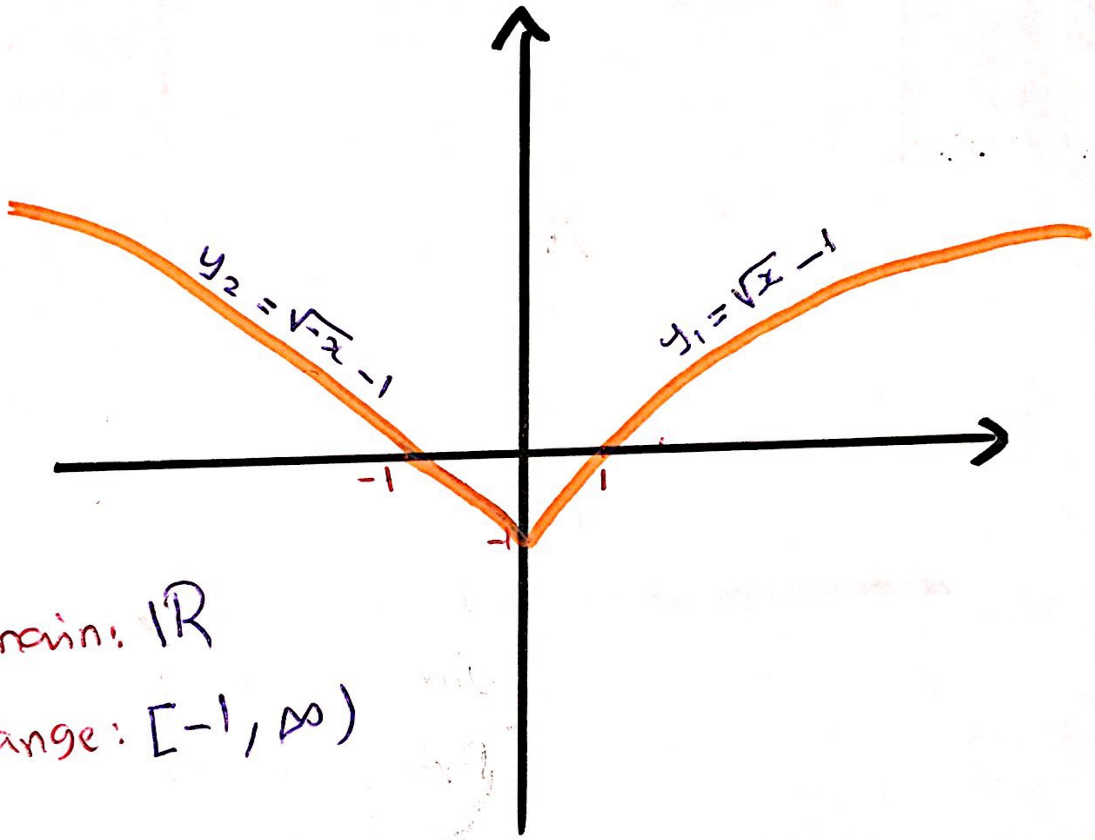
③ 

Domain: $[0, \infty)$

Range: $[0, \infty)$

b) $y = \sqrt{|x|} - 1$

$$y = \begin{cases} \sqrt{x} - 1 & \text{if } x \geq 0 \\ \sqrt{-x} - 1 & \text{if } x < 0 \end{cases}$$



Domain: $\mathbb{R}$

Range: $[-1, \infty)$

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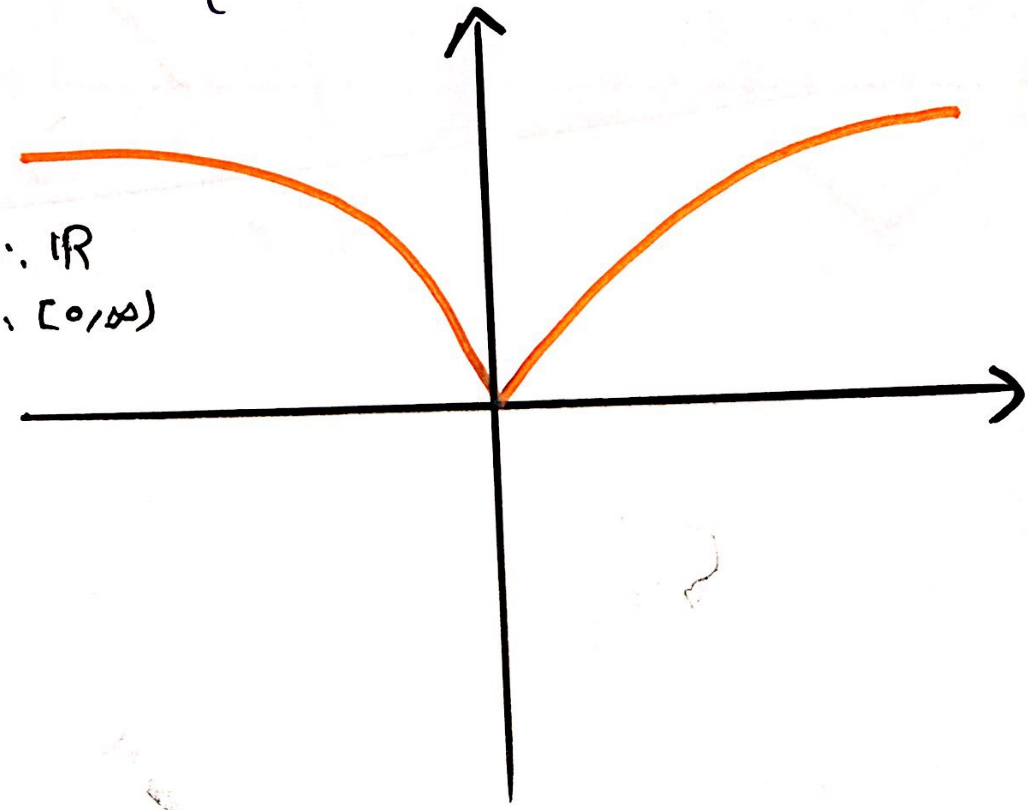
c) $y = \sqrt{|x|}$

$$y = \sqrt{|x|} = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ \sqrt{-x} & \text{if } x < 0 \end{cases}$$



Domain: $\mathbb{R}$

Range: $[0, \infty)$

d) $y = \sin|x|$

$$y = \sin|x| = \begin{cases} \sin x & \text{if } x \geq 0 \\ \sin(-x) & \text{if } x < 0 \end{cases}$$

$$= \begin{cases} \sin x & \text{if } x \geq 0 \\ -\sin x & \text{if } x < 0 \end{cases}$$

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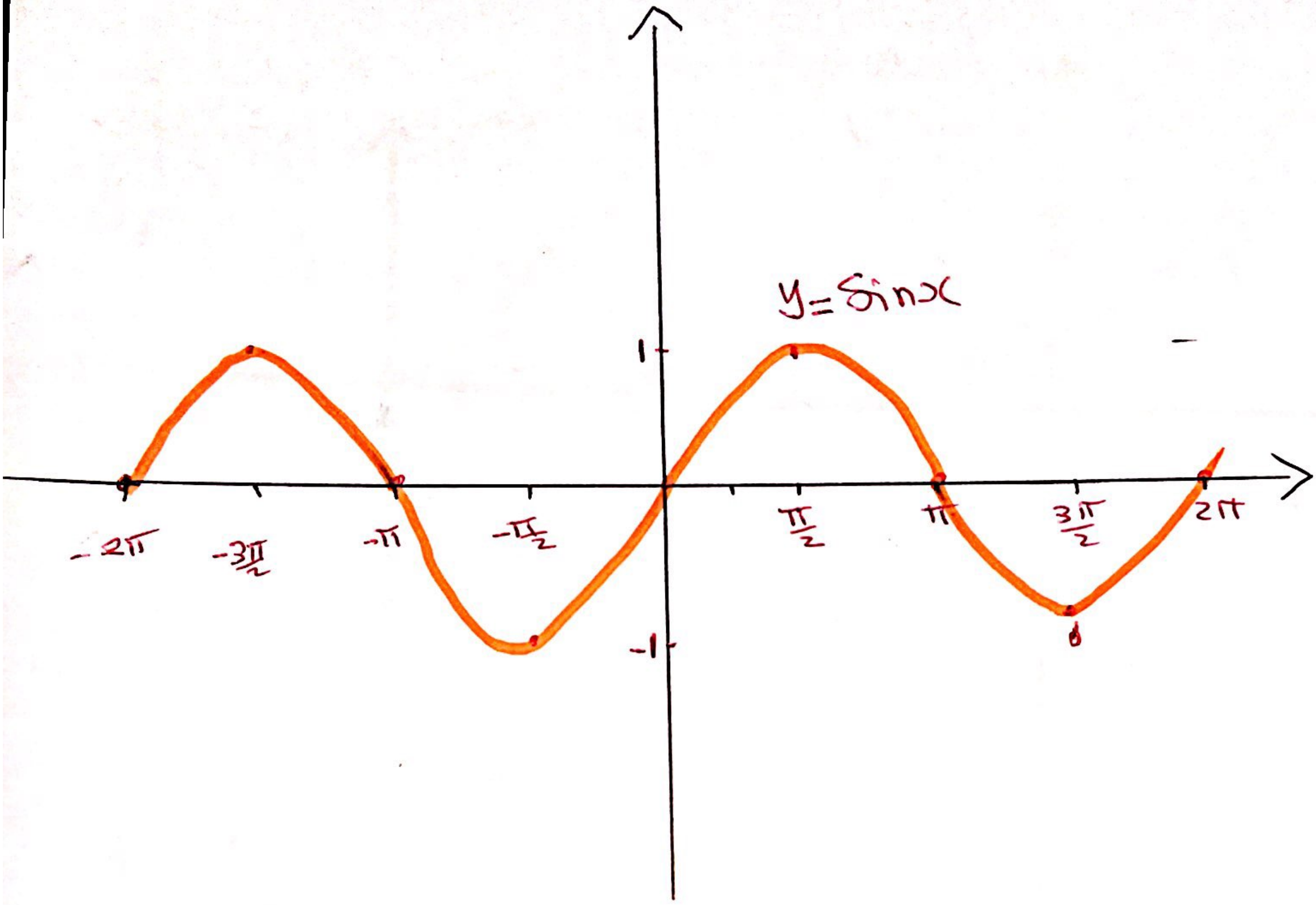
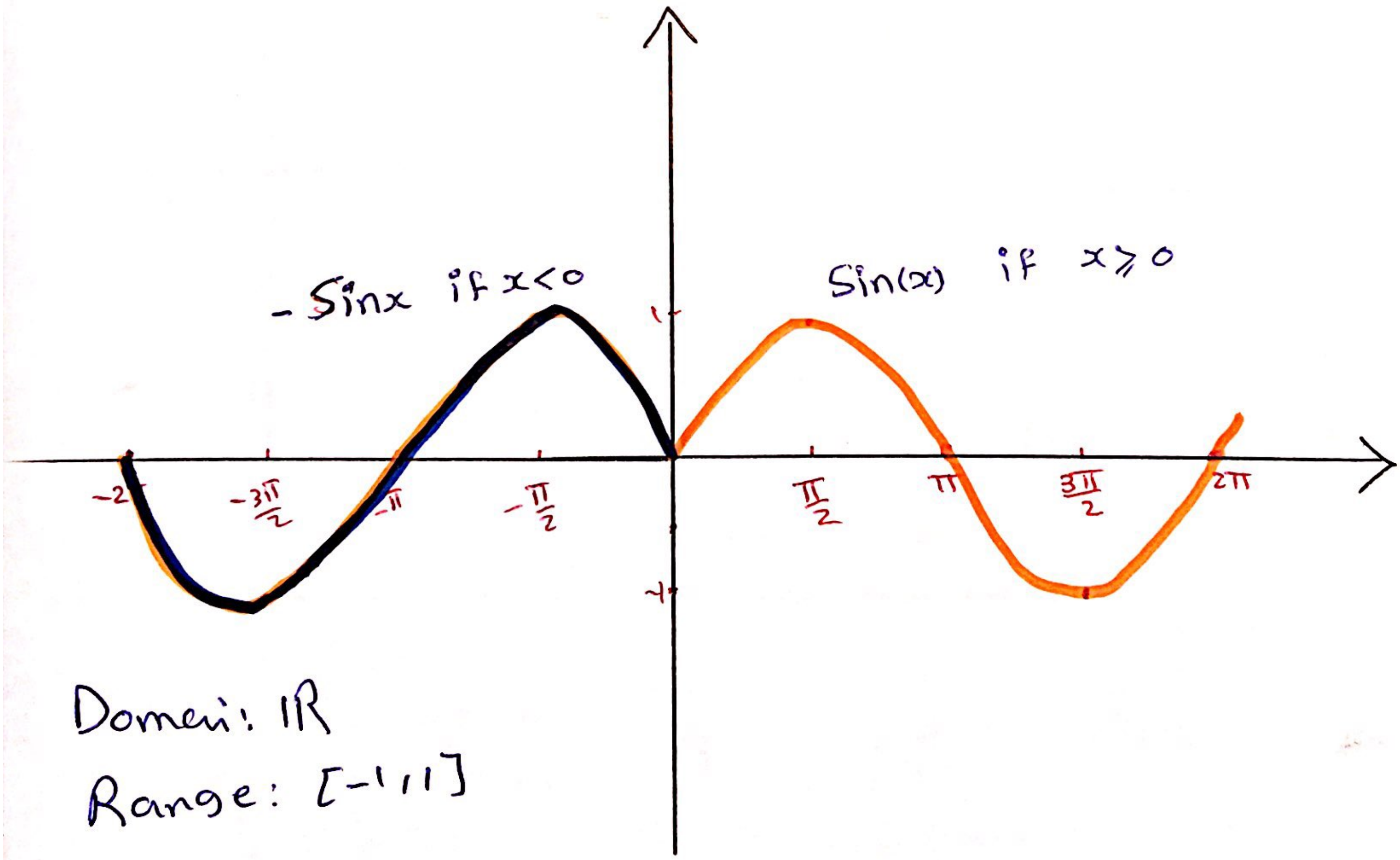
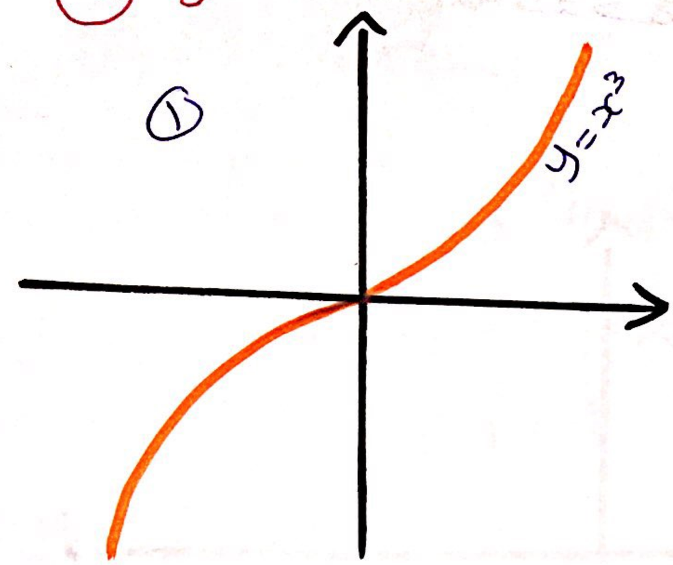


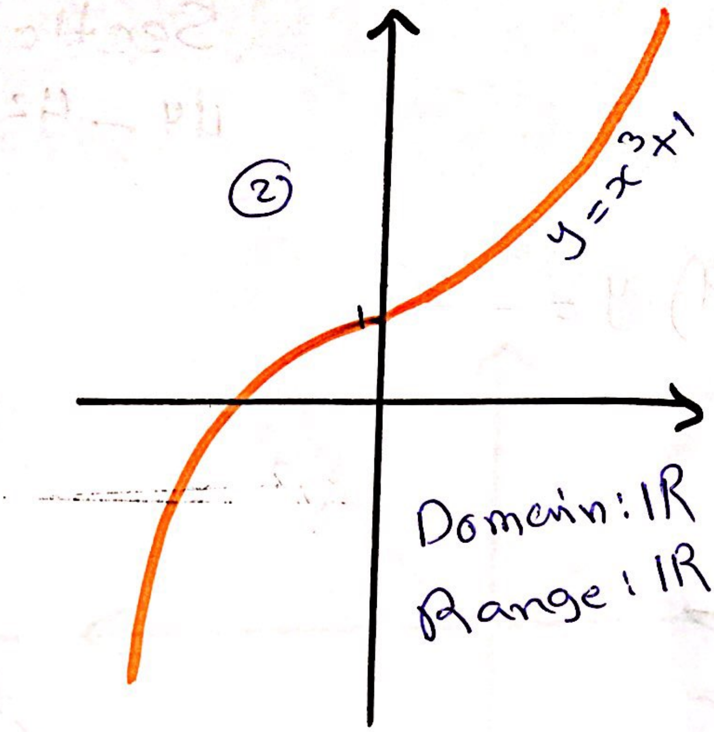


Domain: IR

Range: [-1,1]

(e) $y = x^3 + 1$

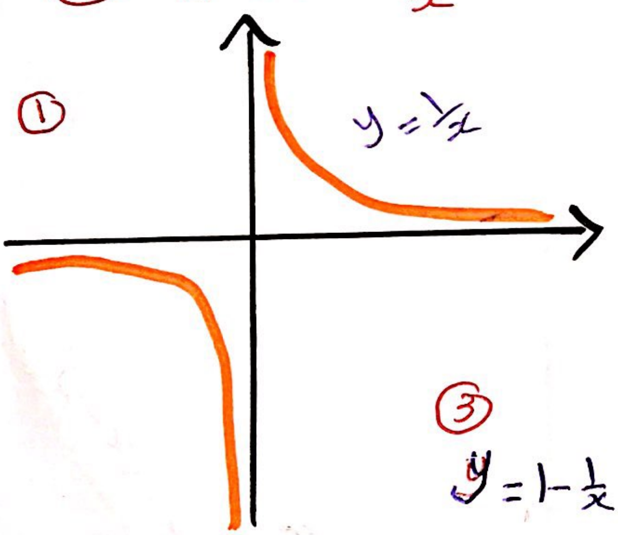


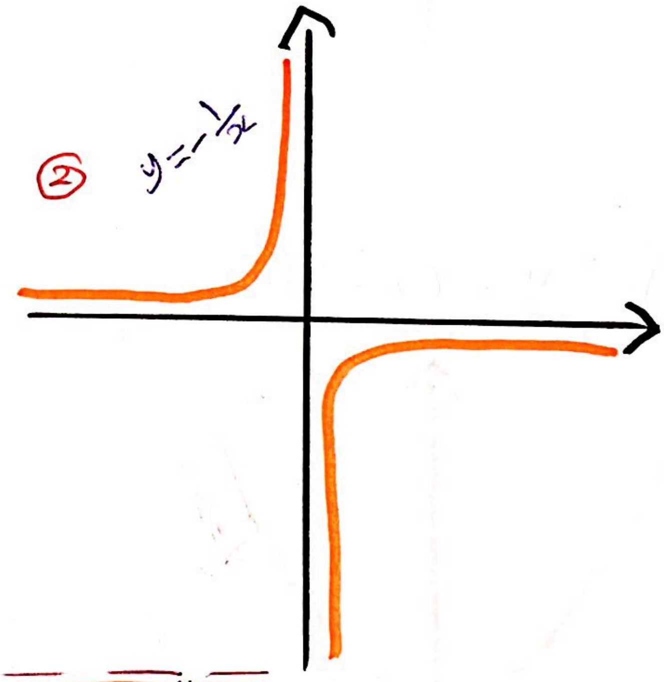


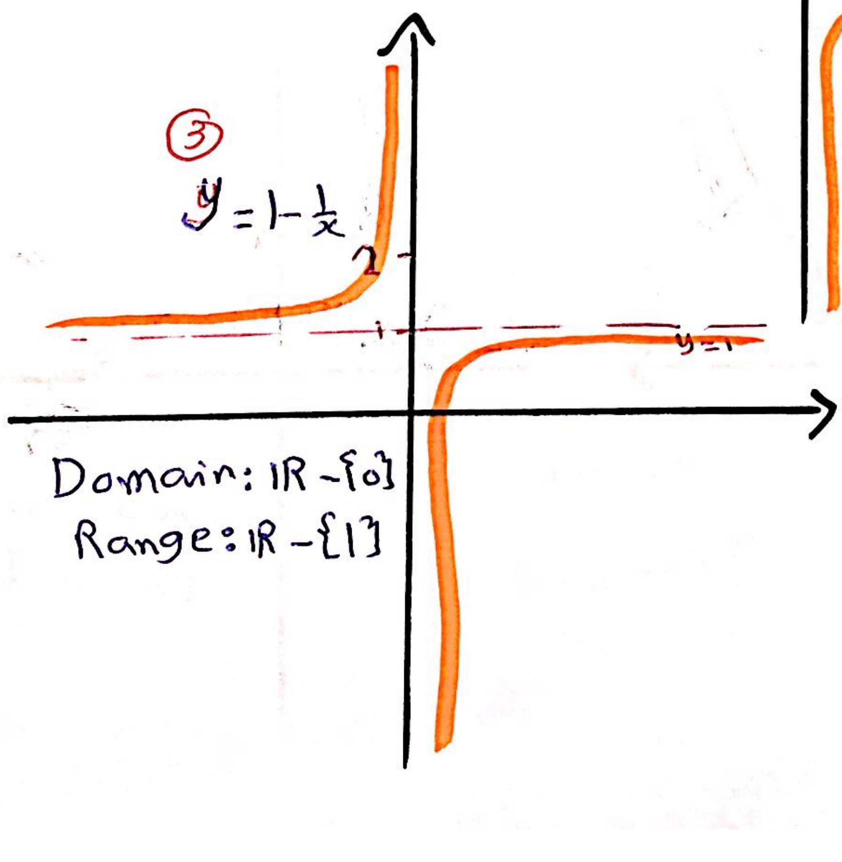
Domain: $\mathbb{R}$

Range: $\mathbb{R}$

(F) $y = 1 - \frac{1}{x}$







Domain: $\mathbb{R}-\{0\}$

Range: $\mathbb{R}-\{1\}$

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(6) $y = x^2 - 4x + 5$

نقوم بإكمال مربع

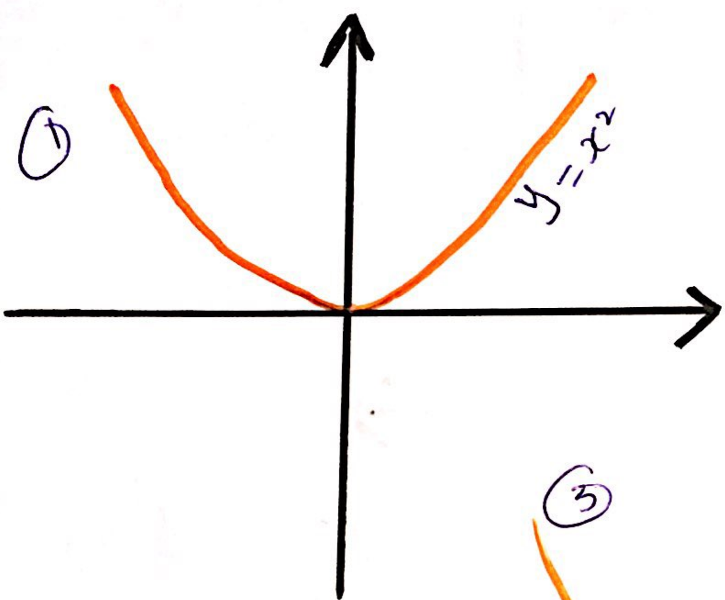
$$y = x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 + 5$$

$$= \underline{x^2 - 4x + 4} \, \underline{-4 + 5}$$

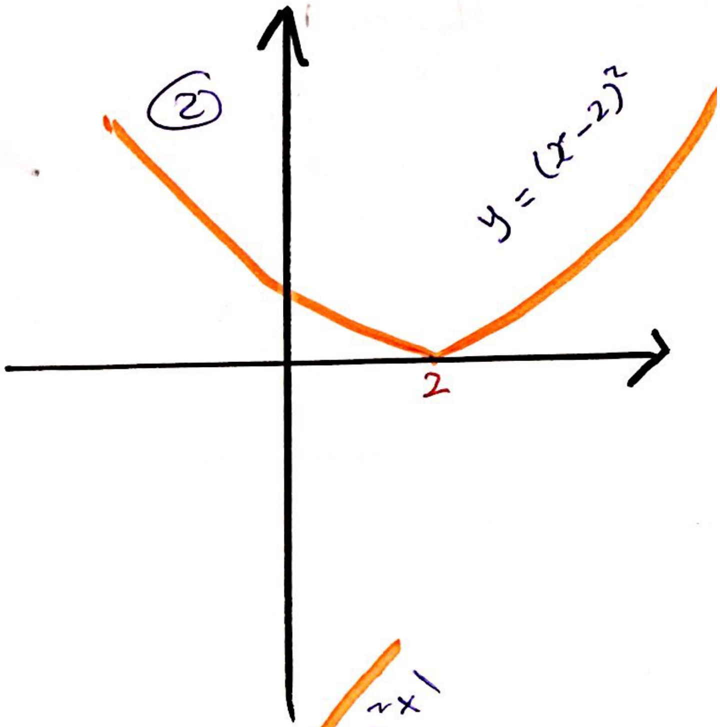
ج.ت ج.ت

$$= (x - 2)^2 + 1$$

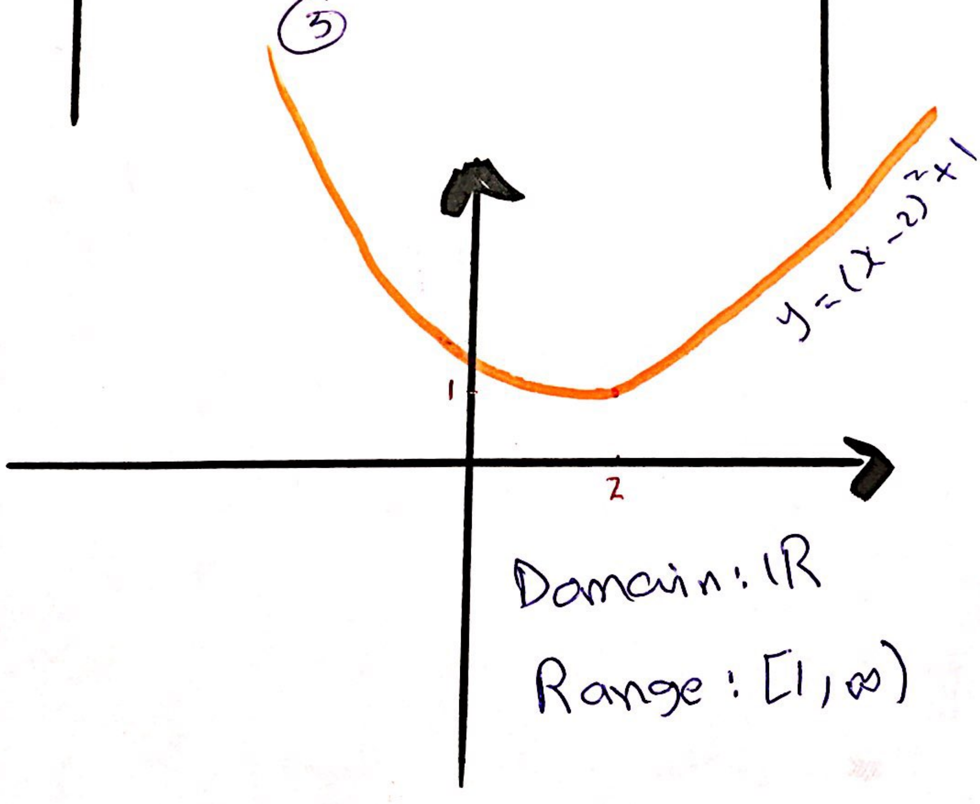
(1)



(2)



(3)



Domain: $\mathbb{R}$

Range: $[1, \infty)$

Example:

سؤال 1 ص 42

Suppose the graph of f is given
Write equation for the graphs that are obtained from the graph of f as follows

a) Shift 3 units up

$y = f(x) + 3$

b) Shift 3 units down

$y = f(x) - 3$

c) shift 3 units to the right

$y = f(x-3)$

d) Shift 3 units to the left

$y = f(x+3)$

e) reflect about the x-axis

$y = -f(x)$

f) reflect about the y-axis

$y = f(-x)$

Exampl (مثال)　　سؤال 2 ص 92

Explain how each graph is obtained from the graph $y = f(x)$

$y = f(x) + 8$, Shift the graph of $y = f(x)$ adistance 8 units up

$y = f(x+8)$, Shift the grap of $y = f(x)$ adistance 8 units to the Left

$y = -f(x) - 1$

reflect of the graph of $y = f(x)$ about x-axis and Shift the graph of $y = -f(x)$ adistance 1 unit down.

Example...

The graph of $y = f(x)$ is given

Match each equation with its graph

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a) $y = f(x-4)$

graph (3)

b) $y = f(x) + 3$

graph (1)

d) $y = -f(x+4)$

grap (5)

Combinations of Functions

let $f(x)$ and $g(x)$ are functions , $D_{f(x)} = A$ and $D_{g(x)} = B$ then.

1) $(f \pm g)(x) = f(x) \pm g(x)$

$$D_{(f \pm g)(x)} = D_{f(x)} \cap D_{g(x)} = A \cap B$$

2) $(fg)(x) = f(x) \cdot g(x)$

$$D_{(fg)(x)} = D_{f(x)} \cap D_{g(x)} = A \cap B$$

3) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$$D_{\left(\frac{f}{g}\right)(x)} = \{x \in A \cap B \mid g(x) \neq 0\}$$

$= A \cap B - \{$ أصفار المقام $\}$

4) $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$

$$D_{\left(\frac{g}{f}\right)(x)} = \{x \in A \cap B \mid f(x) \neq 0\}$$

$= A \cap B - \{$ أصفار المقام. $\}$

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Example

Find a) $f+g$; b) $f-g$; c) fg ; d) $\frac{f}{g}$; e) $\frac{g}{f}$

and state their domains.

1) $f(x) = x^2 - 1$; $g(x) = 2x+1$

a) $(f+g)(x) = f(x) + g(x)$

$= x^2 \cancel{-1} + 2x \cancel{+1}$

$= x^2 + 2x$

b) $(f-g)(x) = f(x) - g(x)$

$= x^2 - 1 - (2x+1)$

$= x^2 \underline{-1} - 2x \underline{-1}$

$= x^2 - 2x \underline{-2}$

c) $(fg)(x) = f(x)\, g(x)$

$= (x^2-1)(2x+1)$

$= x^2(2x+1) - 1(2x+1)$

$= 2x^3 + x^2 - 2x - 1$

$D_{f(x)} = \mathbb{R}$; $D_{g(x)} = \mathbb{R}$

$D_{(f+g)(x)} = D_{(f-g)(x)} = D_{(fg)(x)} = D_{f(x)} \cap D_{g(x)}$

$= \mathbb{R} \cap \mathbb{R}$

$= \mathbb{R}$

$= (-\infty, \infty)$

d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$= \frac{x^2-1}{2x+1}$

$D_{\left(\frac{f}{g}\right)(x)} = D_{f(x)} \cap D_{g(x)} - \{$ أصفار المقام $\}$

أصفار المقام :

$2x+1=0$

$2x=-1$

$\frac{2x}{2} = -\frac{1}{2}$

$\boxed{x = -\frac{1}{2}}$

$\therefore D_{\left(\frac{f}{g}\right)(x)} = \mathbb{R} \cap \mathbb{R} - \{-1/2\}$

$= \mathbb{R} - \{-1/2\}$

or $= \{x \mid x \neq -1/2\}$

or $= (-\infty, -1/2) \cup (-1/2, \infty)$

---

e) $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{2x+1}{x^2-1}$

$D_{\left(\frac{g}{f}\right)(x)} = D_{g(x)} \cap D_{f(x)} - \{$ أصفار المقام $\}$

أصفار المقام :

$x^2-1=0$

$x^2=1$

$\sqrt{x^2} = \sqrt{1}$

$|x| = 1$

$\boxed{x = \pm 1}$

$\therefore D_{\left(\frac{g}{f}\right)(x)} = \mathbb{R} \cap \mathbb{R} - \{\pm 1\}$

$= \mathbb{R} - \{1, -1\}$

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2) $f(x) = x^3 + 2x^2$; $g(x) = 3x^2 - 1$

a) $(f+g)(x) = f(x) + g(x)$

$= x^3 + \underline{2x^2} + \underline{3x^2} - 1$

$= x^3 + \underline{5x^2} - 1$

b) $(f-g)(x) = f(x) - g(x)$

$= x^3 + 2x^2 - (3x^2 - 1)$

$= x^3 + \underline{2x^2} - \underline{3x^2} + 1$

$= x^3 - \underline{x^2} + 1$

c) $(fg)(x) = f(x)\, g(x)$

$= (x^3 + 2x^2)(3x^2 - 1)$

$= x^3(3x^2 - 1) + 2x^2(3x^2 - 1)$

$= 3x^5 - x^3 + 6x^4 - 2x^2$

$= 3x^5 + 6x^4 - x^3 - 2x^2$

$D_{f(x)} = \mathbb{R}$ $\quad D_{g(x)} = \mathbb{R}$

$D_{(f+g)(x)} = D_{(f-g)(x)} = D_{(fg)(x)} = D_{f(x)} \cap D_{g(x)} = \mathbb{R} \cap \mathbb{R}$

$= \mathbb{R}$

$= (-\infty, \infty)$

d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$= \frac{x^3+2x^2}{3x^2-1}$

$D_{\left(\frac{f}{g}\right)(x)} = D_{f(x)} \cap D_{g(x)} - \{$ أصفار المقام $\}$

أصفار المقام : $3x^2-1=0$

$3x^2 = 1$

$\frac{3x^2}{3} = \frac{1}{3}$

$x^2 = \frac{1}{3}$

$\sqrt{x^2} = \sqrt{\frac{1}{3}}$

$|x| = \frac{1}{\sqrt{3}} \Rightarrow x = \pm \frac{1}{\sqrt{3}}$

$\therefore D_{\left(\frac{f}{g}\right)(x)} = \mathbb{R} \cap \mathbb{R} - \{\pm \frac{1}{\sqrt{3}}\} = \mathbb{R} - \{\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\} = \mathbb{R} - \{\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}\}$

e) $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{3x^2-1}{x^3+2x^2}$

$D_{\left(\frac{g}{f}\right)(x)} = D_{f(x)} \cap D_{g(x)} - \{$ أصفار المقام $\}$

أصفار المقام : $x^3+2x^2=0$

$x^2(x+2)=0 \Rightarrow$ $x^2=0 \Rightarrow x=0$ or $x+2=0 \Rightarrow x=-2$

$\therefore D_{\left(\frac{g}{f}\right)(x)} = \mathbb{R} \cap \mathbb{R} - \{0, -2\}$

$= \mathbb{R} - \{0, -2\}$

or $= \{x \mid x \neq 0, x \neq -2\}$

or $= (-\infty, -2) \cup (-2, 0) \cup (0, \infty)$

3) $f(x) = \sqrt{3-x}$ $\qquad g(x) = \sqrt{x^2-1}$

a) $(f+g)(x) = \sqrt{3-x} + \sqrt{x^2-1}$

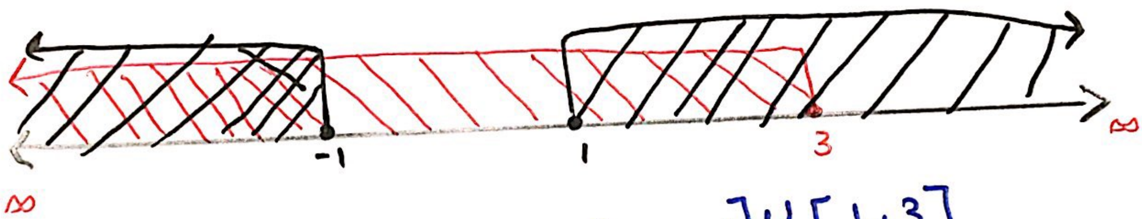
b) $(f-g)(x) = \sqrt{3-x} - \sqrt{x^2-1}$

c) $(fg)(x) = \sqrt{3-x}\,\sqrt{x^2-1}$

$$\left.\begin{aligned} D_{f(x)} &= (-\infty,3] \\ D_{g(x)} &= (-\infty,-1] \cup [1,\infty) \end{aligned}\right\}$$ "مدرس 1.1"

$D_{(f+g)(x)} = D_{(f-g)(x)} = D_{(fg)(x)} = D_{f(x)} \cap D_{g(x)}$

$= (-\infty,3] \cap \left[(-\infty,-1] \cup [1,\infty)\right]$



$= (-\infty,-1] \cup [1,3]$

Note

$(fg)(x) = \sqrt{3-x}\,\sqrt{x^2-1} = \sqrt{(3-x)(x^2-1)}$

$= \sqrt{3x^2-3-x^3+x}$

$= \sqrt{-x^3+3x^2+x-3}$ on $D_{(fg)(x)}$

$= \sqrt{-x^3+3x^2+x-3}$ on $(-\infty,-1] \cup [1,3]$.

d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$$= \frac{\sqrt{3-x}}{\sqrt{x^2-1}}$$

$D_{\left(\frac{f}{g}\right)(x)} = D_{f(x)} \cap D_{g(x)} - \{$ أصفار المقام $\}$

أصفار المقام : $\sqrt{x^2-1} = 0$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$\sqrt{x^2} = \sqrt{1}$$

$$|x| = 1$$

$$\boxed{x = \pm 1}$$

$\therefore D_{\left(\frac{f}{g}\right)(x)} = (-\infty, -1] \cup [1, 3] - \{-1, 1\}$

$$= (-\infty, -1) \cup (1, 3]$$

e) $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{x^2-1}}{\sqrt{3-x}}$

$D_{\left(\frac{g}{f}\right)(x)} = D_{g(x)} \cap D_{f(x)} - \{$ أصفار المقام $\}$

أصفار المقام : $\sqrt{3-x} = 0$

$$3 - x = 0$$

$$-x = -3$$

$$\frac{\cancel{-1}x}{\cancel{-1}} = \frac{-3}{-1}$$

$$\boxed{x = 3}$$

$\therefore D_{\left(\frac{g}{f}\right)(x)} = (-\infty, -1] \cup [1, 3] - \{3\}$

$$= (-\infty, -1] \cup [1, 3)$$

Note : $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{3-x}}{\sqrt{x^2-1}} = \sqrt{\frac{3-x}{x^2-1}}$ on $(-\infty, -1) \cup (1, 3]$

$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{x^2-1}}{\sqrt{3-x}} = \sqrt{\frac{x^2-1}{3-x}}$ on $(-\infty, -1] \cup [1, 3)$

Example

Find the Domain of the following functions :

1) $g(t) = \sqrt{3-t} - \sqrt{2+t}$

let $f_1(t) = \sqrt{3-t}$ and $f_2(t) = \sqrt{2+t}$

$D_{f_1(t)}$: $3 - t \geq 0 \Rightarrow -t \geq -3 \Rightarrow t \leq 3$

$\therefore \boxed{D_{f_1(t)} = (-\infty, 3]}$

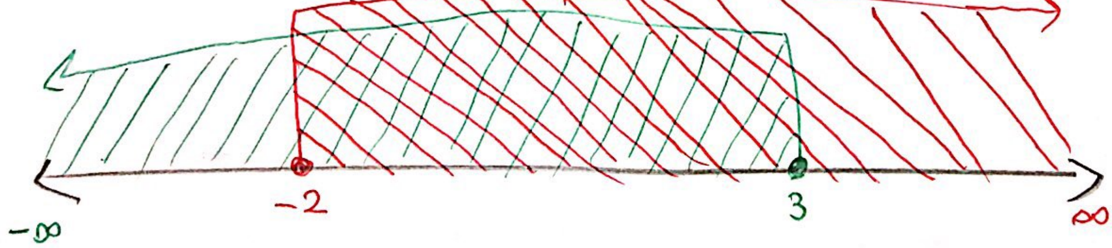
$D_{f_2(t)}$: $2 + t \geq 0 \Rightarrow t \geq -2$

$\therefore \boxed{D_{f_2(t)} = [-2, \infty)}$

$D_{g(t)} = D_{f_1(t)} \cap D_{f_2(t)}$

$= (-\infty, 3] \cap [-2, \infty)$

$= [-2, 3]$



2) $f(x) = \dfrac{1}{\sqrt[6]{2x^2+5}}$

let $f_1(x) = 1$; $f_2(x) = \sqrt[6]{2x^2+5}$

$D_{f_1(x)} = \mathbb{R}$

$D_{f_2(x)}$: $2x^2+5 \geq 0$

$2x^2 \geq -5$

$\dfrac{\cancel{2}x^2}{\cancel{2}} \geq \dfrac{-5}{2}$

$x^2 \geq -\dfrac{5}{2}$

$\sqrt{x^2} \geq \sqrt{\dfrac{-5}{2}}$

$|x| \geq \sqrt{\dfrac{-5}{2}}$

$x \geq \pm\sqrt{\dfrac{-5}{2}} \notin \mathbb{R}$

$\therefore D_{f_2(x)} = \mathbb{R}$

$\Rightarrow D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} -$ {أصفار المقام}

$= \mathbb{R} \cap \mathbb{R}$

$= \mathbb{R}$

أصفار المقام : $2x^2+5=0$

$2x^2 = -5$

$x^2 = -\dfrac{5}{2}$

$\sqrt{x^2} = \sqrt{\dfrac{-5}{2}}$

$|x| = \sqrt{\dfrac{-5}{2}}$

$x = \pm\sqrt{\dfrac{-5}{2}} \notin \mathbb{R}$

لا يوجد أصفار مقام

3) $f(x) = \frac{\ln x}{\sqrt{5-x^2}}$

Let $f_1(x) = \ln(x)$; $f_2(x) = \sqrt{5-x^2}$

$D_{f_1(x)} = (0, \infty)$

$D_{f_2(x)} = [-\sqrt{5}, \sqrt{5}]$

$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{$أصفار المقام$\}$

$= (0,\infty) \cap [-\sqrt{5}, \sqrt{5}] - \{\sqrt{5}, -\sqrt{5}\}$

$= (0, \sqrt{5}] - \{\sqrt{5}, -\sqrt{5}\}$

$= (0, \sqrt{5})$

أصفار المقام :

$5 - x^2 = 0$

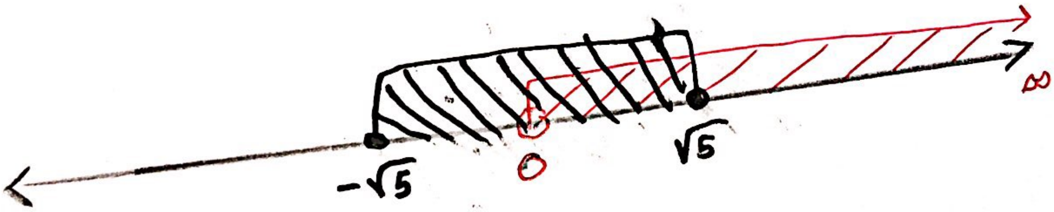
$-x^2 = -5$

$x^2 = 5$

$\sqrt{x^2} = \sqrt{5}$

$|x| = \sqrt{5}$

$\boxed{x = \pm\sqrt{5}}$



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4) $f(x) = \dfrac{\sqrt{9-x^2}}{x-2}$

let $f_1(x) = \sqrt{9-x^2}$; $f_2(x) = x-2$

$D_{f_1(x)} = [-3,3]$ and $D_{f_2(x)} = \mathbb{R}$

أصفار المقام : $x-2=0$

$x=2$

$D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{$ أصفار المقام $\}$

$= [-3,3] \cap \mathbb{R} - \{2\}$

$= [-3,3] - \{2\}$

$= [-3,2) \cup (2,3]$

5) $f(x) = \dfrac{x-2}{\sqrt{9-x^2}}$ "H.W"

let $f_1(x) = x-2 \Rightarrow D_{f_1(x)} = \mathbb{R}$

$f_2(x) = \sqrt{9-x^2} \Rightarrow D_{f_2(x)} = [-3,3]$

أصفار المقام : $9-x^2=0$

$-x^2=-9$

$x^2=9$

$\sqrt{x^2}=\sqrt{9} \Rightarrow |x|=3$

$\Rightarrow \boxed{x=\pm 3}$

$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{$ أصفار المقام $\}$

$= \mathbb{R} \cap [-3,3] - \{-3,3\}$

$= [-3,3] - \{-3,3\}$

$= (-3,3)$

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6) $f(x) = \dfrac{x-1}{\sqrt[3]{x^2-6x+5}}$

let $f_1(x) = x-1$; $f_2(x) = \sqrt[3]{x^2-6x+5}$

$D_{f_1(x)} = \mathbb{R}$; $D_{f_2(x)} = \mathbb{R}$

أصفار المقام : $x^2-6x+5=0$

$(x-1)(x-5)=0$

$x-1=0$, $\boxed{x=1}$; $x-5=0$, $\boxed{x=5}$

$D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{$أصفار المقام$\}$

$= \mathbb{R} \cap \mathbb{R} - \{1, 5\}$

$= \mathbb{R} - \{1,5\}$

7) $f(x) = \dfrac{\sqrt[3]{x^2-6x+5}}{x-1}$ "H.W"

مثال إضافي

* $f(x) = \dfrac{1}{\sqrt[5]{6-2x}}$

7) let $f_1(x) = \sqrt[3]{x^2-6x+5}$ and $f_2(x) = x-1$

$D_{f_1(x)} = \mathbb{R}$; $D_{f_2(x)} = \mathbb{R}$

أصفار المقام : $x-1=0$; $\boxed{x=1}$

$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{$أصفار المقام$\}$

$= \mathbb{R} \cap \mathbb{R} - \{1\}$

$= \mathbb{R} - \{1\} = (-\infty, 1) \cup (1, \infty)$

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حل المثال الإضافي :

$$f(x) = \frac{1}{\sqrt[5]{6-2x}}$$

let $f_1(x) = 1$ and $f_2(x) = \sqrt[5]{6-2x}$

$D_{f_1(x)} = \mathbb{R}$ and $D_{f_2(x)} = \mathbb{R}$

أصفار المقام f

أصفار المقام : $6-2x=0$

$-2x=-6$

$2x=6$

$x=\frac{6}{2}$

$\boxed{x=3}$

$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{$أصفار المقام$\}$

$= \mathbb{R} \cap \mathbb{R} - \{3\}$

$= \mathbb{R} - \{3\}$

or $= \{x \mid x \neq 3\}$

or $= (-\infty, 3) \cup (3, \infty)$

8) $g(x) = \dfrac{x}{e^x + 1}$

Let $g_1(x) = x$; $g_2(x) = e^x + 1$

$\boxed{D_{g_1(x)} = \mathbb{R}}$

$D_{g_2(x)}$: Let $g_3(x) = e^x$; $g_4(x) = 1$

$D_{g_3(x)} = \mathbb{R}$; $D_{g_4(x)} = \mathbb{R}$

$\therefore D_{g_2(x)} = D_{g_3(x)} \cap D_{g_4(x)}$

$= \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

$\Rightarrow \boxed{D_{g_2(x)} = \mathbb{R}}$

$\therefore D_{g(x)} = D_{g_1(x)} \cap D_{g_2(x)} - \{$أصفار المقام$\}$

$= \mathbb{R} \cap \mathbb{R}$

$= \mathbb{R}$

أصفار المقام:

$e^x + 1 = 0$

$e^x = -1$

لا يوجد أصفار مقام

لأن $e^x > 0$

⑨ $G(x) = \dfrac{5^x}{2^x - 1}$

let $G_1(x) = 5^x$; $G_2(x) = 2^x - 1$

$D_{G_1(x)} = \mathbb{R}$ $\qquad$ $D_{G_2(x)} = \mathbb{R}$

$\therefore D_{G(x)} = D_{G_1(x)} \cap D_{G_2(x)} - \{\text{أصفار المقام}\}$

$D_{G(x)} = \mathbb{R} \cap \mathbb{R} - \{0\}$

$= \mathbb{R} - \{0\}$ #

أصفار المقام :

$2^x - 1 = 0$

$2^x = 1$

$2^x = 2^0$

$\boxed{x = 0}$

Note

- if $H(x) = e^{f(x)}$ then $D_{H(x)} = D_{e^x} \cap D_{f(x)}$
- if $H(x) = a^{f(x)}$ then $D_{H(x)} = D_{a^x} \cap D_{f(x)}$
- if $H(x) = \sin(f(x))$ then $D_{H(x)} = D_{\sin x} \cap D_{f(x)}$
- if $H(x) = \cos(f(x))$ then $D_{H(x)} = D_{\cos x} \cap D_{f(x)}$

Example

Find the Domain of the following functions :

a) $f(x) = e^{\sqrt{x}}$

$D_{f(x)} = D_{e^x} \cap D_{\sqrt{x}}$

$= \mathbb{R} \cap [0, \infty)$

$= [0, \infty)$

b) $f(x) = \pi^{2x^2+1}$

$D_{f(x)} = D_{\pi^x} \cap D_{(2x^2+1)}$

$= \mathbb{R} \cap \mathbb{R}$

$= \mathbb{R}$

c) $f(x) = \cos(x^{-3})$

$$D_{f(x)} = D_{\cos x} \cap D_{x^{-3}}$$
$$= D_{\cos x} \cap D_{\frac{1}{x^3}}$$
$$= \mathbb{R} \cap \mathbb{R} - \{0\}$$
$$= \mathbb{R} - \{0\}$$
$$= (-\infty, 0) \cup (0, \infty)$$

d) $\sin(e^{-t}) = f(x)$

$$D_{f(x)} = D_{\sin t} \cap D_{e^{-t}}$$
$$= \mathbb{R} \cap \mathbb{R}$$
$$= \mathbb{R}$$
$$= (-\infty, \infty)$$

d) $f(x) = \cos(\pi^x + \sqrt{x^2-1})$

$D_{\pi^x} = \mathbb{R}$; $D_{\sqrt{x^2-1}} = (-\infty, -1] \cup [1, \infty)$

$$D_{(\pi^x + \sqrt{x^2-1})} = D_{\pi^x} \cap D_{\sqrt{x^2-1}}$$
$$= \mathbb{R} \cap (-\infty, -1] \cup [1, \infty)$$
$$= (-\infty, -1] \cup [1, \infty)$$

e) $f(x) = \dfrac{1 - e^{x^2}}{1 - e^{1-x^2}}$

$D_{(1-e^{x^2})} = \mathbb{R}$; $D_{(1-e^{1-x^2})} = \mathbb{R}$

$$D_{f(x)} = D_{(1-e^{x^2})} \cap D_{(1-e^{1-x^2})} - \{\text{أصفار المقام}\}$$
$$= \mathbb{R} - \{\pm 1\}$$

أصفار المقام : $1 - e^{1-x^2} = 0$

$$-e^{1-x^2} = -1$$
$$e^{1-x^2} = 1$$
$$e^{1-x^2} = e^0$$
$$1 - x^2 = 0 \Longrightarrow \boxed{x = \pm 1}$$

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f) $f(x) = \dfrac{1+x}{e^{\cos x}}$

$D_{(1+x)} = \mathbb{R}$; $D_{e^{\cos x}} = \mathbb{R}$

أصفار المقام: $e^{\cos x} = 0$

لا يوجد أصفار المقام لأنه $e^{\cos x} > 0$

$\therefore D_{f(x)} = D_{(1+x)} \cap D_{e^{\cos x}} - \{\text{أصفار مقام}\}$

$= \mathbb{R} \cap \mathbb{R}$

$= \mathbb{R}$

g) $H(t) = \sqrt{10^t - 100}$

$10^t - 100 \geqslant 0 \Rightarrow 10^t \geqslant 100 \Rightarrow 10^t \geqslant 10^2$

$\Rightarrow t \geqslant 2$

$\therefore D_{H(t)} = [2, \infty)$

h) $g(x) = \sqrt[4]{1 - 2^x}$

$1 - 2^x \geqslant 0 \Rightarrow -2^x \geqslant -1 \Rightarrow 2^x \leq 1 \Rightarrow 2^x \leq 2^0$

$\Rightarrow x \leq 0 \Rightarrow D_{g(x)} = (-\infty, 0]$

i) $f(x) = \sqrt[4]{2^x + 1}$

$D_{f(x)}$: $2^x + 1 \geq 0$

$2^x \geq -1$

مستحيل لأنه دائماً
الدالة الأسية موجبة
أي $2^x > 0$

$\therefore D_{f(x)} = \mathbb{R}$

j) $f(x) = \sqrt{|x| + 1}$

$D_{f(x)}$: $|x| + 1 \geq 0$

$|x| \geq -1$

مستحيل لأنه دائماً
دالة القيمة المطلقة
موجبة أي $|x| \geq 0$

$\therefore D_{f(x)} = \mathbb{R}$

K) $f(x) = \sqrt{|x| - 1}$

$D_{f(x)}$: $|x| - 1 \geq 0$

$|x| \geq 1$

$x \geq 1$ or $x \leq -1$

$\therefore D_{f(x)} = (-\infty, -1] \cup [1, \infty) = \mathbb{R} - (-1, 1)$

L) $f(x) = \dfrac{1}{\sqrt{x} + 2}$

let $f_1(x) = 1$ and

$f_2(x) = \sqrt{x} + 2$

$D_{f_1}(x) = \mathbb{R}$ and

$D_{f_2(x)} = [0, \infty)$

أصفار المقام :

$\sqrt{x} + 2 = 0$

$\sqrt{x} = -2$

مستحيل لأنه $\sqrt{x} \geq 0$
أي أنه $\sqrt{x}$ دائماً
موجبة

إذنه لا يوجد أصفار
مقام .

$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)}$

$= \mathbb{R} \cap [0, \infty)$

$= [0, \infty)$

M) $f(x) = \sqrt{1 - |x|}$

$D_{f(x)}$: $1 - |x| \geq 0$

$-|x| \geq -1$

$|x| \leq 1$

$-1 \leq x \leq 1$

$\therefore D_{f(x)} = [-1, 1]$

N) $f(x) = \dfrac{1}{\sqrt{x} - 2}$

Let $f_1(x) = 1 \implies D_{f_1(x)} = \mathbb{R}$

$f_2(x) = \sqrt{x} - 2 \implies D_{f_2(x)} = [0, \infty)$

أصفار المقام: $\sqrt{x} - 2 = 0$

$\sqrt{x} = 2$

$(\sqrt{x})^2 = (2)^2$

$\boxed{x = 4}$

$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)} - \{$أصفار المقام$\}$

$= \mathbb{R} \cap [0, \infty) - \{4\}$

$= [0, \infty) - \{4\}$

$= [0, 4) \cup (4, \infty)$

p) $f(x) = \sqrt{\cos x + 3}$

$D_{f(x)}$: $\cos x + 3 \geq 0$

$\cos x \geq -3$

مستحيل لأنه دائماً $-1 \leq \cos x \leq 1$

$\therefore D_{f(x)} = \mathbb{R}$

q) $f(x) = \dfrac{1}{\sin x - 3}$

let $f_1(x) = 1$ and $f_2(x) = \sin x - 3$

$D_{f_1}(x) = \mathbb{R}$; $D_{f_2(x)} = \mathbb{R}$

أصفار المقام:

$\sin x - 3 = 0$

$\sin x = 3$

مستحيل لأنه دائماً $-1 \leq \sin x \leq 1$

إذن لا يوجد أصفار مقام

$\therefore D_{f(x)} = D_{f_1(x)} \cap D_{f_2(x)}$

$= \mathbb{R} \cap \mathbb{R}$

$= \mathbb{R}$

Definition:-

$$(f\circ g)(x) = f(g(x))$$

$f\circ g$ is called the Composition of f and g.

Examaple.

If $f(x) = x^2$ and $g(x) = x-3$

then find $f\circ g$ and $g\circ f$?

$$(f\circ g)(x) = f(g(x)) = f(x-3)$$

$$= (x-3)^2 = x^2 - 6x + 9$$

$$(g\circ f)(x) = g(f(x)) = g(x^2) = x^2 - 3$$

Note $f\circ g \neq g\circ f$

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Example: If $f(x) = \frac{x}{x+1}$; $g(x) = x^{10}$ and $h(x) = x+3$ then fing $f \circ g \circ h$ and $h \circ g \circ f$

$$(f \circ g \circ h)(x) = f(g(h(x)))$$
$$= f(g(x+3))$$
$$= f((x+3)^{10})$$
$$= \frac{(x+3)^{10}}{(x+3)^{10}+1}$$

$$(h \circ g \circ f)(x) = h(g(f(x)))$$
$$= h\left(g\left(\frac{x}{x+1}\right)\right)$$
$$= h\left(\left(\frac{x}{x+1}\right)^{10}\right)$$
$$= \left(\frac{x}{x+1}\right)^{10} + 3$$

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Example :: Given $F(x) = \cos^2(x+9)$, find functions f, g and h such that $F = f \circ g \circ h$

$$F(x) = \cos^2(x+9)$$

$$(f \circ g \circ h)(x) = [\cos(x+9)]^2$$

$$f(g(h(x))) = [\cos(x+9)]^2$$

$$h(x) = x+9$$

$$g(x) = \cos(x)$$

$$f(x) = x^2$$

Example: $F(x) = \sqrt[3]{\frac{x}{1+x}}$ find f and g such that

$$F = f \circ g$$

$$F(x) = \sqrt[3]{\frac{x}{1+x}}$$

$$(f \circ g)(x) = \sqrt[3]{\frac{x}{1+x}}$$

$$f(g(x)) = \sqrt[3]{\frac{x}{1+x}}$$

$$g(x) = \frac{x}{x+1} \; ; \; f(x) = \sqrt[3]{x}$$

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Example $F(x) = \dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$ find f and g

Such that $F = F \circ g$

$$F(x) = \frac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$$

$$(f \circ g)(x) = \frac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$$

$$f(g(x)) = \frac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$$

$$g(x) = \sqrt[3]{x}$$

$$f(x) = \frac{x}{1+x}$$

Example $R(x) = \sqrt{\sqrt{x} - 1}$

$$R(x) = \sqrt{\sqrt{x} - 1}$$

$$(F \circ g \circ h)(x) = \sqrt{\sqrt{x} - 1}$$

$$F(g(h(x))) = \sqrt{\sqrt{x} - 1}$$

$h(x) = \sqrt{x}$ $g(x) = x - 1$ $f(x) = \sqrt{x}$

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الواجب في درس 1.3

39) $f(x) = 3x-2$; $g(x) = \sin x$; $h(x) = x^2$

$$(f\circ g\circ h)(x) = f(g(h(x)))$$
$$= f(g(x^2))$$
$$= f(\sin(x^2))$$
$$= 3\sin(x^2) - 2$$

40) $f(x) = |x-4|$; $g(x) = 2^x$; $h(x) = \sqrt{x}$

$$(f\circ g\circ h)(x) = f(g(h(x)))$$
$$= f(g(\sqrt{x}))$$
$$= f(2^{\sqrt{x}})$$
$$= |2^{\sqrt{x}} - 4|$$

41) $f(x) = \sqrt{x-3}$, $g(x) = x^2$; $h(x) = x^3+2$

$$(f\circ g\circ h)(x) = f(g(h(x)))$$
$$= f(g(x^3+2))$$
$$= f((x^3+2)^2)$$
$$= \sqrt{(x^3+2)^2-3}$$

42) $f(x) = \tan x$; $g(x) = \frac{x}{x-1}$, $h(x) = \sqrt[3]{x}$

$$(f \circ g \circ h)(x) = f(g(h(x)))$$
$$= f(g(\sqrt[3]{x}))$$
$$= f\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)$$
$$= \tan\left(\frac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)$$

Express the function in the form $f \circ g$

43) $F(x) = (2x + x^2)^4$

$$(f \circ g)(x) = (2x + x^2)^4$$
$$f(g(x)) = (2x + x^2)^4$$
$$g(x) = 2x + x^2 \; ; \; f(x) = x^4$$

45) $F(x) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$

$$(f \circ g)(x) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$$
$$f(g(x)) = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$$
$$g(x) = \sqrt[3]{x} \; ; \; f(x) = \frac{x}{1+x}$$

47) $v(t) = \sec(t^2)\tan(t^2)$

$(f \circ g)(\cancel{t}) = \sec(t^2)\tan(t^2)$

$f(g(t)) = \sec(t^2)\tan(t^2)$

$g(t) = t^2$; $f(t) = \sec(t)\tan(t)$

48) $u(t) = \dfrac{\tan(t)}{1+\tan(t)}$

$(f \circ g)(t) = \dfrac{\tan(t)}{1+\tan(t)}$

$f(g(t)) = \dfrac{\tan(t)}{1+\tan(t)}$

$g(t) = \tan(t)$; $f(t) = \dfrac{t}{1+t}$

46) $G(x) = \sqrt[3]{\dfrac{x}{1+x}}$

$(f \circ g)(x) = \sqrt[3]{\dfrac{x}{1+x}}$

$f(g(x)) = \sqrt[3]{\dfrac{x}{1+x}}$

$g(x) = \dfrac{x}{1+x}$ $f(x) = \sqrt[3]{x}$

44) $h(x) = \cos^2 x$

$(f \circ g)(x) = [\cos x]^2$

$f(g(x)) = [\cos x]^2$

$g(x) = \cos x$; $f(x) = x^2$

Express the function in the form f∘g∘h

49) $R(x) = \sqrt{\sqrt{x} - 1}$

$(f \circ g \circ h)(x) = \sqrt{\sqrt{x} - 1}$

$f(g(h(x))) = \sqrt{\sqrt{x} - 1}$

$h(x) = \sqrt{x}$; $g(x) =$; $f(x) = \sqrt{x}$

$= x - 1$

50) $H(x) = \sqrt[8]{2 + |x|}$

$(f \circ g \circ h)(x) = \sqrt[8]{2 + |x|}$

$f(g(h(x))) = \sqrt[8]{2 + |x|}$

$h(x) = |x|$; $g(x) = 2 + x$; $f(x) = \sqrt[8]{x}$

51) $S(t) = \sin^2(\cos t)$

$(f \circ g \circ h)(t) = \sin^2(\cos t)$

$f(g(h(t))) = [\sin(\cos t)]^2$

$h(t) = \cos t$; $g(t) = \sin(t)$; $f(t) = t^2$

Example:

if $f(x) = \frac{1}{x^2+5}$, $g(x) = \frac{3}{2}$

then $(f \circ g)(2) = f(g(2))$

$$= f\left(\frac{3}{2}\right)$$

$$= \frac{1}{\left(\frac{3}{2}\right)^2+5} = \frac{1}{\frac{9}{4}+5}$$

$$= \frac{1}{\left(\frac{9+20}{4}\right)}$$

$$= \frac{1}{\left(\frac{29}{4}\right)}$$

$$= \frac{4}{29} \quad \#$$

$(g \circ f)(2) = g(f(2))$

$$= g\left(\frac{1}{2^2+5}\right) = g\left(\frac{1}{4+5}\right) = g\left(\frac{1}{9}\right)$$

$$= \frac{3}{2} \quad \#$$

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Example

if $f(x) = \sqrt{x}$ and $g(x) = x^2$ then find the Domain of the following

① $(f \circ g)(x)$

$(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2} = |x| \Rightarrow$ New function

$D_{New} = \mathbb{R}$ $\qquad D_{g(x)} = \mathbb{R}$

$\therefore D_{(f \circ g)(x)} = D_{New} \cap D_{g(x)}$

$= \mathbb{R} \cap \mathbb{R}$

$= \mathbb{R}$

$= (-\infty, \infty)$

② $(g \circ f)(x)$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 = x \Rightarrow$ New function

$D_{New} = \mathbb{R}$ $\qquad D_{f(x)} = [0, \infty)$

$\therefore D_{(f \circ g)(x)} = D_{New} \cap D_{f(x)}$

$= \mathbb{R} \cap [0, \infty)$

$= [0, \infty)$

Example

if $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$ then find the Domain of the following

① $(f \circ g)(x)$

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{2-x}) = \sqrt[2]{\sqrt[2]{2-x}} = \sqrt[2\times 2]{2-x}$$

$$= \sqrt[4]{2-x} \Rightarrow \text{New function}$$

$D_{g(x)} = D_{\sqrt{2-x}} = (-\infty, 2]$

D_{New} : $2 - x \geq 0$

$-x \geq -2$

$\frac{-x}{-1} \leq \frac{-2}{-1}$

$x \leq 2$

$\therefore D_{New} = (-\infty, 2]$

$\Rightarrow D_{(f \circ g)(x)} = D_{New} \cap D_{g(x)} = (-\infty, 2] \cap (-\infty, 2] = (-\infty, 2]$

② $(f \circ f)(x)$

$$(f \circ f)(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x} \Rightarrow \text{New function}$$

$D_{New} = [0, \infty)$ $\qquad D_{f(x)} = [0, \infty)$

$\therefore D_{(f \circ f)(x)} = D_{New} \cap D_{f(x)} = [0, \infty) \cap [0, \infty) = [0, \infty)$

3) $(g \circ g)(x)$

$$(g \circ g)(x) = g(g(x)) = g(\sqrt{2-x}) = \sqrt{2 - \sqrt{2-x}} \Rightarrow \text{New functio}$$

D_{New}: $2 - \sqrt{2-x} \geq 0$

$$-\sqrt{2-x} \geq -2$$

$$\sqrt{2-x} \leq 2$$

$$0 \leq \sqrt{2-x} \leq 2$$

$$(0)^2 \leq (\sqrt{2-x})^2 \leq 2^2$$

$$0 \leq 2-x \leq 4$$

$$0-2 \leq -x \leq 4-2$$

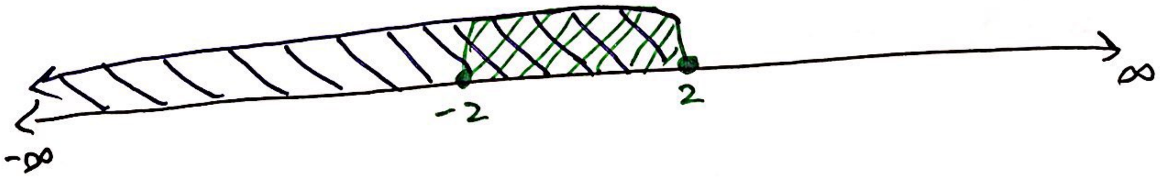
$$-2 \leq -x \leq 2 \Rightarrow 2 \geq x \geq -2$$

$$-2 \leq x \leq 2$$

$\therefore D_{New} = [-2, 2]$; $D_{g(x)} = (-\infty, 2]$

$D_{(g \circ g)(x)} = D_{New} \cap D_{g(x)}$

$$= [-2, 2] \cap (-\infty, 2] = [-2, 2]$$



④ $(g \circ f)(x)$

$$(g \circ f)(x) = g(f(x))$$
$$= g(\sqrt{x})$$
$$= \sqrt{2 - \sqrt{x}} \Rightarrow \text{New function}$$

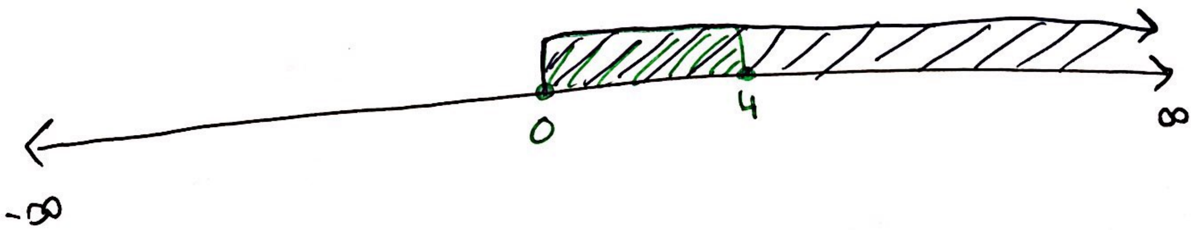
D_{New}: $2 - \sqrt{x} \geq 0$

$$-\sqrt{x} \geq -2$$
$$\sqrt{x} \leq 2$$
$$\sqrt{x} \leq 2$$
$$0 \leq \sqrt{x} \leq 2$$
$$(0)^2 \leq (\sqrt{x})^2 \leq (2)^2$$
$$0 \leq x \leq 4$$

$\therefore D_{New} = [0, 4]$

$\Rightarrow D_{(g \circ f)(x)} = D_{New} \cap D_{f(x)}$

$$= [0,4] \cap [0, \infty) = [0,4]$$

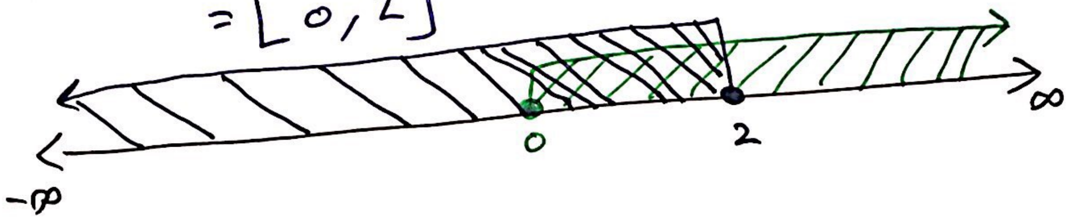


⑤ $(f \pm g)(x)$

$$(f \pm g)(x) = f(x) \pm g(x)$$
$$= \sqrt{x} \pm \sqrt{2-x}$$

$D_{f(x)} = [0, \infty)$ $\quad D_{g(x)} = (-\infty, 2]$

$$\therefore D_{(f+g)(x)} = D_{f(x)} \cap D_{g(x)}$$
$$= [0, \infty) \cap (-\infty, 2]$$
$$= [0, 2]$$



⑥ $(fg)(x) = f(x)\, g(x)$
$$= \sqrt{x}\, \sqrt{2-x}$$

$D_{f(x)} = [0, \infty)$ $\quad D_{g(x)} = (-\infty, 2]$

$$\therefore D_{(fg)(x)} = D_{f(x)} \cap D_{g(x)}$$
$$= [0, \infty) \cap (-\infty, 2]$$
$$= [0, 2]$$

Note

$$(fg)(x) = f(x)\, g(x) = \sqrt{x}\, \sqrt{2-x}$$
$$= \sqrt{x(2-x)}$$
$$= \sqrt{2x - x^2} \quad \text{for all } x \in [0, 2]$$

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$$7)\ \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$= \frac{\sqrt{x}}{\sqrt{2-x}}$$

$D_{f(x)} = [0,\infty)$ $D_{g(x)} = (-\infty,2]$

أصفار المقام : $2-x=0$

$$-x=-2$$

$$\boxed{x=2}$$

$$\therefore D_{\left(\frac{f}{g}\right)(x)} = D_{f(x)} \cap D_{g(x)} - \{\text{أصفار المقام}\}$$

$$= [0,\infty) \cap (-\infty,2] - \{2\}$$

$$= [0,2] - \{2\}$$

$$= [0,2)$$

Not

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\sqrt{2-x}}$$

$$= \sqrt{\frac{x}{2-x}} \quad \text{for all } x \in [0,2)$$

8) $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$

$$= \frac{\sqrt{2-x}}{\sqrt{x}}$$

$D_{g(x)} = (-\infty, 2]$ $\quad D_{f(x)} = [0, \infty)$

أصفار المقام : $\boxed{x = 0}$

$$\Rightarrow D_{\left(\frac{g}{f}\right)(x)} = D_{g(x)} \cap D_{F(x)} - \{\text{أصفار المقام}\}$$

$$= (-\infty, 2] \cap [0, \infty) - \{0\}$$

$$= [0, 2] - \{0\}$$

$$= (0, 2]$$

Note

$$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{2-x}}{\sqrt{x}}$$

$$= \sqrt{\frac{2-x}{x}}$$

$$= \sqrt{\frac{2}{x} - \frac{x}{x}}$$

$$= \sqrt{\frac{2}{x} - 1} \quad \text{for all } x \in (0, 2]$$

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Example

if $f(x) = \sqrt{3-x}$; $g(x) = \sqrt{x^2 - 1}$

then find the domain of the following

① $(f \circ f)(x) = f(f(x)) = f\left(\sqrt{3-x}\right)$

$= \sqrt{3 - \sqrt{3-x}}$ ⇒ New function

D_{New} : $3 - \sqrt{3-x} \geq 0$

$-\sqrt{3-x} \geq -3$

$\sqrt{3-x} \leq 3$

$0 \leq \sqrt{3-x} \leq 3$

$0^2 \leq \left(\sqrt{3-x}\right)^2 \leq 3^2$

$0 \leq 3 - x \leq 9$

$-3 \leq -x \leq 9-3$

$-3 \leq -x \leq 6$

$-6 \leq x \leq 3$ $\therefore D_{New} = [-6, 3]$

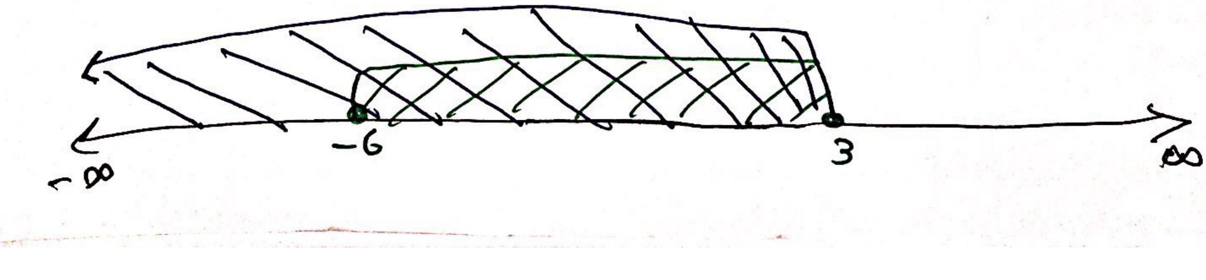
$D_{f(x)}$: $3 - x \geq 0$

$-x \geq -3$

$x \leq 3$

$\therefore D_{f(x)} = (-\infty, 3]$

$\Rightarrow D_{(f \circ f)(x)} = D_{New} \cap D_{f(x)} = [-6, 3] \cap (-\infty, 3] = [-6, 3]$

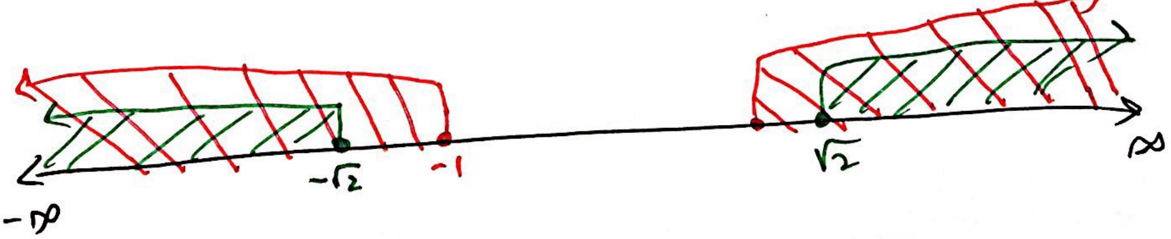


② $(g \circ g)(x)$

$$(g \circ g)(x) = g(g(x))$$
$$= g\left(\sqrt{x^2-1}\right)$$
$$= \sqrt{\left(\sqrt{x^2-1}\right)^2 - 1}$$
$$= \sqrt{x^2-1-1}$$
$$= \sqrt{x^2-2} \Rightarrow \text{New function}$$

$D_{New} = (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$ $\quad D_{g(x)} = (-\infty, -1] \cup [1, \infty)$

$$\Rightarrow D_{(g \circ g)(x)} = D_{New} \cap D_{g(x)}$$
$$= \left((-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)\right) \cap \left((-\infty, -1] \cup [1, \infty)\right)$$
$$= (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$$



④ $(g \circ f)(x)$

$(g \circ f)(x) = g(f(x))$

$= g(\sqrt{3-x})$

$= \sqrt{(\sqrt{3-x})^2 - 1}$

$= \sqrt{3 - x - 1}$

$= \sqrt{2 - x}$ $\Rightarrow$ New function

$D_{f(x)} = (-\infty, 3]$

D_{New}: $2 - x \geq 0$

$-x \geq -2$

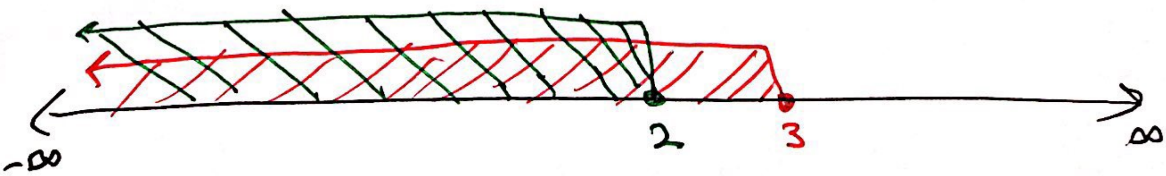
$x \leq 2$

$D_{New} = (-\infty, 2]$

$\Rightarrow D_{(g \circ f)(x)} = D_{New} \cap D_{f(x)}$

$= (-\infty, 2] \cap (-\infty, 3]$

$= (-\infty, 2]$



③ $(f\circ g)(x)$

$$(f\circ g)(x) = f(g(x)) = f\left(\sqrt{x^2-1}\right)$$

$$= \sqrt{3-\sqrt{x^2-1}} \Rightarrow \text{New function}$$

$D_{g(x)} = (-\infty,-1] \cup [1,\infty)$

D_{New}: $3-\sqrt{x^2-1} \geq 0$

$$-\sqrt{x^2-1} \geq -3$$

$$\sqrt{x^2-1} \leq 3$$

$$0 \leq \sqrt{x^2-1} \leq 3$$

$$0^2 \leq \left(\sqrt{x^2-1}\right)^2 \leq 3^2$$

$$0 \leq x^2-1 \leq 9$$

$$0+1 \leq x^2 \leq 9+1$$

$$1 \leq x^2 \leq 10$$

$$1 \leq \sqrt{x^2} \leq \sqrt{10}$$

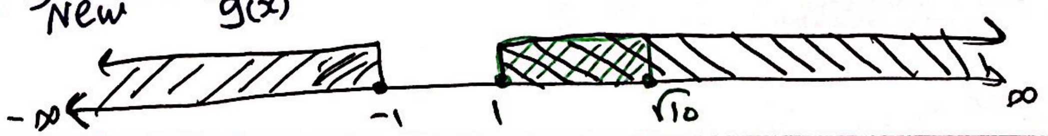
$$1 \leq |x| \leq \sqrt{10}$$

$$1 \leq x \leq \sqrt{10}$$

Note : $|x| = x$
بسبب أنه |x| محصورة
بين عددين موجبين

$D_{New} = [1,\sqrt{10}]$

$\Rightarrow D_{(f\circ g)(x)} = D_{New} \cap D_{g(x)} = [1,\sqrt{10}] \cap ((-\infty,-1] \cup [1,\infty)) = [1,\sqrt{10}$



(33) 1.3 الواجب من درس

$f(x) = 3x + 5 \quad ; \quad g(x) = x^2 + x$

a) $(f \circ g)(x) = f(g(x)) = f(x^2+x) = 3(x^2+x)+5$

$= 3x^2 + 3x + 5$ New function

$D_{(f\circ g)(x)} = D_{New} \cap D_g = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

b) $(g \circ f)(x) = g(f(x)) = g(3x+5)$

$= (3x+5)^2 + (3x+5)$

$= 9x^2 + 30x + 25 + 3x + 5$

$= 9x^2 + 33x + 30$ New function

$D_{(g\circ f)(x)} = D_{New} \cap D_f = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

c) $(f \circ f)(x) = f(f(x)) = f(3x+5) = 3(3x+5)+5$

$= 9x + 15 + 5$

$= 9x + 20$ New function

$D_{(f\circ f)(x)} = D_{New} \cap D_f = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

d) $(g \circ g)(x) = g(g(x)) = g(x^2+x) = (x^2+x)^2 + (x^2+x)$

$= x^4 + 2x^3 + x^2 + x^2 + x$

$= x^4 + 2x^3 + 2x^2 + x$ New Function

$D_{(g\circ g)(x)} = D_{New} \cap D_g = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

35) $f(x) = \sqrt{x+1}$ $g(x) = 4x - 3$

$D_{f(x)} : x+1 \geq 0$

$x \geq -1$

$\therefore \boxed{D_{f(x)} = [-1, \infty)}$

$\boxed{D_{g(x)} = \mathbb{R} = (-\infty, \infty)}$

a) $(f \circ g)(x) = f(g(x)) = f(4x-3) = \sqrt{4x-3+1}$

$= \sqrt{4x-2}$ New function

$D_{New} : 4x-2 \geq 0 \Rightarrow 4x \geq 2 \Rightarrow \frac{4x}{2} \geq \frac{2}{4} \Rightarrow x \geq \frac{1}{2}$

$\therefore \boxed{D_{New} = [1/2, \infty)}$

$D_{(f \circ g)(x)} = D_{New} \cap D_g = [1/2, \infty) \cap \mathbb{R} = [1/2, \infty)$

b) $(g \circ f)(x) = g(f(x)) = g(\sqrt{x+1}) = \underbrace{4\sqrt{x+1} - 3}_{\text{New Function}}$

Domain of New function $=$ (Domain of $4\sqrt{x+1}$) $\cap$ (Domain of -3)

$= [-1, \infty) \cap \mathbb{R}$

$= [-1, \infty)$

$D_{(g \circ f)(x)} = D_{New} \cap D_f = [-1, \infty) \cap [-1, \infty) = [-1, \infty)$

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b) $(f\circ f)(x) = f(f(x)) = f(\sqrt{x+1})$

$= \sqrt{\sqrt{x+1}+1}$ New function

D_{New} : $\sqrt{x+1}+1 \geq 0$

$\sqrt{x+1} \geq -1$ لا مستحيل لأن $\sqrt{x+1}$ أكبر من صفر

$\sqrt{x+1} \geq 0 \Rightarrow (\sqrt{x+1})^2 \geq 0^2$

$\therefore D_{New} = [-1, \infty) \Rightarrow x+1 \geq 0$

$x \geq -1$

$D_{(f\circ f)(x)} = D_{New} \cap D_f = [-1,\infty) \cap [-1,\infty)$

$= [-1, \infty)$

c) $(g\circ g)(x) = g(g(x)) = g(4x-3)$

$= 4(4x-3) - 3$

$= 16x - 12 - 3$

$= 16x - 15$ New function

$D_{New} = \mathbb{R}$

$D_{g\circ g(x)} = D_{New} \cap D_{g(x)} = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$

38) $f(x)=\sqrt{x}$; $g(x)=\sqrt[3]{1-x}$

$D_{f(x)}: x \geq 0 \Rightarrow \boxed{D_{f(x)}=[0,\infty)}$

$D_{g(x)}: \boxed{D_{g(x)}=\mathbb{R}}$

a) $(f\circ g)(x) = f(g(x)) = f(\sqrt[3]{1-x}) = \sqrt[2]{\sqrt[3]{1-x}} = \sqrt[6]{1-x}$ New function

$D_{New}: 1-x \geq 0 \Rightarrow -x \geq -1 \Rightarrow x \leq 1$

$\therefore \boxed{D_{New}=(-\infty,1]}$

$D_{(f\circ g)(x)} = D_{New} \cap D_{g(x)}$

$= (-\infty,1] \cap \mathbb{R} = (-\infty,1]$

b) $(g\circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt[3]{1-\sqrt{x}}$ New function

$D_{New} = D_{\sqrt[3]{x}} \cap D_{(1-\sqrt{x})}$

$= \mathbb{R} \cap [0,\infty)$

$= [0,\infty)$

$D_{(g\circ f)(x)} = D_{New} \cap D_{f(x)}$

$= [0,\infty) \cap [0,\infty)$

$= [0,\infty)$

c) $(f \circ f)(x) = f(f(x))$

$$= f(\sqrt{x})$$

$$= \sqrt[2]{\sqrt[2]{x}}$$

$$= \sqrt[4]{x} \quad \text{New function}$$

$D_{New} : x \geq 0$

$\therefore D_{New} = [0, \infty)$

$$D_{(f \circ f)(x)} = D_{New} \cap D_{f(x)}$$

$$= [0, \infty) \cap [0, \infty)$$

$$= [0, \infty)$$

d) $(g \circ g)(x) = g(g(x)) = g(\sqrt[3]{1-x}) = \sqrt[3]{1 - \sqrt[3]{1-x}}$ New function

$$D_{New} = D_{\sqrt[3]{x}} \cap D_{(1-\sqrt[3]{1-x})}$$

$$= \mathbb{R} \cap \mathbb{R}$$

$$= \mathbb{R}$$

$$\therefore D_{(g \circ g)(x)} = D_{New} \cap D_{g(x)}$$

$$= \mathbb{R} \cap \mathbb{R}$$

$$= \mathbb{R}$$

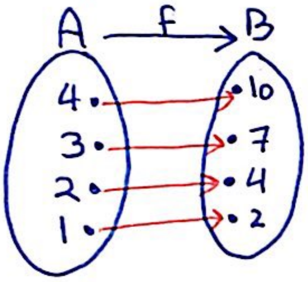
1.5 - Inverse Functions and Logarithms

Definition: A function f is called a One-to-One function if it never takes on the same value twice; that is $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$

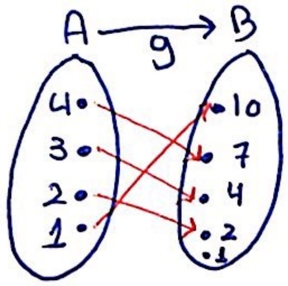
or $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2$

or $x_1 \neq x_2 \Leftrightarrow f(x_1) \neq f(x_2)$

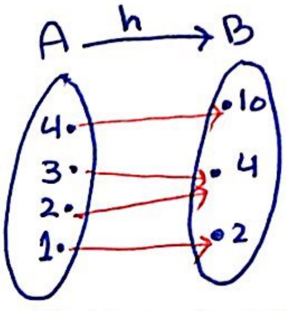
Example:



f is one-to-one function.



g is 1-1 function



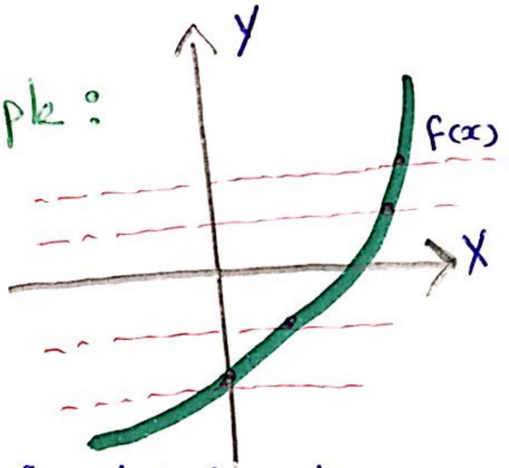
h is not one to one function since: $3 \to 4$, $2 \to 4$

or $h(3) = h(2) = 4$

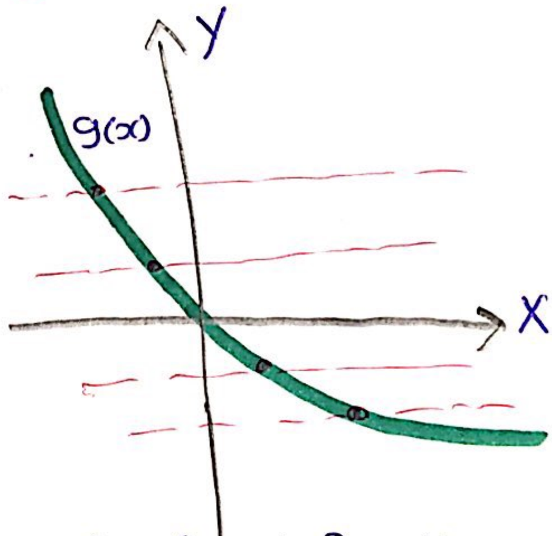
Horizontal Line Test

A function is one-to-one if and only if no horizontal line intersects its graph mor than once

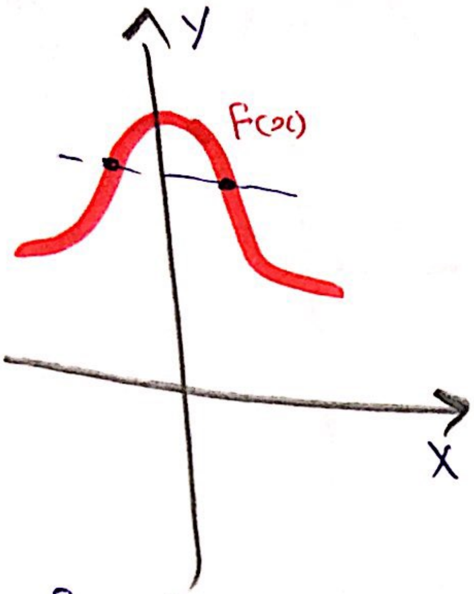
Example:



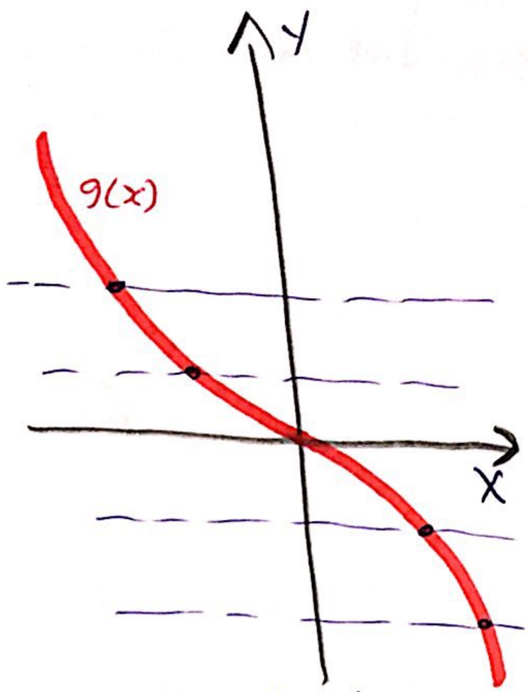
$f(x)$ is 1-1 function.



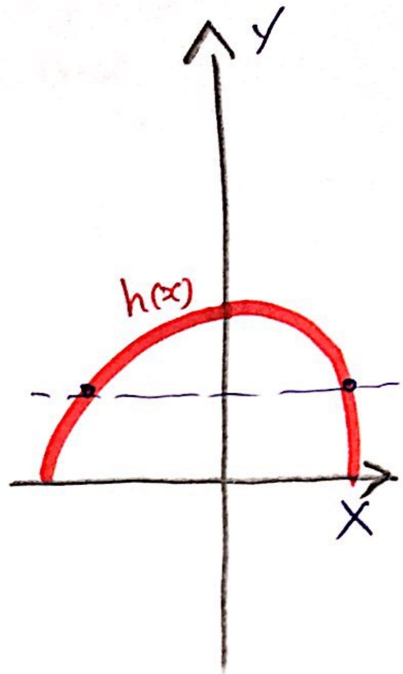
$g(x)$ is 1-1 Function



f(x) is not 1-1 function



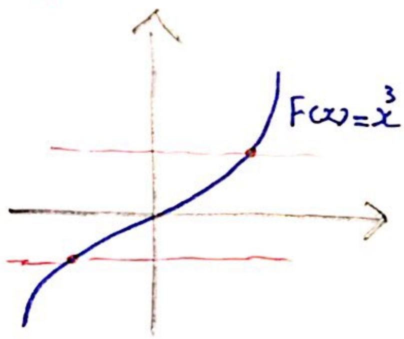
g(x) is 1-1 function



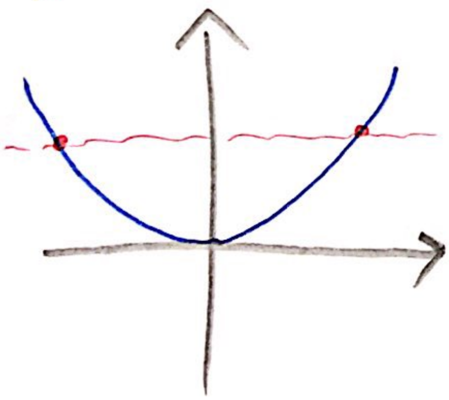
h(x) is not 1-1 function

Example:- A function is given by a formula. Determine whether it is 1-1

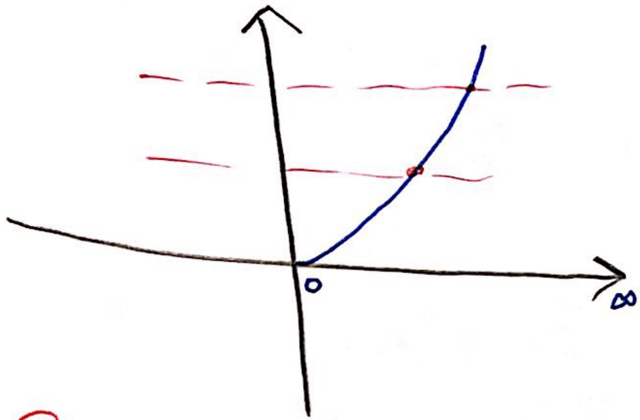
a) $f(x) = x^3$ is 1-1 function



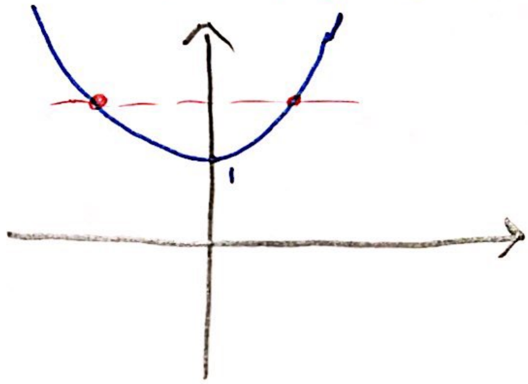
b) $f(x) = x^2$ is not 1-1 function



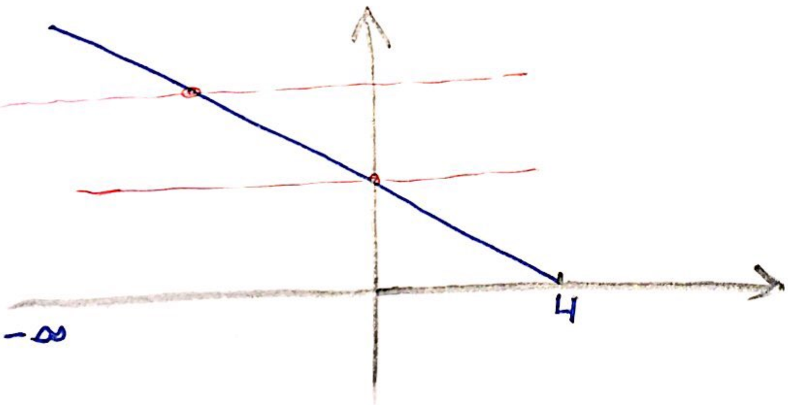
c) $f(x) = x^2$ on $[0, \infty)$ is 1-1 function



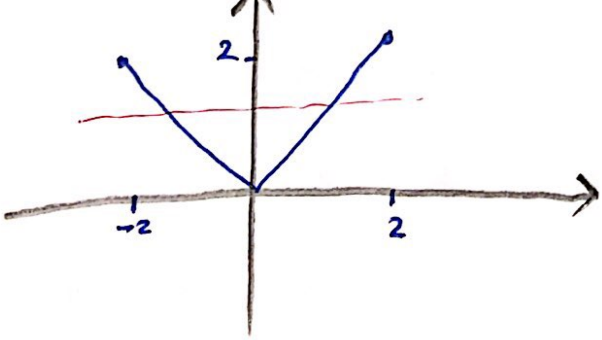
d) $f(x) = x^2 + 1$ is not 1-1 function



e) $f(x) = |x - 4|$ on $(-\infty, 4]$ is 1-1 function



f) $f(x) = |x|$ on $[-2, 2]$ is not 1-1 function



(g) $f(x) = \sqrt[3]{1-x^5}$

let $x_1 \neq x_2$

$x_1^5 \neq x_2^5$

$-x_1^5 \neq -x_2^5$

$1-x_1^5 \neq 1-x_2^5$

$\sqrt[3]{1-x_1^5} \neq \sqrt[3]{1-x_2^5}$

$f(x_1) \neq f(x_2)$

$\because x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$\therefore f(x) = \sqrt[3]{1-x^5}$ is 1-1 function

(h) $f(x) = \dfrac{x-1}{x+3}$

let $f(x_1) = f(x_2)$

$$\frac{x_1-1}{x_1+3} = \frac{x_2-1}{x_2+3}$$

$$(x_1-1)(x_2+3) = (x_1+3)(x_2-1)$$

$$\cancel{x_1x_2} + 3x_1 - x_2 \cancel{-3} = \cancel{x_1x_2} - x_1 + 3x_2 \cancel{-3}$$

$$3x_1 - x_2 = -x_1 + 3x_2$$

$$3x_1 + x_1 = 3x_2 + x_2$$

$$\cancel{4}x_1 = \cancel{4}x_2$$

$$x_1 = x_2$$

$\because f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$\therefore f(x) = \frac{x-1}{x+3}$ is 1-1 function.

(i) $f(x) = \sqrt{x}$

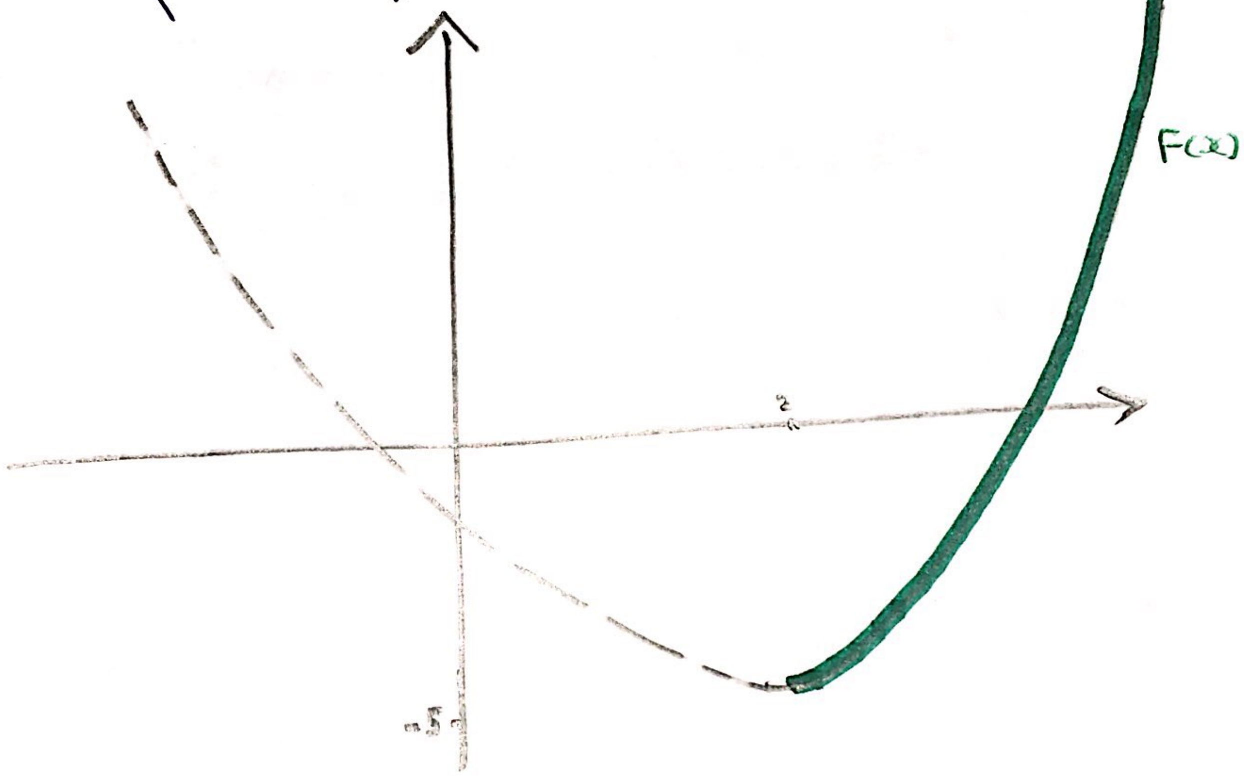
H.W

(J) $f(x) = x^2 - 4x - 1$ on $[2, \infty)$

$= x^2 - 4x + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 - 1$

$= x^2 - 4x + 4 - 4 - 1$

$= (x - 2)^2 - 5$ is 1-1 function on $[2, \infty)$



The inverse function

if $f(x) : A \longrightarrow B$ is 1-1 function with

$D_{f(x)} = A$ and $R_{f(x)} = B$ " $R_{f(x)}$ = Co-Domain "

then $f^{-1}(x) : B \longrightarrow A$ with $D_{f^{-1}(x)} = B$ and $R_{f^{-1}(x)} = A$

Note: ① Domain of f^{-1} = Range of f

Range of f^{-1} = Domain of f

② $f(x) = y \iff f^{-1}(y) = x$

$f(y) = x \iff f^{-1}(x) = y$

③ $f^{-1}(x) \neq \frac{1}{f(x)}$

Example:

① if $f(x) = \sqrt{3-x}$ then find Range of $f^{-1}(x)$

$R_{f^{-1}(x)} = D_{f(x)} = (-\infty, 3]$ " من درس 1.1 "

② if $f(2) = 5$ then $f^{-1}(5) = 2$ ...

if $f^{-1}(3) = 7$ then $f(7) = 3$ ....

③ if $f(x) = \sqrt{x} + 10$ then find Domain of $f^{-1}(x)$

$D_{f^{-1}(x)} = R_{f(x)} = [10, \infty)$ " من درس 1.3 "

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③ if $f(x) = x^3 - 1$ then find $f^{-1}(7)$

$$f^{-1}(7) = x \implies f(x) = 7$$

$$x^3 - 1 = 7$$

$$x^3 = 7 + 1$$

$$x^3 = 8$$

$$\sqrt[3]{x^3} = \sqrt[3]{8}$$

$$x = 2$$

$$\therefore f^{-1}(7) = 2$$

Note

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = x$ for every $x \in D_{f^{-1}(x)} = B$

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$ for every $x \in D_{f(x)} = A$

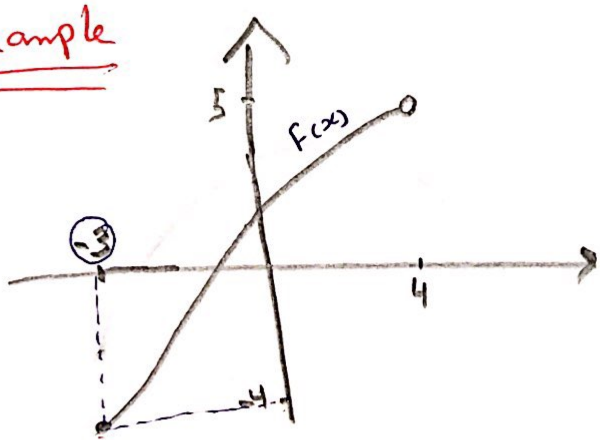
Example

if $f(x) = \sqrt[3]{3-2x}$ then find

a) $(f \circ f^{-1})(2) = f(f^{-1}(2)) = 2 \in D_{f^{-1}(x)} = \mathbb{R}$

b) $(f^{-1} \circ f)(2) = f^{-1}(f(2)) = 2 \in D_{f(x)} = \mathbb{R}$

Example



① $D_{f^{-1}(x)} = R_{f(x)} = [-4, 5)$

② $R_{f^{-1}(x)} = D_{f(x)} = [-3, 4)$

③ $f^{-1}(-4) = -3$

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How to find the inverse function of 1-1 function

① Write $y = f(x)$

② Solve this equation for x in terms of y

③ To express f^{-1} as a function of x,

interchange x and y the resulting equation is $y = f^{-1}(x)$

Example

find the inverse function $f(x) = x^3 + 2$

① $y = x^3 + 2$

② $y - 2 = x^3$

$\sqrt[3]{y-2} = \sqrt[3]{x^3}$

$\sqrt[3]{y-2} = x$

③ $\sqrt[3]{x-2} = y$

④ $f^{-1}(x) = \sqrt[3]{x-2}$

Example

Find $f^{-1}(x)$ of the following functions:

1) $f(x) = 1 + \sqrt{2+3x}$

a) $y = 1 + \sqrt{2+3x}$

b) $y - 1 = \sqrt{2+3x}$

$$(y-1)^2 = \left(\sqrt{2+3x}\right)^2$$

$$(y-1)^2 = 2 + 3x$$

$$(y-1)^2 - 2 = 3x$$

$$\frac{(y-1)^2 - 2}{3} = \frac{3x}{3}$$

$$\frac{(y-1)^2 - 2}{3} = x$$

c) $\dfrac{(x-1)^2 - 2}{3} = y$

d) $f^{-1}(x) = \frac{1}{3}\left[(x-1)^2 - 2\right]$

② $f(x) = \frac{4x-1}{2x+3}$

$y = \frac{4x-1}{2x+3}$

$y(2x+3) = 4x-1$

$2yx + 3y = 4x - 1$

$2yx - 4x = -1 - 3y$

$x(2y-4) = -1 - 3y$

$x = \frac{-1-3y}{2y-4}$

$y = \frac{-1-3x}{2x-4}$

$f^{-1}(x) = \frac{-1-3x}{2x-4} = \frac{-(3x+1)}{-(4-2x)} = \frac{3x+1}{4-2x}$

③ $f(x) = e^{2x-1}$

$y = e^{2x-1}$

$Ln(y) = Ln\left(e^{2x-1}\right)$

$Ln(y) = 2x-1$

$Ln(y) + 1 = 2x$

$\frac{Ln(y)+1}{2} = x$

$\frac{Ln(x)+1}{2} = y \implies f^{-1}(x) = \frac{Ln(x)+1}{2} = \frac{1}{2}\left[Ln(x)+1\right]$

④ $y = \ln(x+3)$

$$y = \ln(x+3)$$

$$e^{y} = e^{\ln(x+3)}$$

$$e^{y} = x+3$$

$$e^{y} - 3 = x$$

$$e^{x} - 3 = y$$

$$f^{-1}(x) = e^{x} - 3$$

⑤ $y = \frac{e^{x}}{1+2e^{x}}$

$$y = \frac{e^{x}}{1+2e^{x}}$$

$$y(1+2e^{x}) = e^{x}$$

$$y + 2ye^{x} = e^{x}$$

$$2ye^{x} - e^{x} = -y$$

$$e^{x}(2y-1) = -y$$

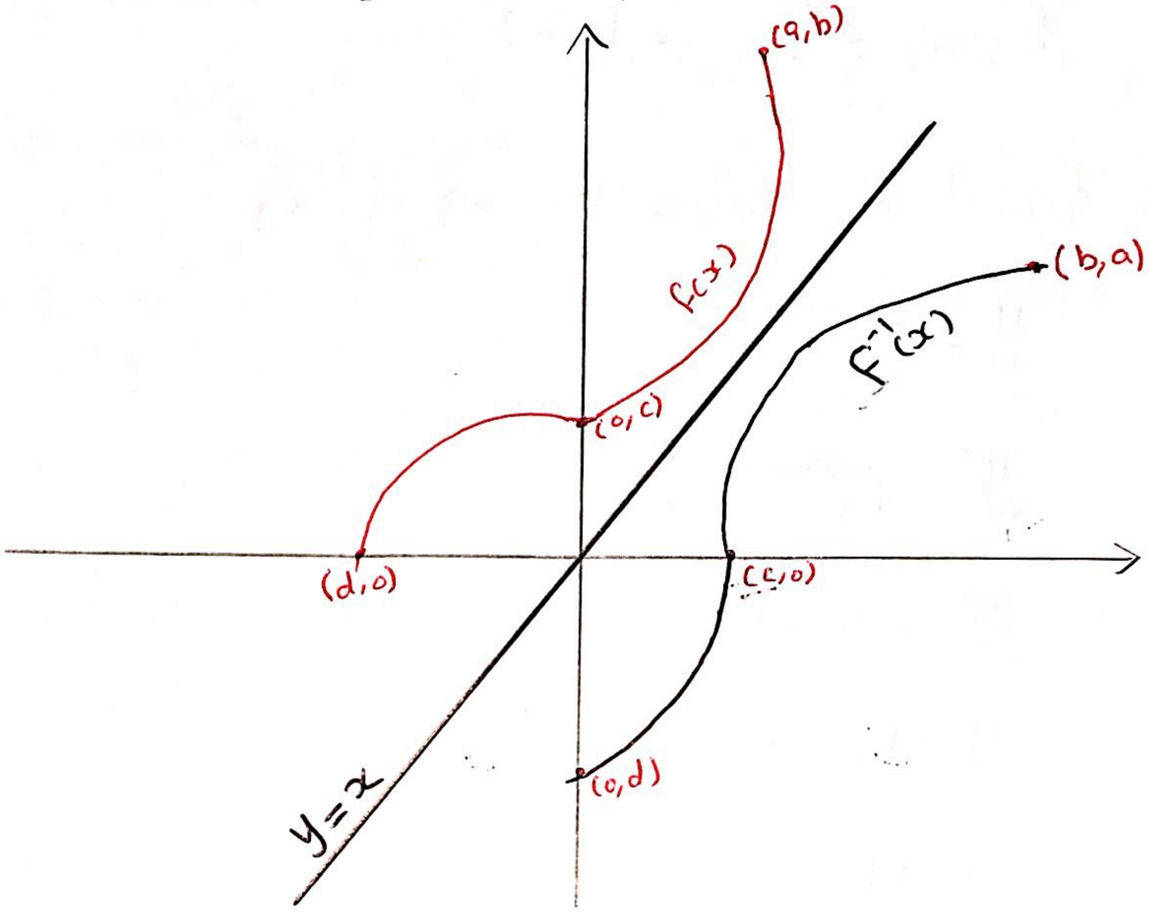
$$e^{x} = \frac{-y}{2y-1}$$

$$\ln e^{x} = \ln\left(\frac{-y}{2y-1}\right)$$

$$x = \ln\left(\frac{y}{1-2y}\right) \implies y = \ln\left(\frac{x}{1-2x}\right) = f^{-1}(x)$$

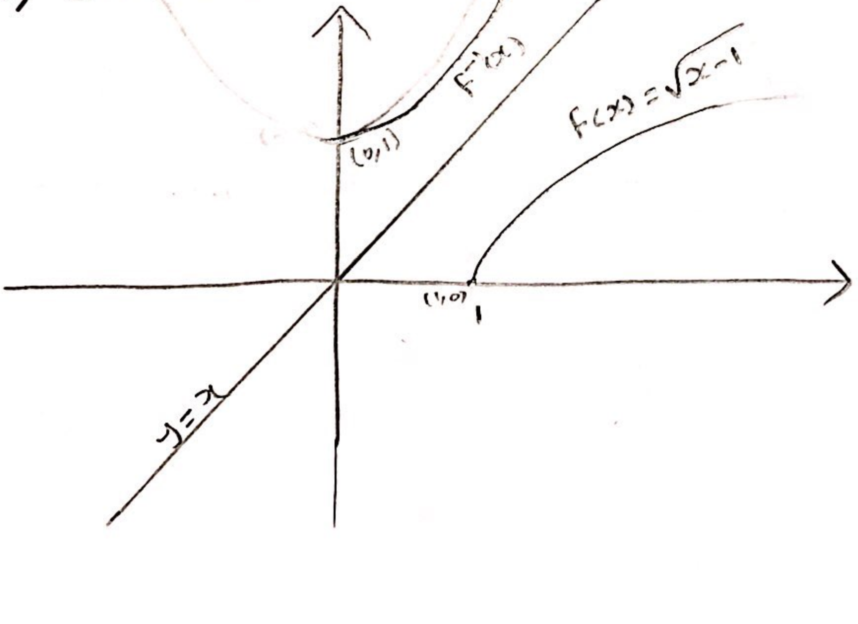
Note

The graph of f^{-1} is obtained by reflecting the graph f about Line $y=x$



Example if $f(x)=\sqrt{x-1}$ then

a) sketch $f(x)$ and $f^{-1}(x)$



b) Find the Domain and Range of $f^{-1}(x)$

$$D_{f^{-1}(x)} = R_{f(x)} = [0, \infty)$$

$$R_{f^{-1}(x)} = D_{f(x)} = [1, \infty)$$

c) Find the formula of $f^{-1}(x)$

$$y = \sqrt{x-1}$$

$$y^2 = (\sqrt{x-1})^2$$

$$y^2 = x - 1$$

$$y^2 + 1 = x$$

$$x^2 + 1 = y$$

$f^{-1}(x) = x^2 + 1$ on $[0, \infty)$ " $D_{f^{-1}(x)}$ "

Logarithmic function

Lows of logarithms

1) $\log_a 1 = 0$; $\ln(1) = 0$

Example: $\log_3 1 = 0$

2) $\log_a a = 1$; $\ln(e) = 1$

Example: $\log_7 7 = 1$

3) $\log_a b = \dfrac{\ln(b)}{\ln(a)}$; $\ln(x) = \log_e x$

Example: $\log_2 3 = \dfrac{\ln 3}{\ln 2}$; $\dfrac{\ln(5x)}{\ln(7)} = \log_7 (5x)$

4) $\log_a x = y \iff a^y = x$

$\ln(x) = y \iff e^y = x$

Example: $\log_a 2b = c \iff a^c = 2b$

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$\ln(5d) = 3+d \iff e^{(3+d)} = 5d$.

5) $\log_a x^n = n\log_a x$

$\ln(x^n) = n\ln(x)$

Example: $\log_4 x^3 = 3\log_4 x$

$\log_3 9 = \log_3 3^2$

$= 2\log_3 3$

$= 2(1)$

$= 2$

$\ln(y^7) = 7\ln(y)$

$\ln(9) = \ln(3^2)$

$= 2\ln(3)$

6) $\log_a(xy) = \log_a x + \log_a y$

$\ln(xy) = \ln(x) + \ln(y)$

7) $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

Note

✓ $\log_a(x \pm y) \neq \log_a x \pm \log_a y$

Example:

$\log_2(x^3 \pm y^4) = 3\log_2 x \pm 4\log_2 y$ ✓ ⓧ

✓ $\ln(x \pm y) \neq \ln(x) \pm \ln(y)$

Example: $\log_3(3x^4) = \log_3 3 + \log_3 x^4 = 1 + 4\log_3 x$

$\ln(3x^4) = \ln(3) + \ln(x^4) = \ln(3) + 4\ln(x)$

$\log_3(15) = \log_3(3 \times 5) = \log_3 3 + \log_3 5 = 1 + \log_3 5$

$\ln(15) = \ln(3 \times 5) = \ln(3) + \ln(5)$

$\log_5\left(\frac{1}{125}\right) = \log_5(1) - \log_5 125 = 0 - \log_5 5^3 = -3\log_5 5$
$= -3(1) = -3$

$\log_2 80 - \log_2 5 = \log_2\left(\frac{80}{5}\right) = \log_2 16 = \log_2 2^4 = 4\log_2 2$
$= 4(1)$
$= 4$

$\ln 80 - \ln 5 = \ln\left(\frac{80}{5}\right)$
$= \ln(16)$
$= \ln(2^4)$
$= 4\ln(2)$

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⑧ $\log_a a^x = x$ for all $x \in \mathbb{R}$

$a^{\log_a x} = x$ for all $x > 0$

⑨ $Ln(e^x) = x$ for all $x \in \mathbb{R}$

$e^{Ln x} = x$ for all $x > 0$

Example: $Ln e^{x+1} = x+1$

$e^{Ln(x+1)} = x+1$

$e^{Ln x + 1} = e^{Ln x} \cdot e^1 = xe$

$Ln e^{x^2} = x^2$

$e^{Ln(\frac{1}{2})} = \frac{1}{2}$

$\log_{11} 11^5 = 5$

$10^{\log \sqrt{2}} = \sqrt{2}$

⑩ • if $f(x) = Ln(g(x))$ or $f(x) = \log_a(g(x))$ then

$D_{f(x)} : g(x) > 0$

Example: Find the Domain of $f(x) = Ln(25 - x^2)$

$25 - x^2 > 0 \Rightarrow -x^2 > -25 \Rightarrow x^2 < 25$

$\Rightarrow \sqrt{x^2} < \sqrt{25}$

$\Rightarrow |x| < 5$

$\Rightarrow -5 < x < +5$

$\therefore D_{f(x)} = (-5, 5)$

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Example: Find the Domain of the following Functions

① $f(x) = \ln(25 - x^2)$

$25 - x^2 > 0$

$-x^2 > -25$

$x^2 < 25$

$\sqrt{x^2} < \sqrt{25}$

$|x| < 5$

$-5 < x < 5$

$\therefore D_{f(x)} = (-5, 5)$

$= \{x \mid -5 < x < 5\}$

② $\ln(x^2 - 100) = g(x)$

$x^2 - 100 > 0$

$x^2 > 100$

$\sqrt{x^2} > \sqrt{100}$

$|x| > 10$

$x > 10$ or $x < -10$

$\therefore D_{g(x)} = (10, \infty) \cup (-\infty, -10)$

or $D_{g(x)} = \{x \mid x > 10 \text{ or } x < -10\}$

or $D_{g(x)} = \mathbb{R} - [-10, 10]$

③ $h(x) = \log_5(x^2 + 16)$

$x^2 + 16 > 0$

$x^2 > -16$

$\sqrt{x^2} > \sqrt{-16}$

$|x| > \sqrt{-16} \notin \mathbb{R}$

$\therefore D_{h(x)} = \mathbb{R}$

④ $f(x) = \log_3(x^3 + 1)$

$x^3 + 1 > 0$

$x^3 > -1$

$\sqrt[3]{x^3} > \sqrt[3]{-1}$

$x > -1$

$\therefore D_{f(x)} = (-1, \infty)$

5) $g(x) = \ln(x^2 - 6)$

H.W

⑥ $\log_7(6 - x^2) = h(x)$

H.W

⑦ $f(x) = \log_2(x^2 + 5)$

H.W

⑧ $f(x) = \ln(x^3 - 8)$

H.W

Example

① Find the Domain of $f(x) = \ln(e^x - 3)$

$$e^x - 3 > 0 \Rightarrow e^x > 3$$

$$\ln e^x > \ln 3$$

$$x > \ln 3$$

$$\therefore D_{f(x)} = (\ln 3, \infty)$$

② Find the rang of $f^{-1}(x)$

$$R_{f^{-1}(x)} = D_{f(x)} = (\ln 3, \infty)$$

③ Find $f^{-1}(x)$ and find the Domain of $f^{-1}(x)$

$$y = \ln(e^x - 3)$$

$$e^y = e^{\ln(e^x - 3)}$$

$$e^y = e^x - 3$$

$$e^y + 3 = e^x$$

$$\ln(e^y + 3) = \ln e^x$$

$$\ln(e^y + 3) = x$$

$$\ln(e^x + 3) = f^{-1}(x)$$

$D_{f^{-1}(x)}$: $e^x + 3 > 0$

$$e^x > -3$$

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$$\therefore D_{f^{-1}(x)} = \mathbb{R} = (-\infty, \infty)$$

④ Find $R_{f(x)}$

$$R_{f(x)} = D_{f^{-1}(x)} = \mathbb{R}$$

Example:

$$\log_2 32 = \log_2 2^5 = 5\log_2 2 = 5(1) = 5$$

$$\ln(32) = \ln(2^5) = 5\ln(2)$$

$$\log 10000 = 4$$

$$\log 0.01 = -2$$

$$\log_4 2 = \frac{\ln(2)}{\ln(4)} = \frac{\ln(2)}{\ln(2^2)} = \frac{1\cancel{\ln(2)}}{2\cancel{\ln(2)}} = \frac{1}{2}$$

$$\log_{27} 3 = \frac{\ln(3)}{\ln(27)} = \frac{\ln(3)}{\ln(3^3)} = \frac{1\cancel{\ln(3)}}{3\cancel{\ln(3)}} = \frac{1}{3}$$

$$\log_{32} 2 = \frac{\ln(2)}{\ln(32)} = \frac{\ln(2)}{\ln(2^5)} = \frac{1\cancel{\ln(2)}}{5\cancel{\ln(2)}} = \frac{1}{5}$$

$$\log 40 + \log 2.5 = \log(40 \times 2.5)$$

$$= \log\left(4\cancel{0} \times \frac{25}{\cancel{10}}\right)$$

$$= \log(4 \times 25)$$

$$= \log(100)$$

$$= \log_{10}(100) = 2$$

$$\log_8 60 - \log_8 3 - \log_8 5 = \log_8\left(\frac{60}{3}\right) - \log_8 5$$

$$= \log_8 20 - \log_8 5$$

$$= \log_8\left(\frac{20}{5}\right) = \log_8 4$$

$$= \frac{\ln(4)}{\ln(8)} = \frac{\ln(2^2)}{\ln(2^3)} = \frac{2\cancel{\ln(2)}}{3\cancel{\ln(2)}}$$

$$= \frac{2}{3}$$

$$\log_6 36 - \log_5 125 + 3\log_7 49 = \log_6 6^2 - \log_5 5^3 + 3\log_7 7^2$$

$$= 2\log_6 6 - 3\log_5 5 + 3(2)\log_7 7$$

$$= 2(1) - 3(1) + 3(2)(1)$$

$$= 2 - 3 + 6$$

$$= -1 + 6$$

$$= 5$$

$$\ln\sqrt{x} = \ln x^{1/2} = \tfrac{1}{2}\ln x$$

$$\ln\left(\tfrac{1}{e^2}\right) = \ln(1) - \ln e^2 = 0 - 2 = -2$$

$$\tfrac{1}{2}\ln(3) - \ln(2) = \ln(3^{1/2}) - \ln(2) = \ln\sqrt{3} - \ln 2 = \ln\left(\tfrac{\sqrt{3}}{2}\right)$$

$$\ln(20) - 2\ln(2) = \ln 20 - \ln 2^2 = \ln 20 - \ln 4 = \ln\left(\tfrac{20}{4}\right) = \ln(5)$$

$$\ln b + 2\ln c - 3\ln d = \ln b + \ln c^2 - \ln d^3$$

$$= \ln(bc^2) - \ln(d^3)$$

$$= \ln\left(\frac{bc^2}{d^3}\right)$$

$$\ln\left(\frac{x^3}{\sqrt[5]{y^2}}\right) = \ln(x^3) - \ln(\sqrt[5]{y^2}) = 3\ln(x) - \ln y^{2/5}$$

$$= 3\ln(x) - \tfrac{2}{5}\ln(y)$$

$$e^{-\ln 7} = e^{\ln 7^{-1}} = 7^{-1} = \tfrac{1}{7}$$

$$e^{-3\ln 2} = e^{\ln 2^{-3}} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$\cancel{e}^{\cancel{\ln}(\ln e^3)} = \ln e^3 = 3 \quad \text{or} \quad e^{\ln(\ln e^3)} = e^{\ln(3)} = 3$$

$$\frac{1}{3}\ln(x+2)^3 + \frac{1}{2}\left[\ln x - \ln(x^2+3x+2)^2\right]$$

$$\ln\left[(x+2)^3\right]^{1/3} + \frac{1}{2}\ln x - \frac{1}{2}\ln(x^2+3x+2)^2$$

$$\ln(x+2) + \ln x^{1/2} - \ln\left[(x^2+3x+2)^2\right]^{1/2}$$

$$\ln(x+2) + \ln\sqrt{x} - \ln(x^2+3x+2)$$

$$\ln(x+2) + \ln\sqrt{x} - \ln\left[(x+1)(x+2)\right]$$

$$\ln(x+2) + \ln\sqrt{x} - \left[\ln(x+1) + \ln(x+2)\right]$$

$$\cancel{\ln(x+2)} + \ln\sqrt{x} - \ln(x+1) \cancel{- \ln(x+2)}$$

$$\ln\sqrt{x} - \ln(x+1)$$

$$\ln\left[\frac{\sqrt{x}}{x+1}\right]$$

Example: Solve each equation for x

$e^{x-1} = e^{2}$

$x-1=2$

$x=2+1$

$\boxed{x=3}$

$5^{x^2-6}=25$

$5^{x^2-6}=5^{2}$

$x^2-6=2$

$x^2=2+6$

$x^2=8$

$\sqrt{x^2}=\sqrt{8}$

$|x|=2\sqrt{2}$

$\boxed{x=\pm 2\sqrt{2}}$

$e^{-x}=5$

$\ln e^{-x}=\ln(5)$

$-x=\ln(5)$

$x=-\ln(5)$

$x=\ln(5^{-1})$

$x=\ln\left(\frac{1}{5}\right)$

$e^{5-3x}=10$

$\ln(e^{5-3x})=\ln(10)$

$5-3x=\ln(10)$

$-3x=\ln(10)-5$

$3x=-\ln(10)+5$

$x=\dfrac{-\ln(10)+5}{3}$

$=\frac{1}{3}\left(5-\ln(10)\right)$

$2^{x-5}=3$

$\ln 2^{x-5}=\ln(3)$

$(x-5)\ln(2)=\ln(3)$

$x-5=\dfrac{\ln(3)}{\ln(2)}$

$x=\dfrac{\ln(3)}{\ln(2)}+5$

$\boxed{x=\log_{2}3+5}$

$e^{3x}=4e^{2x} \Rightarrow \ln(e^{3x})=\ln(4e^{2x})$

$\Rightarrow 3x=\ln(4)+\ln(e^{2x})$

$\Rightarrow 3x=\ln(4)+2x$

$\Rightarrow 3x-2x=\ln(4)$

$\Rightarrow x=\ln 2^{2}=2\ln 2$

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$\log_2 3x - 3 = 0$

$\log_2 3x = 3$

$2^3 = 3x$

$8 = 3x$

$\frac{8}{3} = \frac{3x}{3}$

$\boxed{\frac{8}{3} = x}$

$\log_2(3x-3) = 0$

$2^0 = 3x-3$

$1 = 3x-3$

$1+3 = 3x$

$4 = 3x$

$\frac{4}{3} = \frac{3x}{3}$

$\boxed{\frac{4}{3} = x}$

$\log(3x+10) = 2$

$10^2 = 3x+10$

$100 = 3x+10$

$100-10 = 3x$

$90 = 3x$

$\frac{90}{3} = \frac{3x}{3}$

$\boxed{30 = x}$

$\text{Ln}\,x - 5 = 0$

$\text{Ln}\,x = 5$

$e^{\text{Ln}\,x} = e^5$

$\boxed{x = e^5}$

$\text{Ln}(x-5) = 0$

$e^{\ln(x-5)} = e^0$

$x-5 = 1$

$x = 1+5$

$\boxed{x = 6}$

$\text{Ln}\,x + 2 = 3$

$\text{Ln}\,x = 3-2$

$\text{Ln}\,x = 1$

$e^{\text{Ln}\,x} = e^1$

$\boxed{x = e}$

$\text{Ln}(x+2) = 3$

$e^{\text{Ln}(x+2)} = e^3$

$x+2 = e^3 \Rightarrow \boxed{x = e^3 - 2}$

$2\,\text{Ln}\,x - 1 = 0$

$2\,\text{Ln}\,x = 1$

$\text{Ln}\,x = \frac{1}{2}$

$e^{\text{Ln}\,x} = e^{1/2}$

$x = e^{1/2}$

$\boxed{x = \sqrt{e}}$

$\text{Ln}\,x^2 - 1 = 3$

$\text{Ln}\,x^2 = 3+1$

$2\,\text{Ln}\,x = 4$

$\text{Ln}\,x = \frac{4}{2}$

$\text{Ln}\,x = 2$

$e^{\text{Ln}\,x} = e^2$

$\boxed{x = e^2}$

HOW

$\text{Ln}(5-2x) = 3$

$\text{Ln}\,5 - 2x = 3$

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Solve : a) $\ln(x^2-1)=3$

$$e^{\ln(x^2-1)}=e^3$$

$$x^2-1=e^3$$

$$x^2=e^3+1$$

$$\sqrt{x^2}=\sqrt{e^3+1}$$

$$|x|=\sqrt{e^3+1}$$

$$x=\pm\sqrt{e^3+1}$$

b) $e^{2x}-3e^x+2=0$

$$(e^x)^2-3e^x+2=0$$

$$(e^x-1)(e^x-2)=0$$

$$e^x-1=0 \qquad\qquad e^x-2=0$$

$$e^x=1 \qquad\qquad e^x=2$$

$$\ln e^x=\ln 1 \qquad\qquad \ln e^x=\ln 2$$

$$\boxed{x=0} \qquad\qquad \boxed{x=\ln 2}$$

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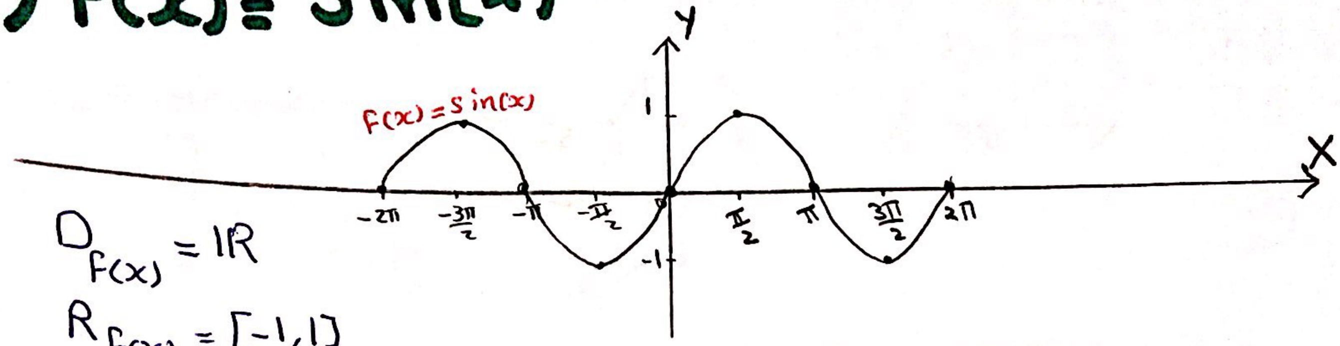
Inverse Trigonometric Functions

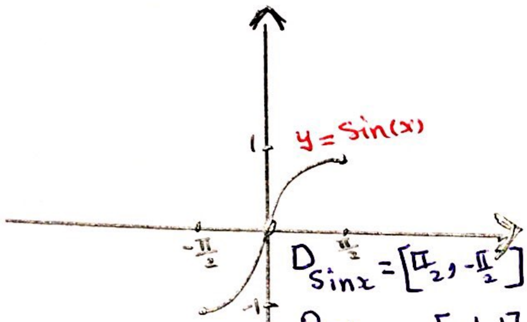
1) $f(x) = \sin(x)$ is not 1-1



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [-1, 1]$

نقوم بتصغير المجال لتكون الدالة 1-1

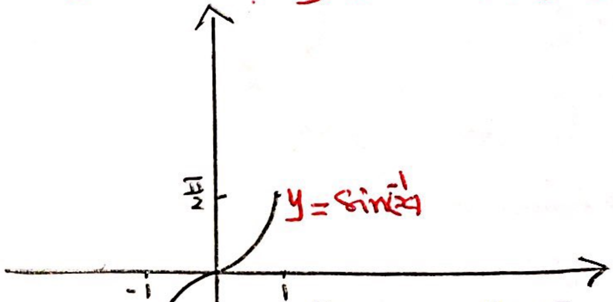


$D_{\sin x} = [\frac{\pi}{2}, -\frac{\pi}{2}]$

$R_{\sin x} = [-1, 1]$

$y = \sin(x)$ is 1-1

$f^{-1}(x) = \sin^{-1} x$ is exist



$D_{\sin^{-1}(x)} = [-1, 1]$

$R_{\sin^{-1}(x)} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

Note

① $y = \sin(x) \iff \sin^{-1}(y) = x$

$x = \sin(y) \iff \sin^{-1}(x) = y$.

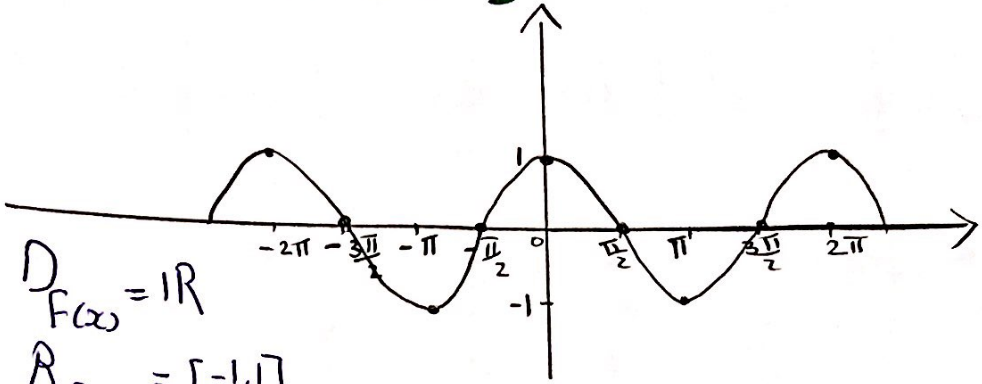
② $f(x) = \sin^{-1}(x)$ is odd function and symmetric about $(0,0)$

i.e $\sin^{-1}(-x) = -\sin^{-1}(x)$

③ $\sin(\sin^{-1} x) = x$ for all $x \in [-1, 1]$ or $-1 \leq x \leq 1$

$\sin^{-1}(\sin x) = x$ for all $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ or $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

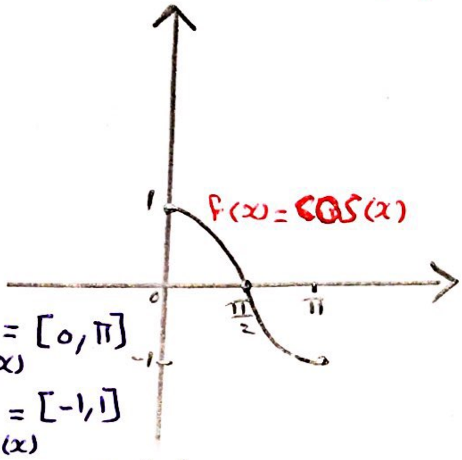
2) $f(x) = \cos(x)$ is not 1-1



$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [-1, 1]$

نقوم بتصغير المجال لتكون الدالة 1_1

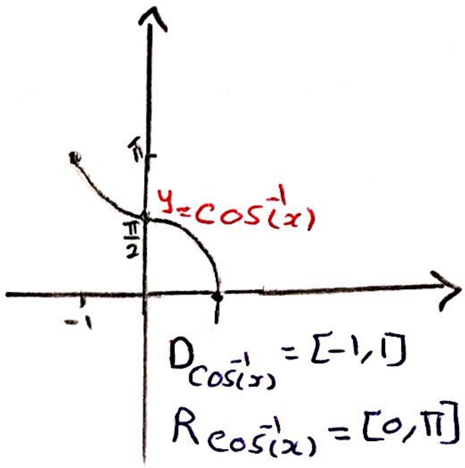


$D_{\cos(x)} = [0, \pi]$

$R_{\cos(x)} = [-1, 1]$

$f(x) = \cos(x)$ is 1-1

$f^{-1}(x) = \cos^{-1}(x)$ is exist



$D_{\cos^{-1}(x)} = [-1, 1]$

$R_{\cos^{-1}(x)} = [0, \pi]$

Note

① $y = \cos(x) \iff \cos^{-1}(y) = x \quad \forall\ 0 \le x \le \pi$

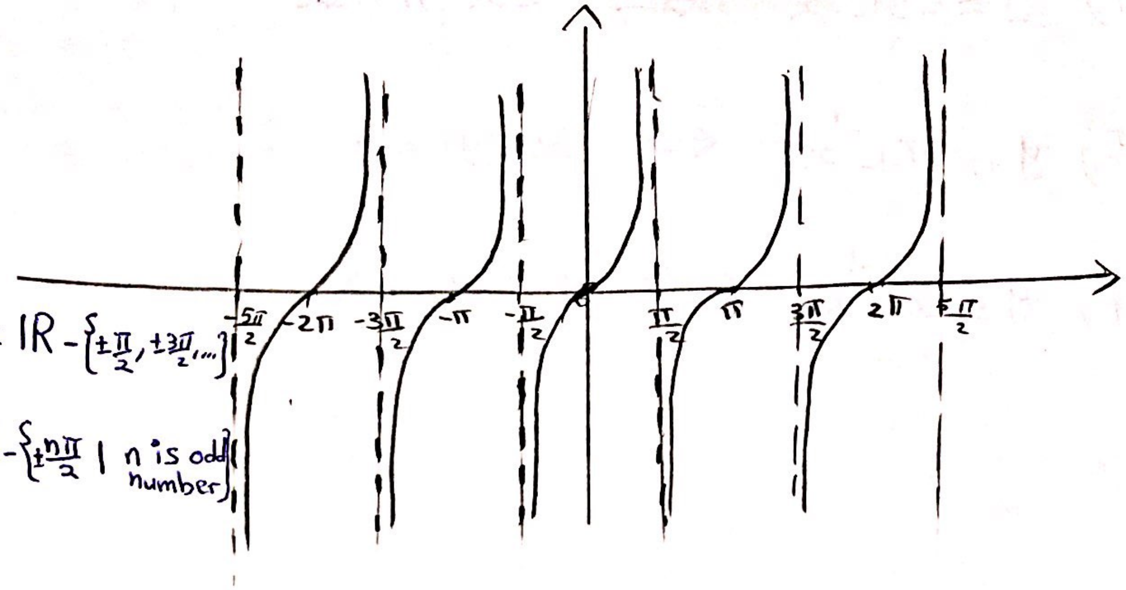
$x = \cos(y) \iff \cos^{-1}(x) = y \quad \forall\ 0 \le y \le \pi$

② $f(x) = \cos^{-1}(x)$ is niether odd nor even

③ $\cos(\cos^{-1}(x)) = x$ for all $x \in [-1, 1]$

$\cos^{-1}(\cos(x)) = x$ for all $x \in [0, \pi]$

3) $f(x) = \tan(x)$ is not 1-1



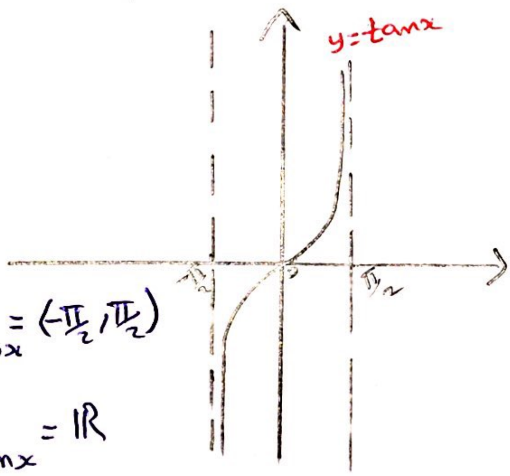
$D_{f(x)} = \mathbb{R} - \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots\}$

$= \mathbb{R} - \{\pm\frac{n\pi}{2} \mid n \text{ is odd number}\}$

$= \mathbb{R} - \{(2n+1)\frac{\pi}{2} \mid n \in \mathbb{Z}\}$

$R_{f(x)} = \mathbb{R}$

نقوم بتصغير المجال لتكون الدالة 1-1

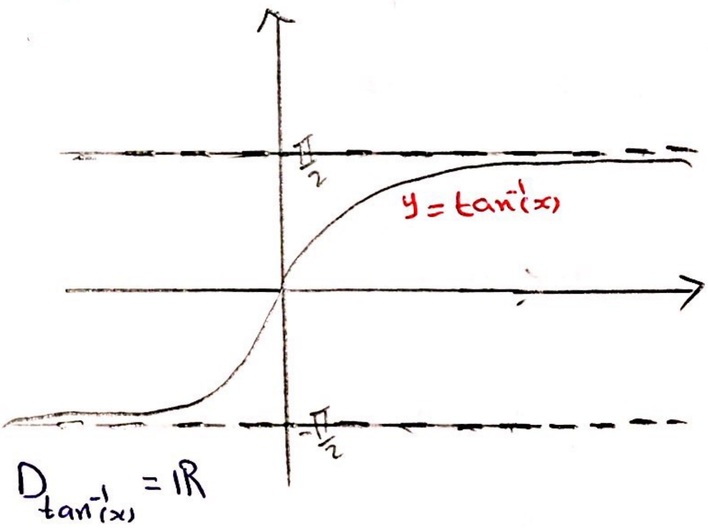


$D_{\tan x} = (-\frac{\pi}{2}, \frac{\pi}{2})$

$R_{\tan x} = \mathbb{R}$

$y = \tan x$ is 1-1

$f^{-1}(x) = \tan^{-1} x$ exist



$D_{\tan^{-1}(x)} = \mathbb{R}$

$R_{\tan^{-1}(x)} = (-\frac{\pi}{2}, \frac{\pi}{2})$

Note

① $y = \tan x \Leftrightarrow \tan^{-1}(y) = x$

$x = \tan y \Leftrightarrow \tan^{-1}(x) = y$

② $f(x) = \tan^{-1}(x)$ is odd function and symmetric about origin

i.e $\tan^{-1}(-x) = -\tan^{-1}(x)$

③ $\tan(\tan^{-1}(x)) = x \quad \forall \; x \in \mathbb{R}$

$\tan^{-1}(\tan(x)) = x \quad \forall \; x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

4) $y = \csc^{-1} x \iff \csc(y) = x$

5) $y = \sec^{-1} x \iff \sec(y) = x$

6) $y = \cot^{-1} x \iff \cot(y) = x$

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Example:

Find the exact value of each expression

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$\cot\theta$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$
$\sec\theta$	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2
$\csc\theta$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

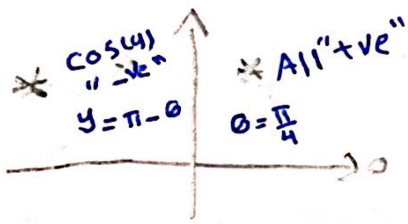
$\tan^{-1}(-1) = -\tan^{-1}(1) = -\frac{\pi}{4}$

$\cos^{-1}(-1) = \pi$

	0	$\frac{\pi}{2}$	π
$\sin\theta$	0	1	0
$\cos\theta$	1	0	-1
$\tan\theta$	0	$\frac{1}{0}$	0
$\cot\theta$	$\frac{1}{0}$	0	$\frac{-1}{0}$
$\sec\theta$	1	$\frac{1}{0}$	-1
$\csc\theta$	$\frac{1}{0}$	1	$\frac{-1}{0}$

$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$

$\cos(y) = -\frac{1}{\sqrt{2}}$



$y = \pi - \theta$

$y = \pi - \frac{\pi}{4}$

$y = \frac{4\pi - \pi}{4}$

$\boxed{y = \frac{3\pi}{4}}$

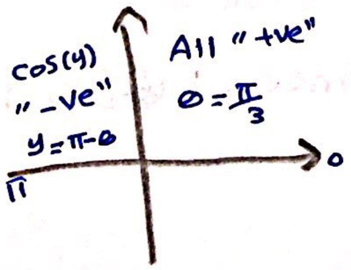
$\cos^{-1}\left(-\frac{1}{2}\right) = y \iff \cos(y) = -\frac{1}{2}$

$y = \pi - \theta$

$y = \pi - \frac{\pi}{3}$

$y = \frac{3\pi - \pi}{3}$

$\boxed{y = \frac{2\pi}{3}}$



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$\cot^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$$\frac{\sqrt{3}}{3} = \frac{\sqrt{3} \times \sqrt{3}}{3 \times \sqrt{3}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \cot^{-1}\left(\frac{\sqrt{3}}{3}\right) = \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3}$$

$$\sec^{-1}(2) = \frac{\pi}{3}$$

$$\csc^{-1}(\sqrt{2}) = \frac{\pi}{4}$$

Example

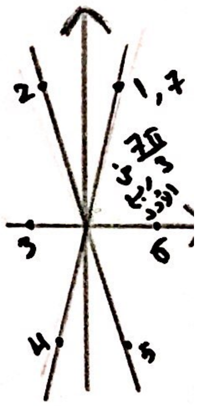
1) $\sin^{-1}\left(\sin\frac{\pi}{6}\right) = \frac{\pi}{6}$ " since $\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ "

$\sin^{-1}\left(\sin\frac{\pi}{12}\right) = \frac{\pi}{12}$ " since $\frac{\pi}{12} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ "

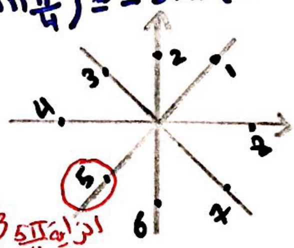
$\sin^{-1}(\sin 0) = 0$ " since $0 \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ "

$\sin^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) = -\frac{\pi}{2}$ " since $-\frac{\pi}{2} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ "

$\sin^{-1}\left(\sin\frac{7\pi}{3}\right) = \sin^{-1}\left(\boxed{+} \sin\frac{\pi}{3}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$



$\sin^{-1}\left(\sin\frac{5\pi}{4}\right) = \sin^{-1}\left(\boxed{-} \sin\frac{\pi}{4}\right) = -\sin^{-1}\left(\sin\frac{\pi}{4}\right) = -\frac{\pi}{4}$



الزاوية $\frac{5\pi}{4}$ في
ربع الثالث

2) $\sin\left(\sin^{-1}\left(\frac{2}{3}\right)\right) = \frac{2}{3}$ " since $\frac{2}{3} \in [-1, 1]$ "

$\sin\left(\sin^{-1}\left(\frac{3}{2}\right)\right) \neq \frac{3}{2}$ " since $\frac{3}{2} \notin [-1, 1]$ "

$\therefore \sin\left(\sin^{-1}\left(\frac{3}{2}\right)\right) = \text{undefined}.$

$\sin\left(\sin^{-1}(-1)\right) = -1$ " since $-1 \in [-1, 1]$

3) $\tan^{-1}\left(\tan\frac{\pi}{14}\right)=\frac{\pi}{14}$ since $\frac{\pi}{14}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\tan^{-1}\left(\tan\left(-\frac{\pi}{5}\right)\right)=-\frac{\pi}{5}$ since $-\frac{\pi}{5}\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

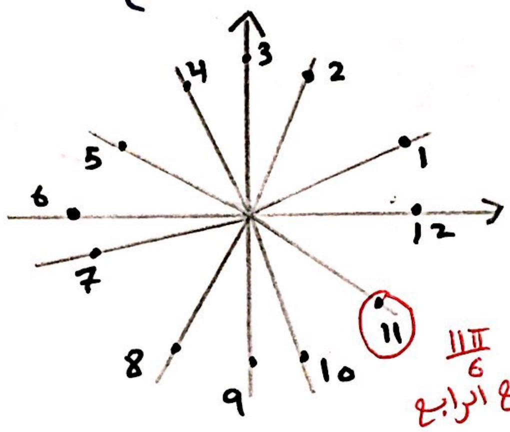
$\tan^{-1}\left(\tan(0)\right)=0$ since $0\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\tan^{-1}\left(\tan\frac{\pi}{2}\right)\neq\frac{\pi}{2}$ since $\frac{\pi}{2}\notin\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore \tan^{-1}\left(\tan\frac{\pi}{2}\right)$ is undefined.

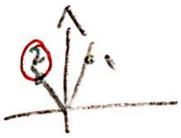
$\tan^{-1}\left(\tan\frac{11\pi}{6}\right)\neq\frac{11\pi}{6}$

$\therefore \tan^{-1}\left(\tan\frac{11\pi}{6}\right)=\tan^{-1}\left(\boxed{-}\tan\frac{\pi}{6}\right)=-\tan^{-1}\left(\tan\frac{\pi}{6}\right)=-\frac{\pi}{6}$



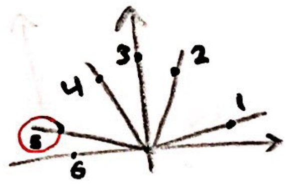
الزاوية $\frac{11\pi}{6}$
في الربع الرابع

(4) $\tan\left(\tan^{-1}(5)\right)=5$ since $5\in\mathbb{R}$



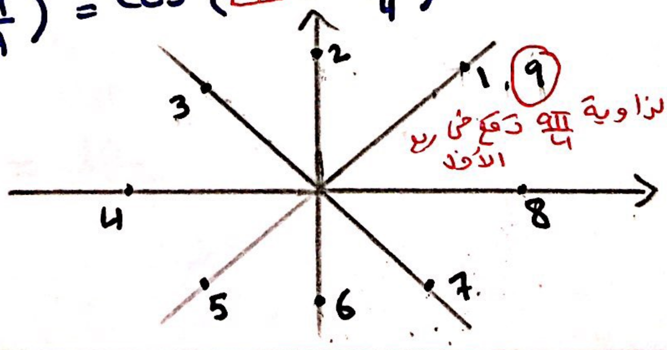
5) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)=\frac{2\pi}{3}$ since $\frac{2\pi}{3}\in[0,\pi]$

$\cos^{-1}\left(\cos\frac{5\pi}{6}\right)=\frac{5\pi}{6}$ since $\frac{5\pi}{6}\in[0,\pi]$



$\cos^{-1}\left(\cos\frac{9\pi}{4}\right)\neq\frac{9\pi}{4}$

$\therefore \cos^{-1}\left(\cos\frac{9\pi}{4}\right)=\cos^{-1}\left(\boxed{+}\cos\frac{\pi}{4}\right)=\cos^{-1}\left(\cos\frac{\pi}{4}\right)=\frac{\pi}{4}$



⑥ $\cos(\cos^{-1}(\tfrac{1}{2})) = \tfrac{1}{2}$ since $\tfrac{1}{2} \in [-1,1]$

$\cos(\cos^{-1}(2)) \neq 2$ since $2 \notin [-1,1]$

$\therefore \cos(\cos^{-1}(2))$ is undefined.

$\cos(\cos^{-1}(1)) = 1$ since $1 \in [-1,1]$

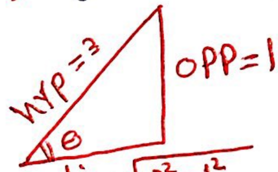
$\cos(\cos^{-1}(-1)) = -1$ since $-1 \in [-1,1]$

$\cos(\cos^{-1}(0)) = 0$ since $0 \in [-1,1]$

Example

① $\tan(\sin^{-1}\tfrac{1}{3})$

Let $\theta = \sin^{-1}(1/3) \iff \sin\theta = \dfrac{1}{3} = \dfrac{opp}{hyp}$



$adj = \sqrt{3^2 - 1^2}$
$= \sqrt{9-1}$
$= \sqrt{8}$
$= 2\sqrt{2}$

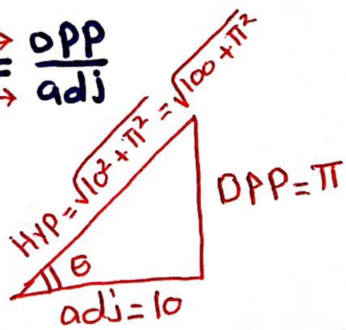
$\therefore \tan(\sin^{-1}\tfrac{1}{3}) = \tan(\theta)$

$$= \frac{opp}{adj} = \frac{1}{2\sqrt{2}}$$

$$= \frac{1 \times \sqrt{2}}{2\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2(2)} = \frac{\sqrt{2}}{4}$$

② $\cos(\tan^{-1}(\tfrac{\pi}{10}))$

Let $\theta = \tan^{-1}(\tfrac{\pi}{10}) \Rightarrow \tan(\theta) = \dfrac{\pi}{10} = \dfrac{opp}{adj}$



$\therefore \cos(\tan^{-1}(\tfrac{\pi}{10})) = \cos(\theta)$

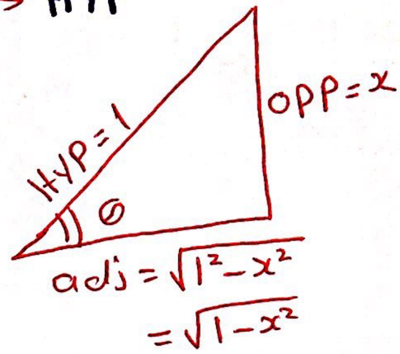
$$= \frac{adj}{Hyp}$$

$$= \frac{10}{\sqrt{100+\pi^2}}$$

$$= \frac{10\sqrt{100+\pi^2}}{\sqrt{100+\pi^2}\,\sqrt{100+\pi^2}} = \frac{10\sqrt{100+\pi^2}}{(100+\pi^2)}$$

③ $\tan(\sin^{-1}x)$

let $\theta = \sin^{-1}x \iff \sin\theta = \dfrac{x}{1} = \dfrac{opp}{HYP}$



$\therefore \tan(\sin^{-1}x) = \tan(\theta)$

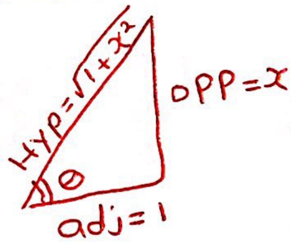
$$= \frac{opp}{adj}$$

$$= \frac{x}{\sqrt{1-x^2}}$$

$$= \frac{x \times \sqrt{1-x^2}}{\sqrt{1-x^2}\times\sqrt{1-x^2}} = \frac{x\sqrt{1-x^2}}{(1-x^2)}$$

④ $\sin(\tan^{-1}x)$

let $\theta = \tan^{-1}x \iff \tan\theta = \dfrac{x}{1} = \dfrac{opp}{adj}$



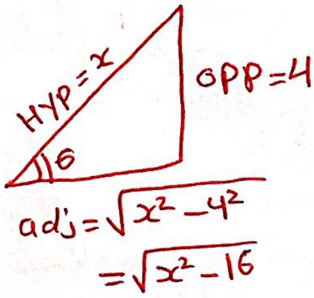
$\therefore \sin(\tan^{-1}x) = \sin(\theta)$

$$= \frac{opp}{HYP}$$

$$= \frac{x}{\sqrt{1+x^2}}$$

⑤ $\sec(\csc^{-1}\frac{x}{4})$

let $\theta = \csc^{-1}\frac{x}{4} \Longleftrightarrow \csc\theta = \frac{x}{4} = \frac{Hyp}{opp}$



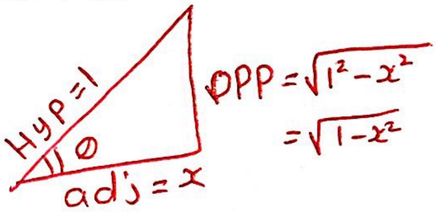
$\therefore \sec(\csc^{-1}\frac{x}{4}) = \sec(\theta)$

$= \frac{Hyp}{adj}$

$= \frac{x}{\sqrt{x^2-16}}$

⑥ $\sin(2\cos^{-1}x)$

let $\theta = \cos^{-1}x \Longleftrightarrow \cos\theta = \frac{x}{1} = \frac{adj}{Hyp}$



$\therefore \sin(2\cos^{-1}x) = \sin(2\theta)$

$= 2\sin\theta.\cos\theta$

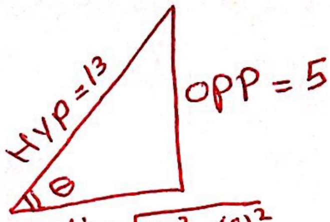
$= 2\left(\frac{opp}{Hyp}\right)\left(\frac{adj}{Hyp}\right)$

$= 2\left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{x}{1}\right)$

$= 2x\sqrt{1-x^2}$

⑦ $\cos\left(2\arcsin\left(\frac{5}{13}\right)\right) = \cos\left(2\sin^{-1}\left(\frac{5}{13}\right)\right)$

Let $\theta = \sin^{-1}\left(\frac{5}{13}\right) \Leftrightarrow \sin\theta = \frac{5}{13} = \frac{\text{OPP}}{\text{HYP}}$



$\text{adj} = \sqrt{(13)^2 - (5)^2}$

$= \sqrt{169 - 25}$

$= \sqrt{144}$

$= 12$

$\therefore \cos\left(2\sin^{-1}\left(\frac{5}{13}\right)\right) = \cos(2\theta)$

$= \cos^2\theta - \sin^2\theta$

$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{144}{169} - \frac{25}{169}$

$= \frac{144-25}{169}$

$= \frac{119}{169}$

Example:

Find the Domain of the following

① $f(x) = \tan^{-1}\left(\sqrt[4]{5-x^2}\right)$

$D_{f(x)} = D_{\tan^{-1}x} \cap D_{\sqrt[4]{5-x^2}}$

$= \mathbb{R} \cap [-\sqrt{5}, \sqrt{5}]$

$= [-\sqrt{5}, \sqrt{5}]$

② $g(x) = \sin^{-1}(3x+1)$

$$-1 \leq 3x+1 \leq 1$$

$$-1-1 \leq 3x \leq 1-1$$

$$-2 \leq 3x \leq 0$$

$$\frac{-2}{3} \leq \frac{3x}{3} \leq \frac{0}{3}$$

$$\frac{-2}{3} \leq x \leq 0$$

$$\therefore D_{g(x)} = \left[-\tfrac{2}{3}, 0\right]$$

Example

if $f(x) = \cos(2x-6)+1$ then find $f^{-1}(x)$

1) $y = \cos(2x-6)+1$

2) $y-1 = \cos(2x-6)$

$$\cos^{-1}(y-1) = 2x-6$$

$$\cos^{-1}(y-1)+6 = 2x$$

$$\frac{1}{2}\left[\cos^{-1}(y-1)+6\right] = \frac{2x}{2}$$

$$\frac{1}{2}\left[\cos^{-1}(y-1)+6\right] = x$$

3) $\frac{1}{2}\left[\cos^{-1}(x-1)+6\right] = y$

4) $f^{-1}(x) = \frac{1}{2}\left[\cos^{-1}(x-1)+6\right] = \frac{1}{2}\cos^{-1}(x-1)+\frac{6}{2} = \frac{1}{2}\cos^{-1}(x-1)+3$ #

Appendix D : Trigonometry

الهدف الأول

كيفية التحويل الزاوية من درجات الى راديان ومن راديان الى درجات

Exercise (1)

convert from degrees to radians

✓ $$1)\ 210° = \frac{210 \times \pi}{180} = \frac{21\pi}{18} = \frac{(3)(7)\pi}{(2)(3)(3)} = \frac{7\pi}{(2)(3)} = \frac{7\pi}{6}$$

$$2)\ 300° = \frac{300 \times \pi}{180} = \frac{30\pi}{18} = \frac{(2)(3)(5)\pi}{(2)(3)(3)} = \frac{5\pi}{3}$$

$$3)\ 9° = \frac{9 \times \pi}{180} = \frac{(3)(3)\pi}{(2)(2)(3)(3)(5)} = \frac{\pi}{(2)(2)(5)} = \frac{\pi}{20}$$

✓ $$4)\ -315° = -\frac{315 \times \pi}{180} = -\frac{(3)(3)(5)(7)\pi}{(2)(2)(3)(3)(5)} = -\frac{7\pi}{(2)(2)} = -\frac{7\pi}{4}$$

$$5)\ 900° = \frac{900 \times \pi}{180} = \frac{90\pi}{18} = \frac{(2)(3)(3)(5)\pi}{(2)(3)(3)} = 5\pi$$

✓ $$6)\ 36° = \frac{36 \times \pi}{180} = \frac{(2)(2)(3)(3)\pi}{(2)(2)(3)(3)(5)} = \frac{\pi}{5}$$

Exercise (2)

convert from radians to degrees

✓ $1)\ 4\pi = 4 \times 180 = 720°$

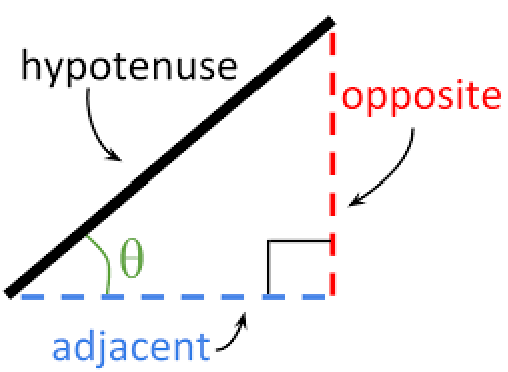
✓ $2)\ -\frac{7\pi}{2} = -\frac{7 \times 180}{2} = -7 \times 90 = -630°$

$3)\ \frac{8\pi}{3} = \frac{8 \times 180}{3} = 8 \times 60 = 480°$

✓ $4)\ \frac{5\pi}{12} = -\frac{5 \times 180}{12} = 5 \times 15 = 75°$

الهدف الثاني

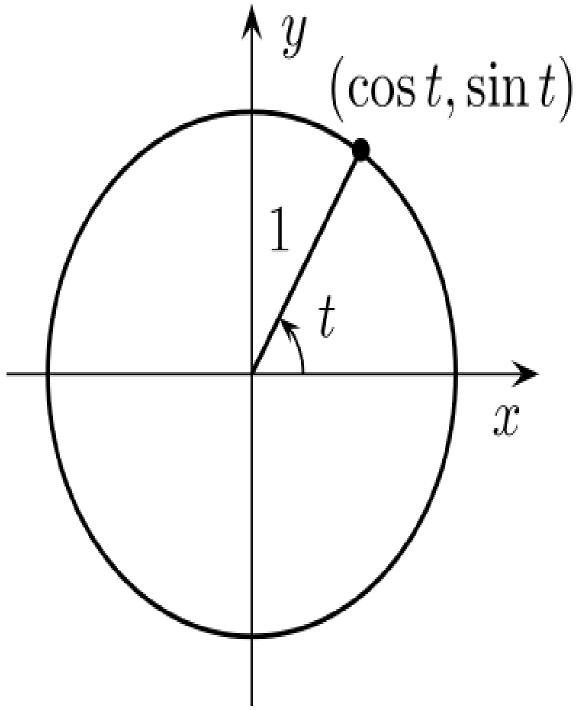
معرفة قوانين الدوال المثلثية وكيفية استخدامها



$\sin(\theta) = \frac{opp}{hyp}$ $\qquad \cos(\theta) = \frac{adj}{hyp}$

$\csc(\theta) = \frac{hyp}{opp}$ $\qquad \sec(\theta) = \frac{hyp}{adj}$

$\tan(\theta) = \frac{opp}{adj}$ $\qquad \cot(\theta) = \frac{adj}{opp}$



Trig Functions using Unit Circle

$$\sin\Theta = \frac{y}{1} = y \qquad \csc\Theta = \frac{1}{y}$$

$$\cos\Theta = \frac{x}{1} = x \qquad \sec\Theta = \frac{1}{x}$$

$$\tan\Theta = \frac{y}{x} \qquad \cot\Theta = \frac{x}{y}$$

قاعدة الاشارات

الربع الأول قيم الدوال
المثلثية موجبة

فقط في ربع الثاني قيم الـ
Sin(x) و csc(x) موجبة
وباقي الدوال المثلثية قيمها سالبة

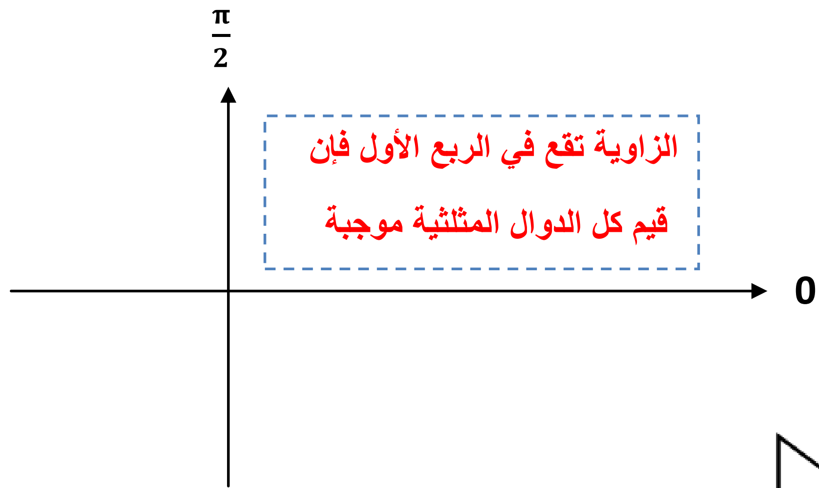
فقط في ربع الثالث قيم الـ
tan(x) و cot(x) موجبة
وباقي الدوال المثلثية قيمها سالبة

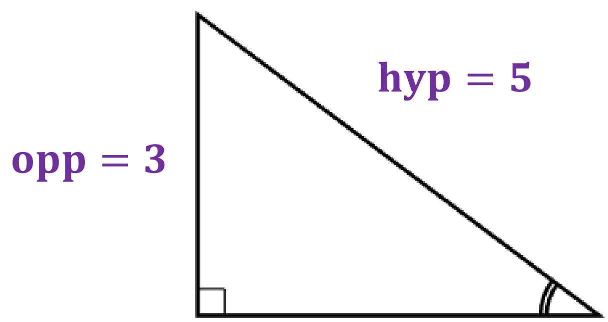
فقط في ربع الرابع قيم الـ
cos(x) و sec(x) موجبة
وباقي الدوال المثلثية قيمها سالبة

Exercise (3)

Find the remaining trigonometric ratio

✓ $\mathbf{1)}\ \sin(\theta) = \frac{3}{5}\ , 0 < \theta < \frac{\pi}{2}$





$$\sin(\theta) = \frac{3}{5} = \frac{\text{opp}}{\text{hyp}}$$

$$\text{adj} = \sqrt{5^2 - 3^2}$$
$$= \sqrt{25 - 9}$$
$$= \sqrt{16} = 4$$

$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$$

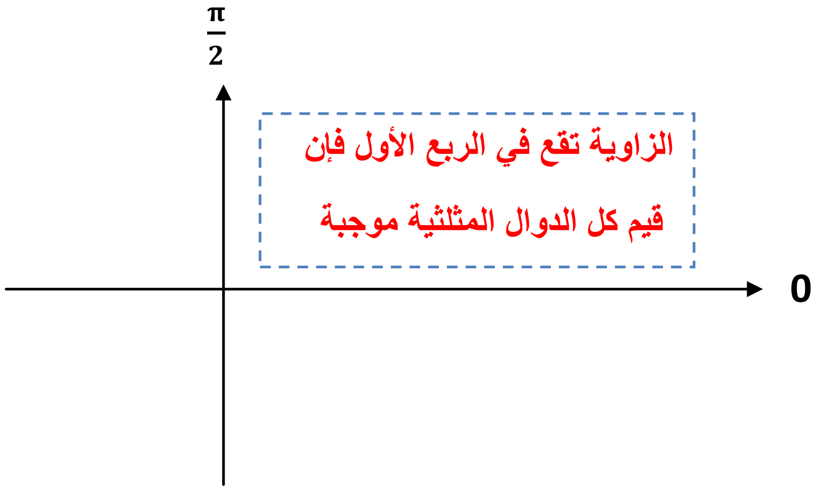
$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$$

$$\cot(\theta) = \frac{\text{adj}}{\text{opp}} = \frac{4}{3}$$

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = \frac{5}{3}$$

$$\sec(\theta) = \frac{\text{hyp}}{\text{adj}} = \frac{5}{4}$$

$2)\ \tan(\theta) = 2\ , 0 < \theta < \frac{\pi}{2}$



$$\tan(\theta) = \frac{2}{1} = \frac{\text{opp}}{\text{adj}}$$

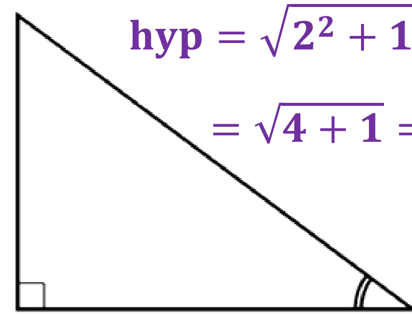
$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{5}}$$

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{2}{\sqrt{5}}$$

$$\cot(\theta) = \frac{\text{adj}}{\text{opp}} = \frac{1}{2}$$

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{2}$$

$$\sec(\theta) = \frac{\text{hyp}}{\text{adj}} = \sqrt{5}$$



$$\text{hyp} = \sqrt{2^2 + 1^2}$$

$$= \sqrt{4+1} = \sqrt{5}$$

$$\text{opp} = 2$$

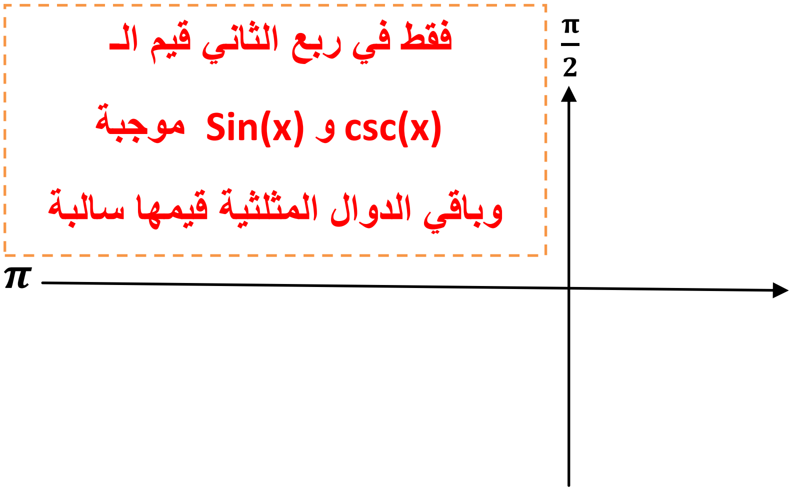
$$\text{adj} = 1$$

✓ $3)\ \sec(\theta) = -1.5\ ,\ \frac{\pi}{2} < \theta < \pi$

فقط في ربع الثاني قيم الـ

Sin(x) و csc(x) موجبة

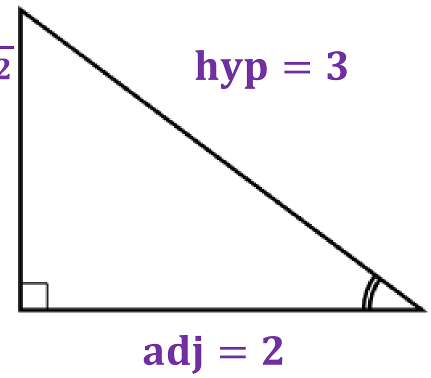
وباقي الدوال المثلثية قيمها سالبة



$$sec(\theta) = -1.5 = -\frac{15}{10} = -\frac{3\times5}{2\times5} = -\frac{3}{2}$$

$$sec(\theta) = -\frac{3}{2} = \frac{\text{hyp}}{\text{adj}}$$

$$\text{opp} = \sqrt{3^2 - 2^2}$$
$$= \sqrt{9-4}$$
$$= \sqrt{5}$$



$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = -\frac{2}{3}$$

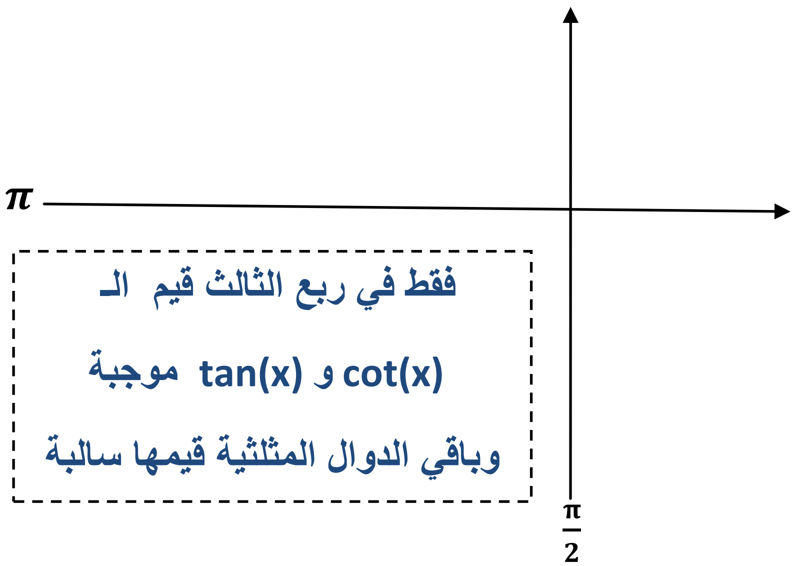
$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = -\frac{\sqrt{5}}{2}$$

$$\cot(\theta) = \frac{\text{adj}}{\text{opp}} = -\frac{2}{\sqrt{5}}$$

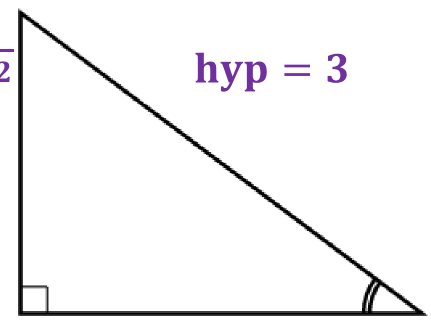
$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = \frac{3}{\sqrt{5}}$$

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{5}}{3}$$

$$4)\ \cos(\theta) = -\frac{1}{3}\ ,\ \pi < \theta < \frac{3\pi}{2}$$



$$\text{opp} = \sqrt{3^2 - 1^2} = \sqrt{9-1} = \sqrt{8} = 2\sqrt{2}$$



$$\cos(\theta) = -\frac{1}{3} = \frac{\text{adj}}{\text{hyp}}$$

$$\sec(\theta) = \frac{\text{hyp}}{\text{adj}} = -3$$

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = 2\sqrt{2}$$

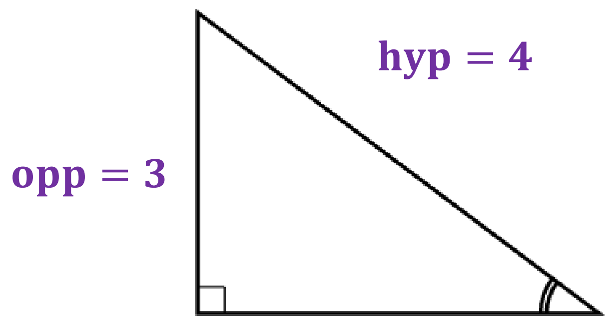
$$\cot(\theta) = \frac{\text{adj}}{\text{opp}} = \frac{1}{2\sqrt{2}} = \frac{1\times\sqrt{2}}{2\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = -\frac{3}{2\sqrt{2}} = -\frac{3\times\sqrt{2}}{2\sqrt{2}\times\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = -\frac{2\sqrt{2}}{3}$$

✓ 5) $\csc(\theta) = -\frac{4}{3}$, $\frac{3\pi}{2} < \theta < 2\pi$

$$\csc(\theta) = -\frac{4}{3} = \frac{\text{hyp}}{\text{opp}}$$



$$\text{adj} = \sqrt{4^2 - 3^2}$$
$$= \sqrt{16 - 9}$$
$$= \sqrt{7}$$

$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{7}}{4}$$

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = -\frac{3}{\sqrt{7}}$$

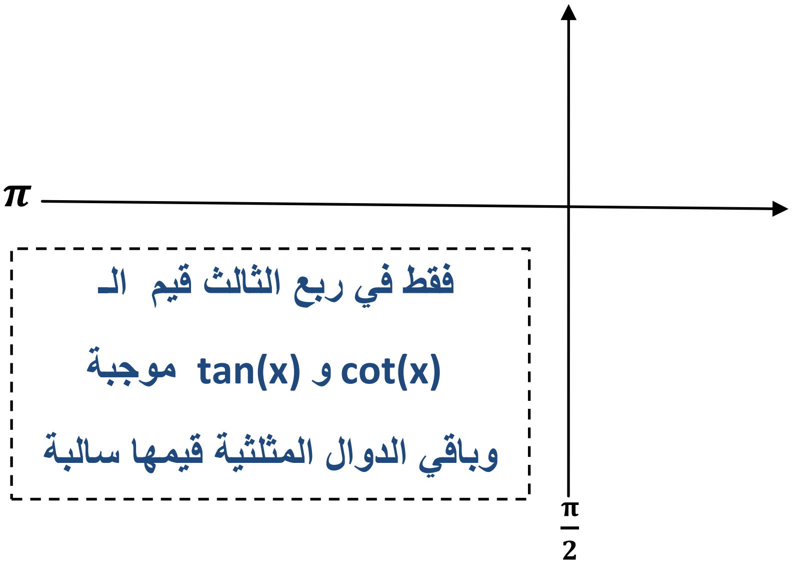
$$\cot(\theta) = \frac{\text{adj}}{\text{opp}} = -\frac{\sqrt{7}}{3}$$

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = -\frac{3}{4}$$

$$\sec(\theta) = \frac{\text{hyp}}{\text{adj}} = \frac{4}{\sqrt{7}}$$

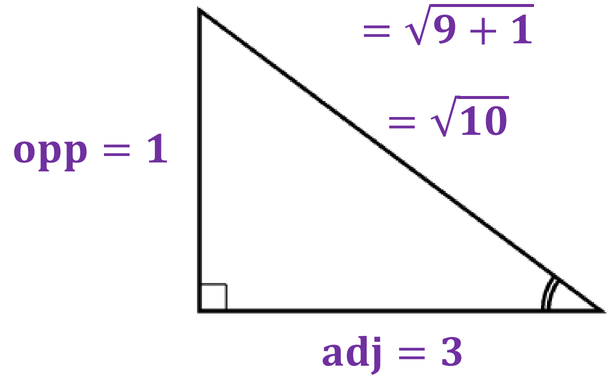
✓ 6) $\cot(\theta) = 3 \;, \qquad \pi < \theta < 2\pi$

الزاوية تقع في الربع الثالث والرابع ولكن بحكم أن قيمة الـ $\cot(\theta)$ موجبة إذن الزاوية تقع في ربع الثالث



$$\cot(\theta) = \frac{3}{1} = \frac{\text{adj}}{\text{opp}}$$

$$\text{hyp} = \sqrt{3^2 + 1^2}$$
$$= \sqrt{9 + 1}$$
$$= \sqrt{10}$$



$$\sec(\theta) = \frac{\text{hyp}}{\text{adj}} = -\frac{\sqrt{10}}{3}$$

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{1}{3}$$

$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = -\frac{3}{\sqrt{10}} = -\frac{3\times\sqrt{10}}{\sqrt{10}\times\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

$$\csc(\theta) = \frac{\text{hyp}}{\text{opp}} = -\sqrt{10}$$

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = -\frac{1}{\sqrt{10}} = -\frac{1\times\sqrt{10}}{\sqrt{10}\times\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

الهدف الثالث

ايجاد قيم بعض الدوال المثلثية للزوايا الأساسية

الزوايا الأساسية هي :

Degrees	radians	Degrees	radians
$0°$	0	$90°$	$\frac{\pi}{2}$
$30°$	$\frac{\pi}{6}$	$180°$	π
$45°$	$\frac{\pi}{4}$	$270°$	$\frac{3\pi}{2}$
$60°$	$\frac{\pi}{3}$	$360°$	2π

	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$
$\sin(\theta)$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos(\theta)$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan(\theta)$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

	$0^\circ = 0$	$90^\circ = \frac{\pi}{2}$	$180^\circ = \pi$	$270^\circ = \frac{3\pi}{2}$	$180^\circ = 2\pi$
$\sin(\theta)$	0	1	0	-1	0
$\cos(\theta)$	1	0	-1	0	1
$\tan(\theta)$	0	undefined	0	undefined	0

Exercise (4)

1) $\sin(0) = 0$

$\sin(\pi) = 0$

$\sin(2\pi) = 0$

Note: $\sin(n\pi) = 0$ for all $n \in \mathbb{Z}$

For example: $\sin(4\pi) = 0$, $\sin(5\pi) = 0$

2) $\cos(0) = 1$

$\cos(\pi) = -1$

$\cos(2\pi) = 1$

Note: $\cos(n\pi) = \begin{cases} 1 & \text{if } n \text{ is even number} \\ -1 & \text{if } n \text{ is odd number} \end{cases}$

For example: $\cos(4\pi) = 1$, $\cos(5\pi) = -1$

3) $\cos\left(\frac{\pi}{2}\right) = 0$

$\cos\left(\frac{3\pi}{2}\right) = 0$

$\cos\left(\frac{5\pi}{2}\right) = 0$

Note: $\cos\left(\frac{n\pi}{2}\right) = 0$ for all n is odd number

Or : $\cos\left(\frac{(2n+1)\pi}{2}\right) = 0$ for all $n \in \mathbb{Z}$

For example: $\cos\left(\frac{9\pi}{2}\right) = 0$, $\cos\left(\frac{15\pi}{2}\right) = 0$

4) $\sin\left(\frac{\pi}{2}\right) = 1$

$\sin\left(\frac{3\pi}{2}\right) = -1$

$\sin\left(\frac{5\pi}{2}\right) = 1$

Note:

$$\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 1 & \text{if} \quad \text{n is odd number and n=1,5,9,13,...} \\ -1 & \text{if} \quad \text{n is odd number and n=3,7,11,15,...} \end{cases}$$

For example: $\sin\left(\frac{9\pi}{2}\right) = 1$, $\sin\left(\frac{15\pi}{2}\right) = -1$

Exercise (5)

$$\sec(7\pi) = \frac{1}{\cos(7\pi)} = \frac{1}{-1} = -1$$

$$\cot\left(\frac{5\pi}{2}\right) = \frac{\cos\left(\frac{5\pi}{2}\right)}{\sin\left(\frac{5\pi}{2}\right)} = \frac{0}{1} = 0$$

$$\csc(6\pi) = \frac{\cos(6\pi)}{\sin(6\pi)} = \frac{1}{0} = \text{undefined}$$

Exercise (6)

$$1)\ \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2}$$

$$\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2} \text{ or } \frac{2}{\sqrt{2}}$$

$$2)\ \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\cot\left(\frac{\pi}{6}\right) = \frac{\cos\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$$

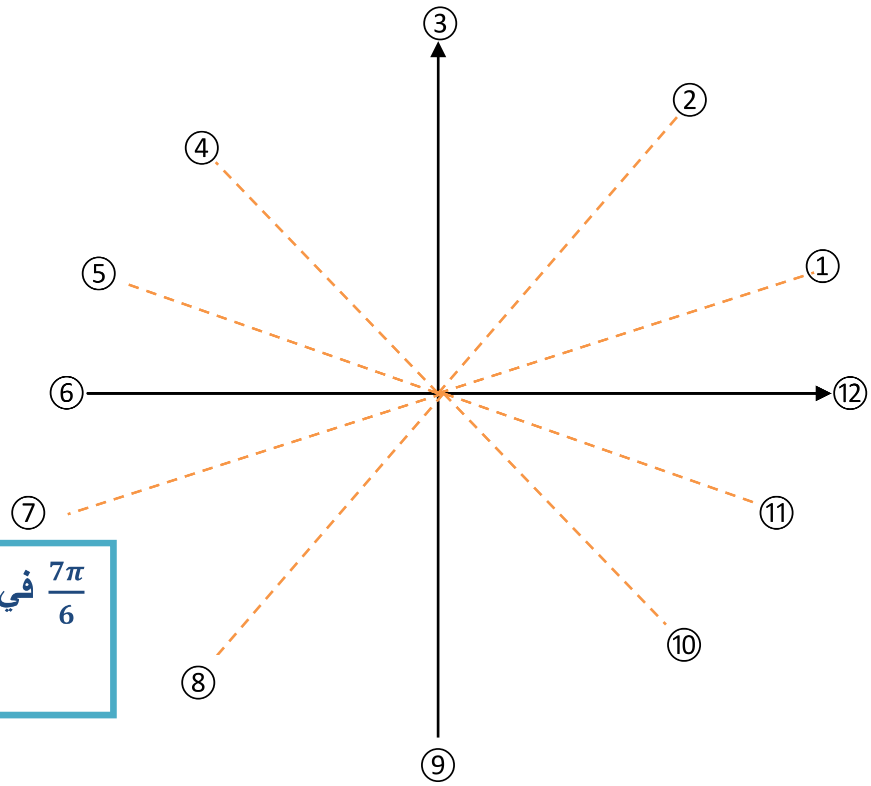
$$3)\ \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\csc\left(\frac{\pi}{3}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

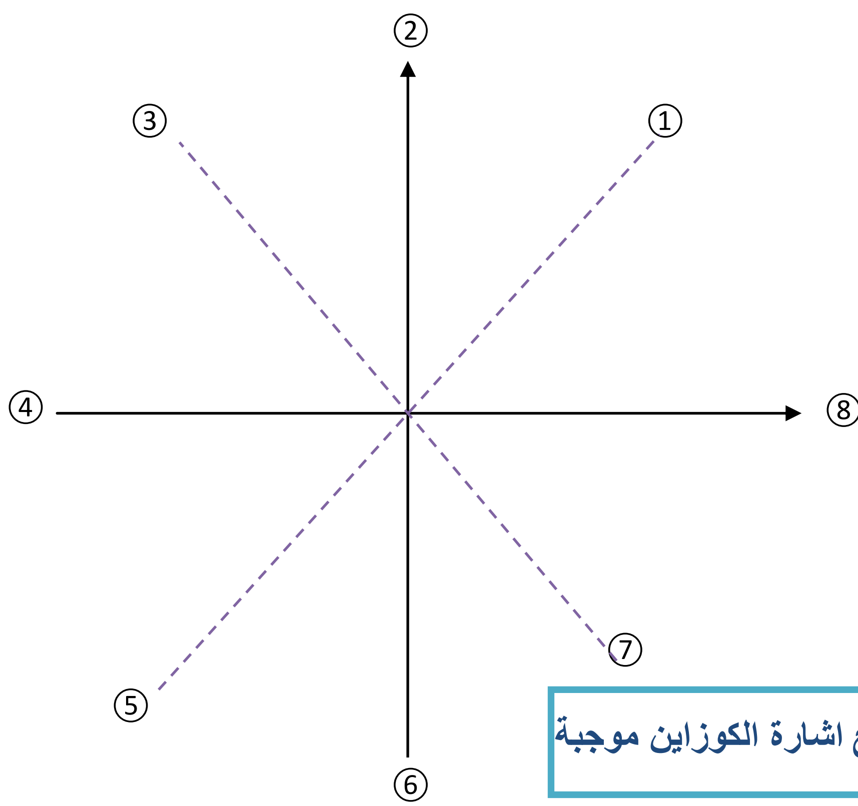
Exercise (7)

1) $\sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$



$\frac{7\pi}{6}$ في ربع الثالث اشارة الساين سالبة

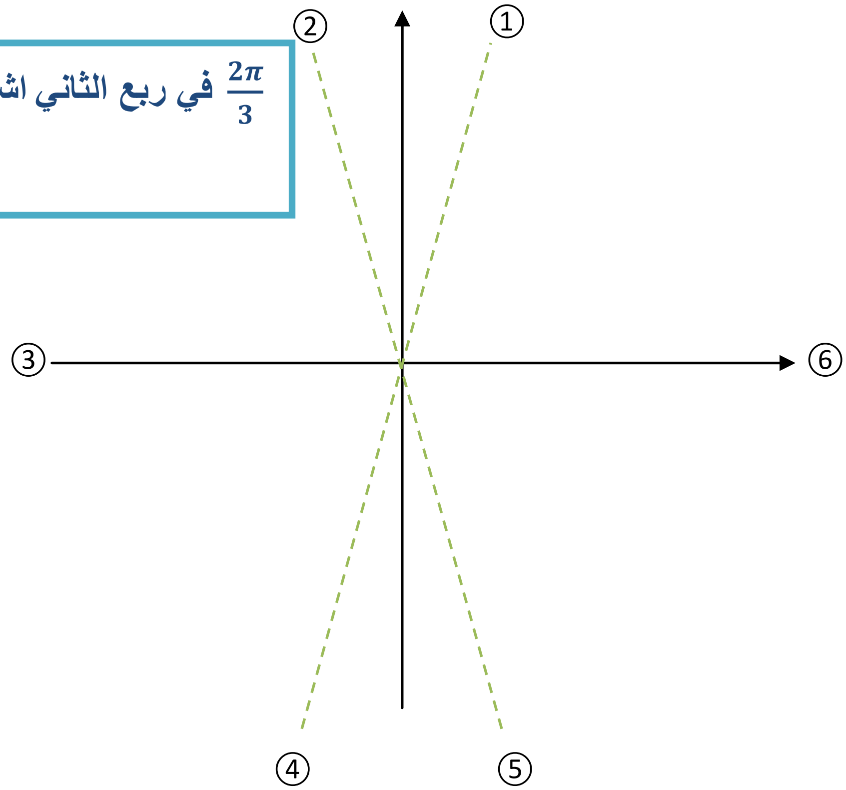
2) $\cos\left(\frac{7\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$



$\frac{7\pi}{4}$ في ربع الرابع اشارة الكوزاين موجبة

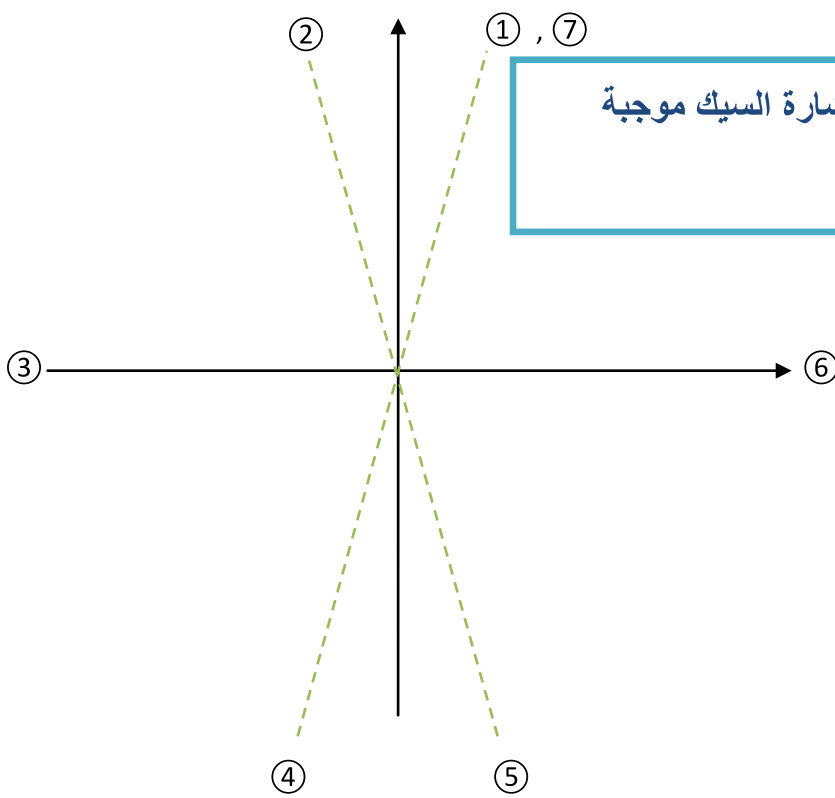
$$3)\ \tan\left(\frac{2\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right) = -\frac{\sin\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}\right)} = -\frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = -\frac{\sqrt{3}}{2} \times \frac{2}{1} = -\sqrt{3}$$

في ربع الثاني اشارة التان سالبة $\frac{2\pi}{3}$



$$\mathrm{Sec}\left(\frac{7\pi}{3}\right) = \mathrm{Sec}\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = \frac{2}{1} = 2$$

في ربع الأول اشارة السيك موجبة $\frac{7\pi}{3}$



Trigonometric identities

Trigonometric Identities

Reciprocal Identities	Quotient Identities
$\cot\theta = \dfrac{1}{\tan\theta}$ $\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Exercise (8)

$\mathbf{sec(\theta)\,cos(\theta) = 1}$

$\mathbf{tan(\theta)\,cos(\theta) = sin(\theta)}$

$\mathbf{1 - csc^2(\theta) = -cot^2(\theta)}$

$$\sin(-\theta) = -\sin(\theta)$$

$$\csc(-\theta) = -\csc(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

$$\cot(-\theta) = -\cot(\theta)$$

The odd functions

$$\cos(-\theta) = \cos(\theta)$$

$$\sec(-\theta) = \sec(\theta)$$

The even functions

Exercise (9)

$$\cos(-\pi) = \cos(\pi) = -1$$

$$\sin\left(-\frac{3\pi}{2}\right) = -\sin\left(\frac{3\pi}{2}\right) = -(-1) = 1$$

$$\cos(\theta+2\pi) = \cos(\theta)$$

$$\sec(\theta+2\pi) = \sec(\theta)$$

$$\sin(\theta+2\pi) = \sin(\theta)$$

$$\csc(\theta+2\pi) = \csc(\theta)$$

$$\tan(\theta+\pi) = \tan(\theta)$$

$$\cot(\theta+\pi) = \cot(\theta)$$

جميع الدوال المثلثية الـ period : 2π

ماعدا الـ tan(θ) و الـ cot(θ) الـ period: π

Exercise (10)

$$\tan\left(\frac{\pi}{4}+\pi\right) = \tan\left(\frac{\pi}{4}\right) = 1$$

$$\csc\left(\frac{\pi}{6}+2\pi\right) = \csc\left(\frac{\pi}{6}\right) = \frac{1}{\sin\left(\frac{\pi}{6}\right)} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

Exercise (11)

If $\sin(\theta) = \frac{3}{4}$ and $\cos(\theta) = \frac{\sqrt{7}}{4}$ then find $\sin(2\theta)$ and $\cos(2\theta)$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

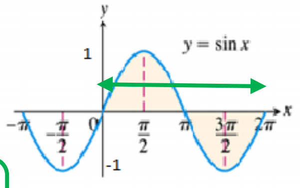
$$\sin(2\theta) = 2 \times \frac{3}{4} \times \frac{\sqrt{7}}{4} = \frac{3}{2} \times \frac{\sqrt{7}}{4} = \frac{3\sqrt{7}}{8}$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\cos(2\theta) = \left(\frac{\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2 = \frac{7}{16} - \frac{9}{16} = -\frac{2}{16} = -\frac{1}{8}$$

Trigonometric Functions

$y = \sin x$



Domain: $\mathbb{R}$

Range: $[-1, 1]$

Period: 2π

$\sin(\theta + 2\pi) = \sin\theta$

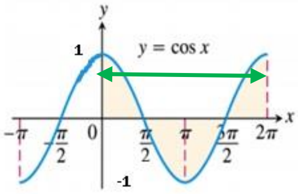
Odd function:

$\sin(-\theta) = -\sin\theta$

$$-1 \le \sin x \le 1 \iff |\sin x| \le 1$$

$$\sin x = 0 \quad when \quad x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots.$$

$y = \cos x$



Domain: $\mathbb{R}$

Range: $[-1, 1]$

Period: 2π

$\cos(\theta + 2\pi) = \cos\theta$

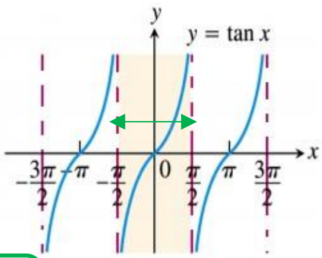
Even function:

$\cos(-\theta) = \cos\theta$

$$-1 \le \cos x \le 1 \iff |\cos x| \le 1$$

$$\cos x = 0 \quad when \quad x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots.$$

$y = \tan x$



Domain:

$\mathbb{R} - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots\right\}$

Range: $\mathbb{R}$

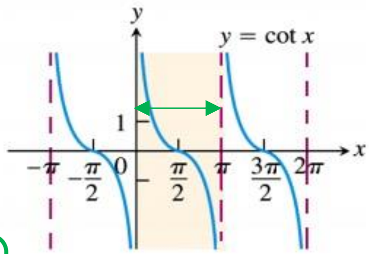
Period: π

$\tan(\theta + \pi) = \tan\theta$

Odd function:

$\tan(-\theta) = -\tan\theta$

$y = \cot x$



Domain:

$\mathbb{R} - \{0, \pm\pi, \pm 2\pi, \ldots\}$

Range: $\mathbb{R}$

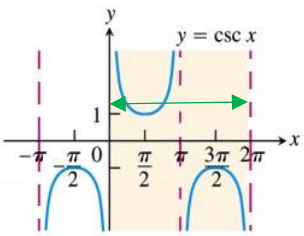
Period: π

$\cot(\theta + \pi) = \cot\theta$

Odd function:

$\cot(-\theta) = -\cot\theta$

$y = \csc x$



Domain:

$\mathbb{R} - \{0, \pm\pi, \pm 2\pi, \ldots\}$

Range:

$\mathbb{R} - (-1, 1)$

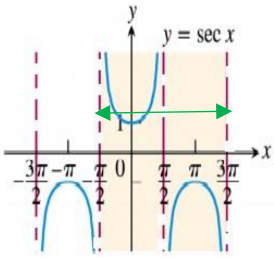
Period: 2π

$\csc(\theta + 2\pi) = \csc\theta$

Odd function:

$\csc(-\theta) = -\csc\theta$

$y = \sec x$



Domain:

$\mathbb{R} - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots.\right\}$

Range:

$\mathbb{R} - (-1, 1)$

Period: 2π

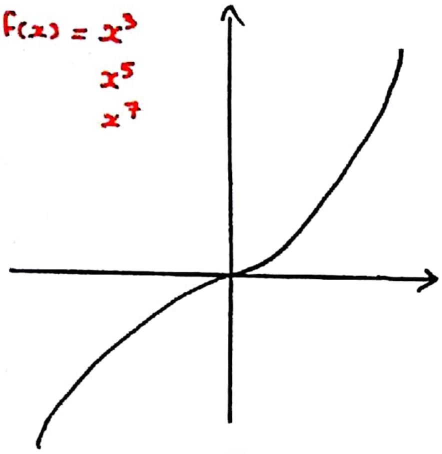
$\sec(\theta + 2\pi) = \sec\theta$

Even function:

$\sec(-\theta) = \sec\theta$

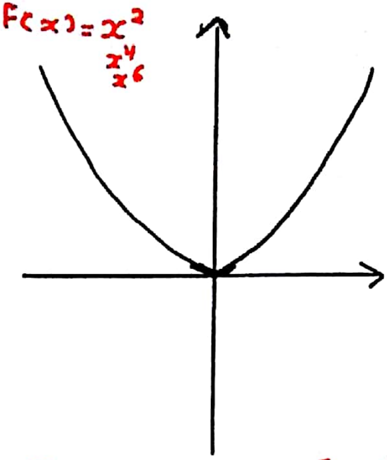
أ/ هناء الحطامي

$f(x) = x^3$
x^5
x^7



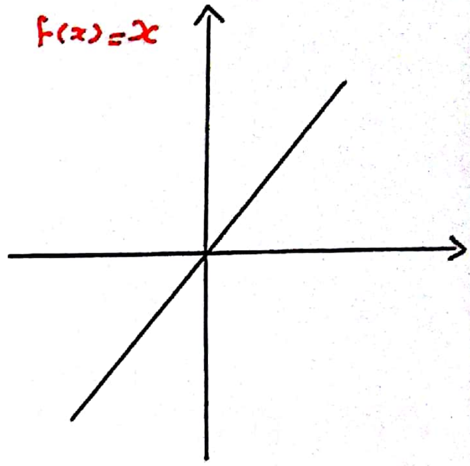
$D_{f(x)} = \mathbb{R}$ $\quad R_{f(x)} = \mathbb{R}$

$f(x) = x^2$
x^4
x^6



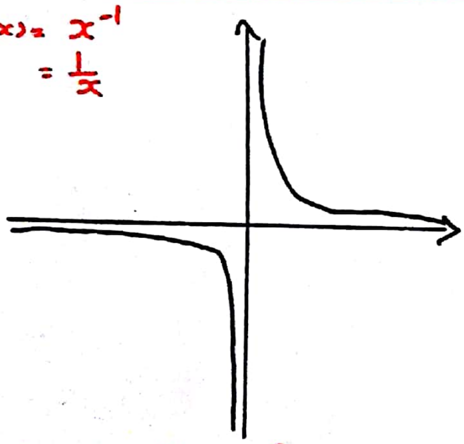
$D_{f(x)} = \mathbb{R}$, $R_{f(x)} = [0, \infty)$

$f(x) = x$



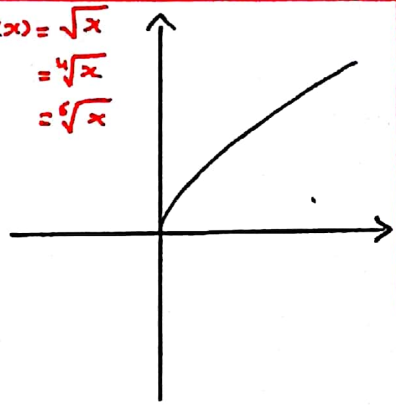
$D_{f(x)} = \mathbb{R}$; $R_{f(x)} = \mathbb{R}$

$f(x) = x^{-1}$
$= \frac{1}{x}$



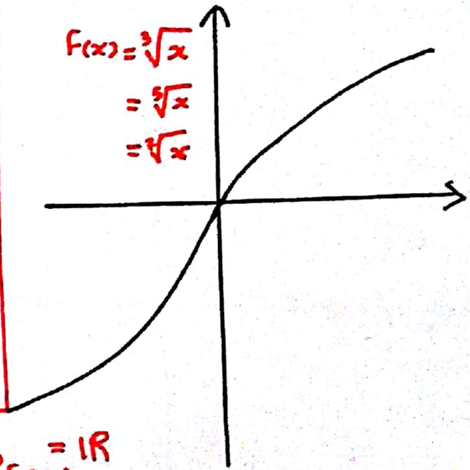
$D_{f(x)} = \mathbb{R} - \{0\}$
$R_{f(x)} = \mathbb{R} - \{0\}$

$f(x) = \sqrt{x}$
$= \sqrt[4]{x}$
$= \sqrt[6]{x}$



$D_{f(x)} = [0, \infty)$
$R_{f(x)} = [0, \infty)$

$f(x) = \sqrt[3]{x}$
$= \sqrt[5]{x}$
$= \sqrt[7]{x}$



$D_{f(x)} = \mathbb{R}$
$R_{f(x)} = \mathbb{R}$

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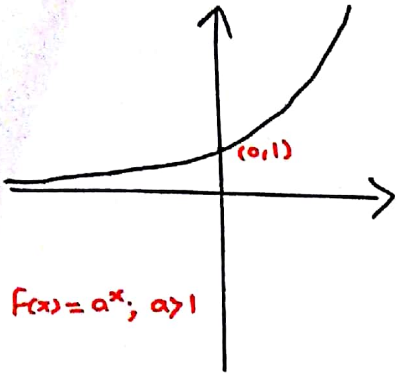
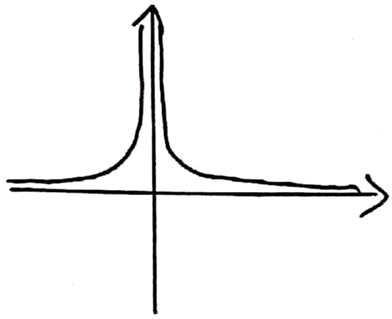


$F(x) = a^x;\ a > 1$

$D_{F(x)} = \mathbb{R}$

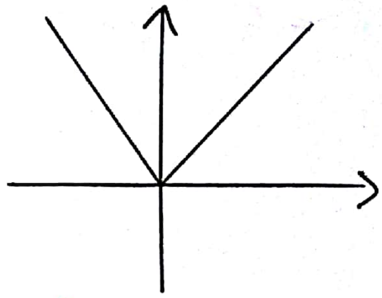
$R_{F(x)} = (0, \infty)$



$F(x) = x^{-2}$

$= \frac{1}{x^2}$

$D_{F(x)} = \mathbb{R} - \{0\}$; $R_{F(x)} = (0, \infty)$

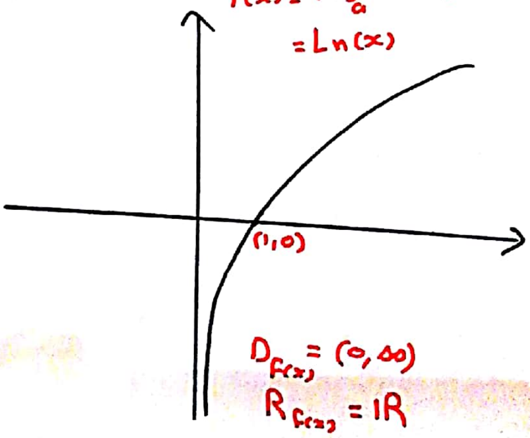


$F(x) = |x|$

$D_{F(x)} = \mathbb{R}$; $R_{F(x)} = [0, \infty)$

$F(x) = \log_a x \quad a > 1$

$= \mathrm{Ln}(x)$

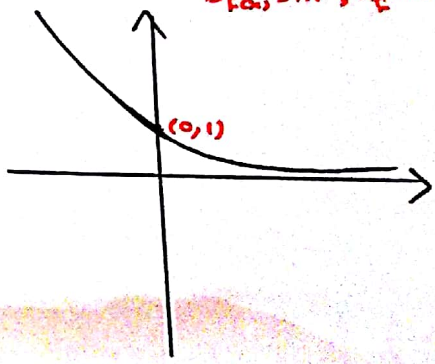


$D_{F(x)} = (0, \infty)$

$R_{F(x)} = \mathbb{R}$

$F(x) = a^x;\ 0 < a < 1$

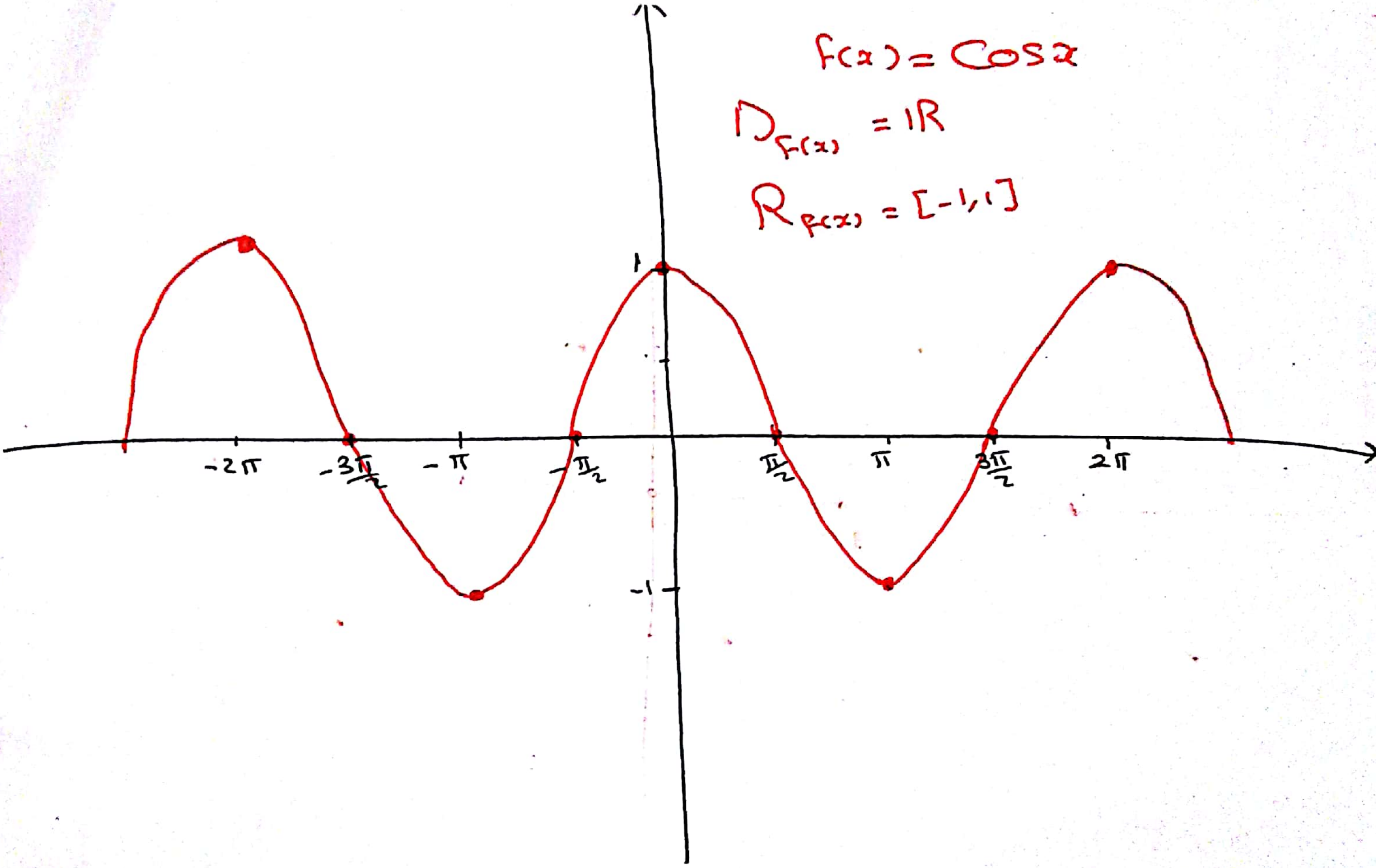
$D_{F(x)} = \mathbb{R}$; $R_F = (0, \infty)$



$f(x) = \cos x$

$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [-1, 1]$



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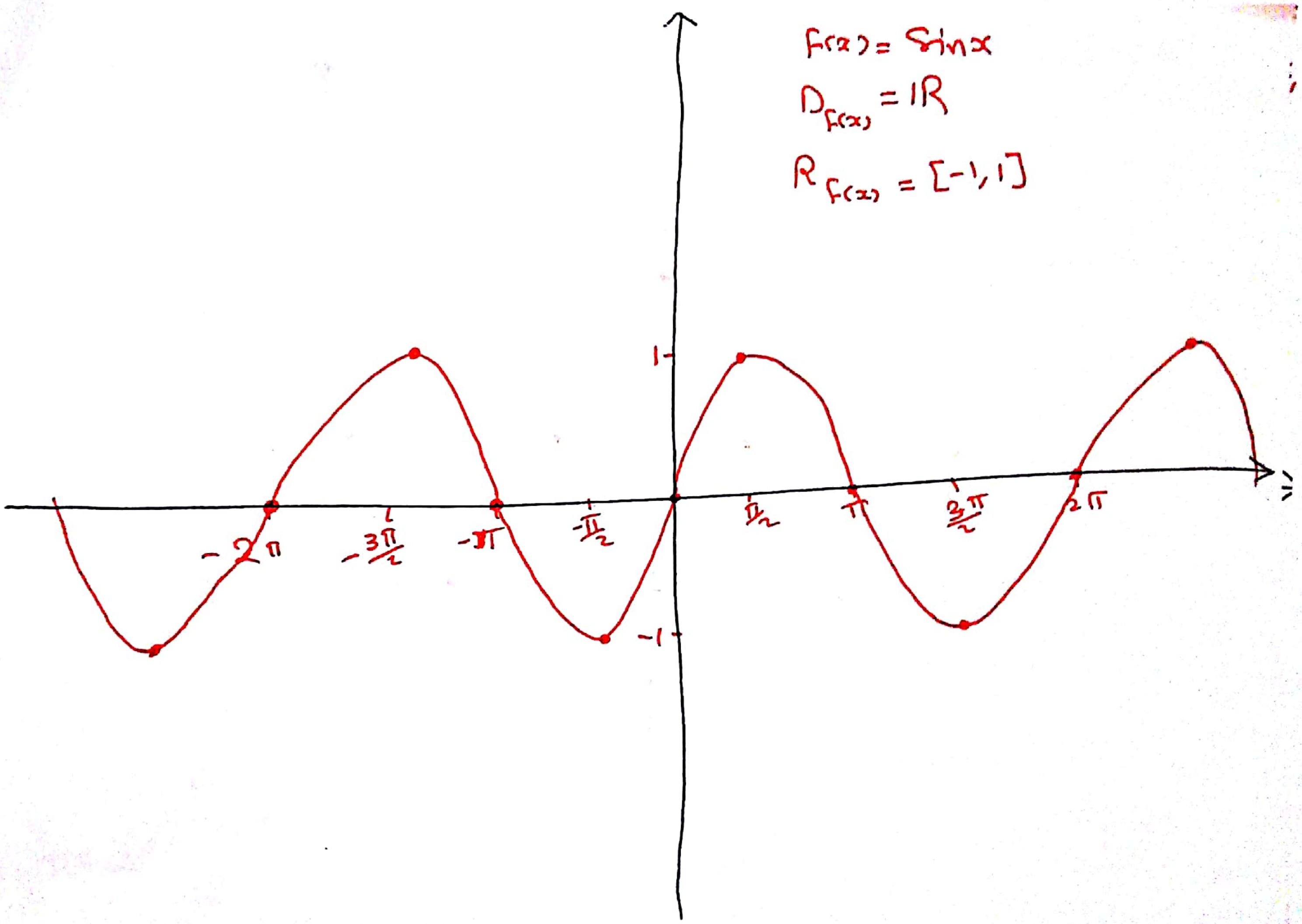
$f(x) = \sin x$

$D_{f(x)} = \mathbb{R}$

$R_{f(x)} = [-1, 1]$



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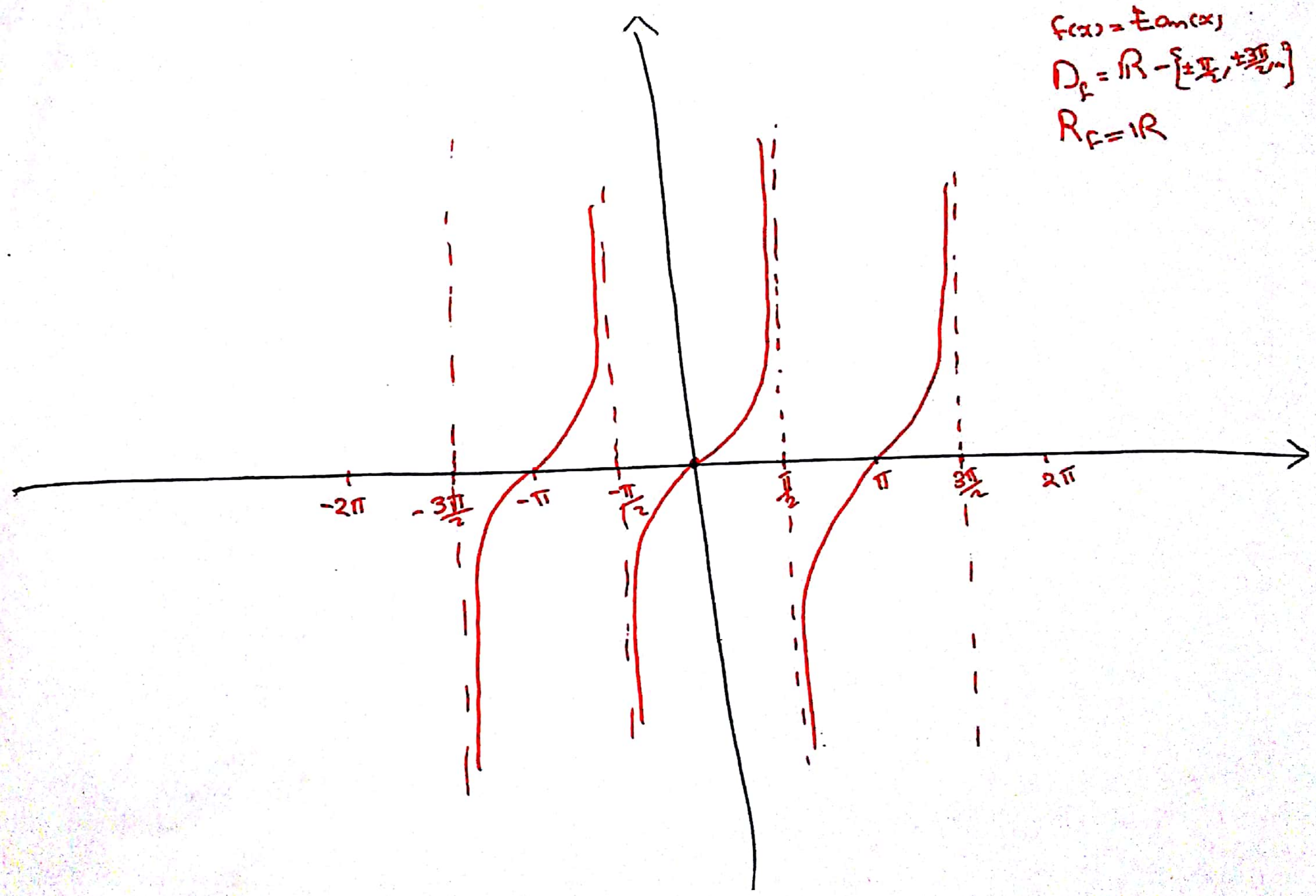

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Workshop Solutions to Sections 2.1 and 2.2(1.1 & 1.2)

1) Find the domain of the function $f(x) = 9 - x^2$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$ **Note:** The domain of any polynomial is $\mathbb{R}$.	2) Find the range of the function $f(x) = 9 - x^2$. Solution: $R_f = (-\infty, 9]$
3) Find the domain of the function $f(x) = 6 - 2x$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	4) Find the range of the function $f(x) = 6 - 2x$. Solution: Since $f(x)$ is a polynomial of degree one ($i.e.$ is of an odd degree), then $R_f = \mathbb{R} = (-\infty, \infty)$
5) Find the domain of the function $f(x) = x^2 - 2x - 3$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	6) Find the domain of the function $f(x) = 1 + 2x^3 - x^5$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$
7) Find the domain of the function $f(x) = 5$. Solution: Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	8) Find the range of the function $f(x) = 5$. Solution: $R_f = \{5\}$
9) Find the domain of the function $f(x) = \|x - 1\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$ **Note:** The domain of an absolute value of any polynomial is $\mathbb{R}$.	10) Find the domain of the function $f(x) = \|x + 5\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$
11) Find the domain of the function $f(x) = \|x\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	12) Find the range of the function $f(x) = \|x\|$. Solution: $R_f = [0, \infty)$ **Note:** The range of an absolute value of any polynomial is always $[0, \infty)$.
13) Find the domain of the function $f(x) = \|3x - 6\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$	14) Find the domain of the function $f(x) = \|9 - 3x\|$. Solution: Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$
15) Find the domain of the function $f(x) = \dfrac{x+2}{x-3}$ Solution: $f(x)$ is defined when $x - 3 \neq 0 \implies x \neq 3$. So, $D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$	16) Find the domain of the function $f(x) = \dfrac{x-2}{x+3}$ Solution: $f(x)$ is defined when $x + 3 \neq 0 \implies x \neq -3$. So, $D_f = \mathbb{R} \setminus \{-3\} = (-\infty, -3) \cup (-3, \infty)$

17) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-9}$$

Solution:

$f(x)$ is defined when $x^2 - 9 \neq 0 \Longrightarrow x^2 \neq 9 \Longrightarrow x \neq \pm 3$. So,

$$D_f = \mathbb{R} \setminus \{-3,3\} = (-\infty,-3) \cup (-3,3) \cup (3,\infty)$$

18) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-5x+6}$$

Solution:

$f(x)$ is defined when $x^2 - 5x + 6 \neq 0$

$\Longrightarrow (x-2)(x-3) \neq 0 \Longrightarrow x \neq 2$ or $x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{2,3\} = (-\infty,2) \cup (2,3) \cup (3,\infty)$$

19) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-x-6}$$

Solution:

$f(x)$ is defined when $x^2 - x - 6 \neq 0$

$\Longrightarrow (x+2)(x-3) \neq 0 \Longrightarrow x \neq -2$ or $x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{-2,3\} = (-\infty,-2) \cup (-2,3) \cup (3,\infty)$$

20) Find the domain of the function

$$f(x) = \frac{x+2}{x^2+9}$$

Solution:

$f(x)$ is defined when $x^2 + 9 \neq 0$ but for any value x the denominator $x^2 + 9$ cannot be 0. So,

$$D_f = \mathbb{R} = (-\infty,\infty)$$

21) Find the domain of the function

$$f(x) = \sqrt[3]{x-3}$$

Solution:

$$D_f = \mathbb{R} = (-\infty,\infty)$$

Note: The domain of an odd root of any polynomial is $\mathbb{R}$.

22) Find the domain of the function

$$f(x) = \sqrt{x-3}$$

Solution:

$f(x)$ is defined when $x - 3 \geq 0 \Longrightarrow x \geq 3$ because $f(x)$ is an even root. So,

$$D_f = [3,\infty)$$

23) Find the domain of the function

$$f(x) = \sqrt{3-x}$$

Solution:

$f(x)$ is defined when $3 - x \geq 0 \Longrightarrow -x \geq -3 \Longrightarrow x \leq 3$ because $f(x)$ is an even root. So,

$$D_f = (-\infty,3]$$

24) Find the domain of the function

$$f(x) = \sqrt{x+3}$$

Solution:

$f(x)$ is defined when $x + 3 \geq 0 \Longrightarrow x \geq -3$ because $f(x)$ is an even root. So,

$$D_f = [-3,\infty)$$

25) Find the domain of the function

$$f(x) = \sqrt{-x}$$

Solution:

$f(x)$ is defined when $-x \geq 0 \Longrightarrow x \leq 0$ because $f(x)$ is an even root. So,

$$D_f = (-\infty,0]$$

26) Find the range of the function

$$f(x) = \sqrt{-x}$$

Solution:

$$R_f = [0,\infty)$$

Note: The range of an even root is always ≥ 0.

27) Find the domain of the function

$$f(x) = \sqrt{9-x^2}$$

Solution:

$f(x)$ is defined when $9 - x^2 \geq 0 \Longrightarrow -x^2 \geq -9 \Longrightarrow$

$x^2 \leq 9 \Longrightarrow \sqrt{x^2} \leq \sqrt{9} \Longrightarrow |x| \leq 3 \Longrightarrow -3 \leq x \leq 3$. So,

$$D_f = [-3,3]$$

28) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{x-3}}$$

Solution:

$f(x)$ is defined when $x - 3 > 0 \Longrightarrow x > 3$. So,

$$D_f = (3,\infty)$$

29) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{9-x^2}}$$

Solution:

$f(x)$ is defined when $9 - x^2 > 0 \Longrightarrow -x^2 > -9$

$\Longrightarrow x^2 < 9 \Longrightarrow \sqrt{x^2} < \sqrt{9} \Longrightarrow |x| < 3 \Longrightarrow -3 < x < 3$. So,

$$D_f = (-3,3)$$

30) Find the domain of the function

$$f(x) = \sqrt{x^2-9}$$

Solution:

$f(x)$ is defined when $x^2 - 9 \geq 0 \Longrightarrow x^2 \geq 9$

$\Longrightarrow \sqrt{x^2} \geq \sqrt{9} \Longrightarrow |x| \geq 3 \Longrightarrow x \geq 3$ or $x \leq -3$. So,

$$D_f = (-\infty,-3] \cup [3,\infty)$$

31) Find the range of the function

$$f(x) = \sqrt{x^2 - 9}$$

<u>Solution:</u>

$$R_f = [0, \infty)$$

32) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{x^2 - 9}}$$

<u>Solution:</u>

$f(x)$ is defined when $x^2 - 9 > 0 \implies x^2 > 9$

$\implies \sqrt{x^2} > \sqrt{9} \implies |x| > 3 \implies x > 3 \text{ or } x < -3$.

So,

$$D_f = (-\infty, -3) \cup (3, \infty)$$

33) Find the domain of the function

$$f(x) = \sqrt{9 + x^2}$$

<u>Solution:</u>

$f(x)$ is defined when $9 + x^2 \geq 0$ but it is always true for any value x . So,

$$D_f = \mathbb{R}$$

34) Find the domain of the function

$$f(x) = \sqrt[4]{x^2 - 25}$$

<u>Solution:</u>

$f(x)$ is defined when $x^2 - 25 \geq 0 \implies x^2 \geq 25$

$\implies \sqrt{x^2} \geq \sqrt{25} \implies |x| \geq 5 \implies x \geq 5 \text{ or } x \leq -5$.

So,

$$D_f = (-\infty, -5] \cup [5, \infty)$$

35) Find the domain of the function

$$f(x) = \sqrt[6]{16 - x^2}$$

<u>Solution:</u>

$f(x)$ is defined when $16 - x^2 \geq 0 \implies -x^2 \geq -16 \implies$

$x^2 \leq 16 \implies \sqrt{x^2} \leq \sqrt{16} \implies |x| \leq 4 \implies -4 \leq x \leq 4$.

So,

$$D_f = [-4, 4]$$

36) Find the range of the function

$$f(x) = \sqrt{16 - x^2}$$

<u>Solution:</u>

We know that $f(x)$ is defined when $16 - x^2 \geq 0$

$\implies -x^2 \geq -16 \implies x^2 \leq 16 \implies \sqrt{x^2} \leq \sqrt{16}$

$\implies |x| \leq 4 \implies -4 \leq x \leq 4$. So,

$$D_f = [-4, 4]$$

Using D_f we find the outputs vary from 0 to 4 . Hence,

$$R_f = [0, 4]$$

37) Find the domain of the function

$$f(x) = \frac{x + |x|}{x}$$

<u>Solution:</u>

$f(x)$ is defined when $x \neq 0$. So,

$$D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$$

38) Find the domain of the function

$$f(x) = \begin{cases} -\dfrac{1}{x} \,, & x < 0 \\ x \,, & x \geq 0 \end{cases}$$

<u>Solution:</u>

It is clear from the definition of the function $f(x)$ that

$$D_f = \mathbb{R} = (-\infty, \infty)$$

39) Find the domain of the function

$$f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$$

<u>Solution:</u>

$f(x)$ is defined when

1- $x \geq 0 \implies D_{\sqrt{x}} = [0, \infty)$

2- $x^2 + 1 > 0$ but this is always true for all x

$\implies D_{\sqrt{x^2+1}} = \mathbb{R}$.

Hence,

$$D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2+1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$$

40) Find the domain of the function

$$f(x) = \sqrt{x - 1} + \sqrt{x + 3}$$

<u>Solution:</u>

$f(x)$ is defined when

1- $x - 1 \geq 0 \implies x \geq 1 \implies D_{\sqrt{x-1}} = [1, \infty)$

2- $x + 3 \geq 0 \implies x \geq -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$

Hence,

$$D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$$

41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function.

42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function.

43) The function $f(x) = -3x^2 + 7$ is a quadratic function.

44) The function $f(x) = 2x + 3$ is a linear function.

45) The function $f(x) = x^7$ is a power function.

46) The function $f(x) = \frac{2x+3}{x^2-1}$ is a rational function.

47) The function $f(x) = \frac{x-3}{x+2}$ is a rational function and we can say it is an algebraic function as well.

48) The function $f(x) = \sin x$ is a trigonometric function.

49) The function $f(x) = e^x$ is a natural exponential function.	50) The function $f(x) = 3^x$ is a general exponential function.
51) The function $f(x) = x^2 + \sqrt{x-2}$ is an algebraic function.	52) The function $f(x) = -3$ is a constant function.
53) The function $f(x) = \log_3 x$ is a general logarithmic function.	54) The function $f(x) = \ln x$ is a natural logarithmic function.
55) The function $f(x) = 3x^4 + x^2 + 1$ is <u>Solution:</u> $f(-x) = 3(-x)^4 + (-x)^2 + 1 = 3x^4 + x^2 + 1 = f(x)$ Hence, $f(x)$ is an even function.	56) The function $f(x) = 9 - x^2$ is <u>Solution:</u> $f(-x) = 9 - (-x)^2 = 9 - x^2 = f(x)$ Hence, $f(x)$ is an even function.
57) The function $f(x) = x^5 - x$ is <u>Solution:</u> $f(-x) = (-x)^5 - (-x) = -x^5 + x$ $= -(x^5 - x) = -f(x)$ Hence, $f(x)$ is an odd function.	58) The function $f(x) = 2 - \sqrt[5]{x}$ is <u>Solution:</u> $f(-x) = 2 - \sqrt[5]{(-x)} = 2 - \sqrt[5]{-x} = 2 + \sqrt[5]{x}$ $= -\left(-2 - \sqrt[5]{x}\right)$ Hence, $f(x)$ is neither even nor odd.
59) The function $f(x) = 3x + \frac{2}{\sqrt{x^2+9}}$ is <u>Solution:</u> $f(-x) = 3(-x) + \dfrac{2}{\sqrt{(-x)^2+9}} = -3x + \dfrac{2}{\sqrt{x^2+9}}$ $= -\left(3x - \dfrac{2}{\sqrt{x^2+9}}\right)$ Hence, $f(x)$ is neither even nor odd.	60) The function $f(x) = \frac{3}{\sqrt{x^2+9}}$ is <u>Solution:</u> $f(-x) = \dfrac{3}{\sqrt{(-x)^2+9}} = \dfrac{3}{\sqrt{x^2+9}} = f(x)$ Hence, $f(x)$ is an even function.
61) The function $f(x) = \sqrt{4 + x^2}$ is <u>Solution:</u> $f(-x) = \sqrt{4 + (-x)^2} = \sqrt{4 + x^2} = f(x)$ Hence, $f(x)$ is an even function.	62) The function $f(x) = 3$ is <u>Solution:</u> Since the graph of the constant function 3 is symmetric about the $y - axis$, then $f(x)$ is an even function.
63) The function $f(x) = \frac{9-x^2}{x-2}$ is <u>Solution:</u> $f(-x) = \dfrac{9-(-x)^2}{(-x)-2} = \dfrac{9-x^2}{-x-2}$ $= -\left(\dfrac{9-x^2}{x+2}\right)$ Hence, $f(x)$ is neither even nor odd.	64) The function $f(x) = \frac{x^2-4}{x^2+1}$ is <u>Solution:</u> $f(-x) = \dfrac{(-x)^2-4}{(-x)^2+1} = \dfrac{x^2-4}{x^2+1} = f(x)$ Hence, $f(x)$ is an even function.
65) The function $f(x) = 3\lvert x\rvert$ is <u>Solution:</u> $f(-x) = 3\lvert(-x)\rvert = 3\lvert x\rvert = f(x)$ Hence, $f(x)$ is an even function.	66) The function $f(x) = x^{-2}$ is <u>Solution:</u> $f(x) = x^{-2} = \dfrac{1}{x^2}$ $f(-x) = \dfrac{1}{(-x)^2} = \dfrac{1}{x^2} = f(x)$ Hence, $f(x)$ is an even function.

67) The function $f(x) = x^3 - 2x + 5$ is Solution: $f(-x) = (-x)^3 - 2(-x) + 5 \;= -x^3 + 2x + 5$ $\qquad\qquad\qquad\qquad\qquad\;\; = -(x^3 - 2x - 5)$ Hence, $f(x)$ is neither even nor odd.	68) The function $f(x) = \sqrt[3]{x^5} - x^3 + x$ is Solution: $f(-x) = \sqrt[3]{(-x)^5} - (-x)^3 + (-x) \;= -\sqrt[3]{x^5} + x^3 - x$ $= -\left(\sqrt[3]{x^5} - x^3 + x\right) = -f(x)$ Hence, $f(x)$ is an odd function.
69) The function $f(x) = 7$ is Solution: Since the graph of the constant function 7 is symmetric about the $y - axis$, then $f(x)$ is an even function.	70) The function $f(x) = \frac{x^3-4}{x^3+1}$ is Solution: $$f(-x) = \frac{(-x)^3 - 4}{(-x)^3 + 1} = \frac{-x^3 - 4}{-x^3 + 1} = -\frac{x^3 + 4}{-x^3 + 1}$$ Hence, $f(x)$ is neither even nor odd.
71) The function $f(x) = \frac{x^2-1}{x^3+3}$ is Solution: $$f(-x) = \frac{(-x)^2 - 1}{(-x)^3 + 3} = \frac{x^2 - 1}{-x^3 + 3} = -\frac{x^2 - 1}{x^3 - 3}$$ Hence, $f(x)$ is neither even nor odd.	72) The function $f(x) = x^6 - 4x^2 + 1$ is Solution: $f(-x) = (-x)^6 - 4(-x)^2 + 1 = x^6 - 4x^2 + 1 = f(x)$ Hence, $f(x)$ is an even function.
73) The function $f(x) = x^2$ is increasing on $(0, \infty)$.	74) The function $f(x) = x^2$ is decreasing on $(-\infty, 0)$.
75) The function $f(x) = x^3$ is increasing on $(-\infty, \infty)$.	76) The function $f(x) = x^3$ is not decreasing at all.
77) The function $f(x) = \sqrt{x}$ is increasing on $(0, \infty)$.	78) The function $f(x) = \sqrt{x}$ is not decreasing at all.
79) The function $f(x) = \frac{1}{x}$ is not increasing at all.	80) The function $f(x) = \frac{1}{x}$ is decreasing on $(-\infty, \infty)\backslash\{0\}$

Workshop Solutions to Sections 2.3 and 2.4 (1.3 & app D)

1) If $f(x)=x^2$ and $g(x)=\sqrt{4-x}$, then $(f+g)(x)=$ Solution: $(f+g)(x)=x^2+\sqrt{4-x}$	2) If $f(x)=x^2$ and $g(x)=\sqrt{4-x}$, then $D_{f+g}=$ Solution: $D_f=\mathbb{R}$ $g(x)$ is defined when $4-x\geq 0 \iff x\leq 4$. Thus, $D_g=(-\infty,4]$ $D_{f+g}=D_f\cap D_g=\mathbb{R}\cap(-\infty,4]=(-\infty,4]$
3) If $f(x)=x^2$ and $g(x)=\sqrt{4-x}$, then $(f-g)(x)=$ Solution: $(f-g)(x)=x^2-\sqrt{4-x}$	4) If $f(x)=x^2$ and $g(x)=\sqrt{4-x}$, then $D_{f-g}=$ Solution: $D_f=\mathbb{R}$ $g(x)$ is defined when $4-x\geq 0 \iff x\leq 4$. Thus, $D_g=(-\infty,4]$ $D_{f-g}=D_f\cap D_g=\mathbb{R}\cap(-\infty,4]=(-\infty,4]$
5) If $f(x)=x^2$ and $g(x)=\sqrt{4-x}$, then $(fg)(x)=$ Solution: $(fg)(x)=x^2\sqrt{4-x}$	6) If $f(x)=x^2$ and $g(x)=\sqrt{4-x}$, then $D_{fg}=$ Solution: $D_f=\mathbb{R}$ $g(x)$ is defined when $4-x\geq 0 \iff x\leq 4$. Thus, $D_g=(-\infty,4]$ $D_{fg}=D_f\cap D_g=\mathbb{R}\cap(-\infty,4]=(-\infty,4]$
7) If $f(x)=x^2$ and $g(x)=\sqrt{4-x}$, then $(f\circ g)(x)=$ Solution: $(f\circ g)(x)=f(g(x))$ $=f(\sqrt{4-x})=(\sqrt{4-x})^2=4-x$	8) If $f(x)=x^2$ and $g(x)=\sqrt{4-x}$, then $D_{f\circ g}=$ Solution: $(f\circ g)(x)=f(g(x))$ $=f(\sqrt{4-x})=(\sqrt{4-x})^2=4-x$ $D_g=(-\infty,4]$ $D_{f(g(x))}=\mathbb{R}$ $D_{f\circ g}=D_g\cap D_{f(g(x))}=(-\infty,4]\cap\mathbb{R}=(-\infty,4]$
9) If $f(x)=x^2$ and $g(x)=\sqrt{4-x}$, then $(g\circ f)(x)=$ Solution: $(g\circ f)(x)=g(f(x))=g(x^2)=\sqrt{4-x^2}$	10) If $f(x)=x^2$ and $g(x)=\sqrt{4-x}$, then $D_{g\circ f}=$ Solution: $(g\circ f)(x)=g(f(x))=g(x^2)=\sqrt{4-x^2}$ $D_f=\mathbb{R}$ $D_{g(f(x))}=[-2,2]$ $D_{g\circ f}=D_f\cap D_{g(f(x))}=\mathbb{R}\cap[-2,2]=[-2,2]$
11) If $f(x)=x^2$, then $(f\circ f)(x)=$ Solution: $(f\circ f)(x)=f(f(x))=f(x^2)=(x^2)^2=x^4$	12) If $f(x)=x^2$, then $D_{f\circ f}=$ Solution: $(f\circ f)(x)=f(f(x))=f(x^2)=(x^2)^2=x^4$ $D_f=\mathbb{R}$ $D_{f(f(x))}=\mathbb{R}$ $D_{f\circ f}=D_f\cap D_{f(f(x))}=\mathbb{R}\cap\mathbb{R}=\mathbb{R}$

13) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $\left(\frac{f}{g}\right)(x) =$

Solution:

$$\left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{4-x}}$$

14) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{\frac{f}{g}} =$

Solution:

$$\left(\frac{f}{g}\right)(x) = \frac{x^2}{\sqrt{4-x}}$$

$D_f = \mathbb{R}$

$g(x)$ is defined when $4 - x \geq 0 \quad \Leftrightarrow \quad x \leq 4$. Thus,

$D_g = (-\infty, 4]$

$$\begin{aligned} D_{\frac{f}{g}} &= \{x \in D_f \cap D_g | g(x) \neq 0\} \\ &= \mathbb{R} \cap (-\infty, 4) = (-\infty, 4) \end{aligned}$$

15) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $\left(\frac{g}{f}\right)(x) =$

Solution:

$$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$$

16) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$, then $D_{\frac{g}{f}} =$

Solution:

$$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$$

$D_f = \mathbb{R}$

$g(x)$ is defined when $4 - x \geq 0 \quad \Leftrightarrow \quad x \leq 4$. Thus,

$D_g = (-\infty, 4]$

$$\begin{aligned} D_{\frac{g}{f}} &= \{x \in D_f \cap D_g | f(x) \neq 0\} \\ &= \mathbb{R} \setminus \{0\} \cap (-\infty, 4] = (-\infty, 0) \cup (0, 4] \end{aligned}$$

17) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f+g)(x) =$

Solution:

$$\begin{aligned} (f+g)(x) = (9-x^2) + (10) &= 9 - x^2 + 10 \\ &= 19 - x^2 \end{aligned}$$

18) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f-g)(x) =$

Solution:

$$\begin{aligned} (f-g)(x) = (9-x^2) - (10) &= 9 - x^2 - 10 \\ &= -x^2 - 1 \end{aligned}$$

19) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g-f)(x) =$

Solution:

$$\begin{aligned} (g-f)(x) = (10) - (9-x^2) &= 10 - 9 + x^2 \\ &= 1 + x^2 \end{aligned}$$

20) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(fg)(x) =$

Solution:

$$(fg)(x) = (9-x^2)(10) = 90 - 10x^2$$

21) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f \circ g)(x) =$

Solution:

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) = f(10) \\ &= 9 - 10^2 = 9 - 100 = -91 \end{aligned}$$

22) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g \circ f)(x) =$

Solution:

$(g \circ f)(x) = g(f(x)) = g(9 - x^2) = 10$

23) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(f \circ f)(x) =$

Solution:

$$\begin{aligned} (f \circ f)(x) &= f(f(x)) = f(9 - x^2) \\ &= 9 - (9 - x^2)^2 \end{aligned}$$

24) If $f(x) = 9 - x^2$ and $g(x) = 10$, then $(g \circ g)(x) =$

Solution:

$$(g \circ g)(x) = g(g(x)) = g(10) = 10$$

25) If $f(x) = 9 - x^2$, $g(x) = \sin x$ and $h(x) = 3x + 2$, then $(f \circ g \circ h)(x) =$

Solution:

$$\begin{aligned} (f \circ g \circ h)(x) &= f\big(g(h(x))\big) \\ &= f\big(g(3x+2)\big) \\ &= f(\sin(3x+2)) \\ &= 9 - (\sin(3x+2))^2 \\ &= 9 - sin^2(3x+2) \end{aligned}$$

26) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$, then $(f+g)(x) =$

Solution:

$$(f+g)(x) = \sqrt{25 + x^2} + x^3$$

27) If $f(x)=\sqrt{25+x^2}$ and $g(x)=x^3$, then $(f-g)(x)=$ Solution: $(f-g)(x)=\sqrt{25+x^2}-x^3$	28) If $f(x)=\sqrt{25+x^2}$ and $g(x)=x^3$, then $(fg)(x)=$ Solution: $(fg)(x)=x^3\sqrt{25+x^2}$
29) If $f(x)=\sqrt{25+x^2}$ and $g(x)=x^3$, then $\left(\frac{f}{g}\right)(x)=$ Solution: $\left(\frac{f}{g}\right)(x)=\frac{\sqrt{25+x^2}}{x^3}$	30) If $f(x)=\sqrt{25+x^2}$ and $g(x)=x^3$, then $(f\circ g)(x)=$ Solution: $(f\circ g)(x)=f(g(x))=f(x^3)=\sqrt{25+(x^3)^2}$ $=\sqrt{25+x^6}$
31) If $f(x)=\sqrt{25+x^2}$ and $g(x)=x^3$, then $(g\circ f)(x)=$ Solution: $(g\circ f)(x)=g(f(x))=g\left(\sqrt{25+x^2}\right)=\left(\sqrt{25+x^2}\right)^3$ $=\sqrt{(25+x^2)^3}$	32) If $f(x)=\sqrt{x}$ and $g(x)=x-2$, then $(f\circ g)(x)=$ Solution: $(f\circ g)(x)=f(g(x))=f(x-2)=\sqrt{x-2}$
33) If $f(x)=\sqrt{x}$ and $g(x)=x-2$, then $(g\circ f)(x)=$ Solution: $(g\circ f)(x)=g(f(x))=g(\sqrt{x})=\sqrt{x}-2$	34) If $f(x)=\sqrt{x}$ and $g(x)=x-2$, then $(g\circ g)(x)=$ Solution: $(g\circ g)(x)=g(g(x))=g(x-2)=(x-2)-2$ $=x-2-2=x-4$
35) If $f(x)=\sqrt{x}$ and $g(x)=x-2$, then $(fg)(x)=$ Solution: $(fg)(x)=(\sqrt{x})(x-2)=(x-2)\sqrt{x}$	36) If $f(x)=\sin 5x$ and $g(x)=x^2+3$, then $(f\circ g)(x)=$ Solution: $(f\circ g)(x)=f(g(x))=f(x^2+3)=\sin 5(x^2+3)$
37) If $f(x)=\sin 5x$ and $g(x)=x^2+3$, then $(g\circ f)(x)=$ Solution: $(g\circ f)(x)=g(f(x))=g(\sin 5x)=(\sin 5x)^2+3$ $=sin^2 5x+3$	38) If $f(x)=\sin 5x$ and $g(x)=x^2+3$, then $(fg)(x)=$ Solution: $(fg)(x)=(\sin 5x)(x^2+3)=(x^2+3)\sin 5x$
39) If $f(x)=\sqrt{x}$ and $g(x)=\cos x$, then $(g\circ f)(x)=$ Solution: $(g\circ f)(x)=g(f(x))=g(\sqrt{x})=\cos\sqrt{x}$	40) If $f(x)=x+\frac{1}{x}$ and $g(x)=1-x^2$, then $(f\circ g)(x)=$ Solution: $(f\circ g)(x)=f(g(x))=f(1-x^2)=(1-x^2)+\frac{1}{1-x^2}$
41) If $f(x)=x+\frac{1}{x}$ and $g(x)=1-x^2$, then $(g\circ f)(x)=$ Solution: $(g\circ f)(x)=g(f(x))=g\left(x+\frac{1}{x}\right)=1-\left(x+\frac{1}{x}\right)^2$	42) If $f(x)=x+\frac{1}{x}$ and $g(x)=1-x^2$, then $(fg)(x)=$ Solution: $(fg)(x)=\left(x+\frac{1}{x}\right)(1-x^2)$
43) If the graph of the function $f(x)=x^2$ is shifted a distance 2 units upwards, then the new graph represented the graph of the function is Solution: x^2+2	44) If the graph of the function $f(x)=x^2$ is shifted a distance 2 units downwards, then the new graph represented the graph of the function is Solution: x^2-2
45) If the graph of the function $f(x)=x^2$ is shifted a distance 2 units to the right, then the new graph represented the graph of the function is Solution: $(x-2)^2=x^2-4x+4$	46) If the graph of the function $f(x)=x^2$ is shifted a distance 2 units to the left, then the new graph represented the graph of the function is Solution: $(x+2)^2=x^2+4x+4$

47) If the graph of the function $f(x) = \cos x$ is stretched vertically by a factor of 2 , then the new graph represented the graph of the function is

<u>Solution:</u>

$$2\cos x$$

48) If the graph of the function $f(x) = \cos x$ is compressed vertically by a factor of $\frac{1}{2}$, then the new graph represented the graph of the function is

<u>Solution:</u>

$$\frac{1}{2}\cos x$$

49) If the graph of the function $f(x) = \cos x$ is compressed horizontally by a factor of 2 , then the new graph represented the graph of the function is

<u>Solution:</u>

$$\cos 2x$$

50) If the graph of the function $f(x) = \cos x$ is stretched horizontally by a factor of $\frac{1}{2}$, then the new graph represented the graph of the function is

<u>Solution:</u>

$$\cos\frac{x}{2}$$

51) The graph of the function $f(x) = \sqrt{x}$ is reflected about the $x - axis$ if

<u>Solution:</u>

$$f(x) = -\sqrt{x}$$

52) The graph of the function $f(x) = \sqrt{x}$ is reflected about the $y - axis$ if

<u>Solution:</u>

$$f(x) = \sqrt{-x}$$

53) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units upwards , then the new graph represented the graph of the function is

<u>Solution:</u>

$$e^x + 2$$

54) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units downwards , then the new graph represented the graph of the function is

<u>Solution:</u>

$$e^x - 2$$

55) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units to the right , then the new graph represented the graph of the function is

<u>Solution:</u>

$$e^{x-2}$$

56) If the graph of the function $f(x) = e^x$ is shifted a distance 2 units to the left , then the new graph represented the graph of the function is

<u>Solution:</u>

$$e^{x+2}$$

57) $\frac{2\pi}{3}$ rad $= \frac{2\pi}{3} \times \frac{180^\circ}{\pi} = 120^\circ$

58) $\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

59) $\frac{7\pi}{6}$ rad $= \frac{7\pi}{6} \times \frac{180^\circ}{\pi} = 210^\circ$

60) $\frac{3\pi}{2}$ rad $= \frac{3\pi}{2} \times \frac{180^\circ}{\pi} = 270^\circ$

61) $120^\circ = 120 \times \frac{\pi}{180^\circ} = \frac{2\pi}{3}$ rad

62) $270^\circ = 270 \times \frac{\pi}{180^\circ} = \frac{3\pi}{2}$ rad

63) $\frac{5\pi}{12}$ rad $= \frac{5\pi}{12} \times \frac{180^\circ}{\pi} = 75^\circ$

64) $\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$ (Repeated)

65) $150^\circ = 150 \times \frac{\pi}{180^\circ} = \frac{5\pi}{6}$ rad

66) $210^\circ = 210 \times \frac{\pi}{180^\circ} = \frac{7\pi}{6}$ rad

67) $\frac{1}{\sec x} = \cos x$

68) $\frac{1}{\csc x} = \sin x$

69) $\frac{1}{\cot x} = \tan x$

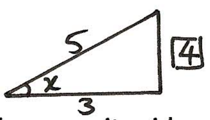
70) $\frac{\sin x}{\cos x} = \tan x$

71) $\frac{\cos x}{\sin x} = \cot x$

72) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\cot x =$

<u>Solution:</u>

$$\cos x = \frac{3}{5} = \frac{adj}{hyp}$$



Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

$$\therefore \cot x = \frac{1}{\tan x} = \frac{adj}{opp} = \frac{3}{4}$$

73) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\tan x =$

<u>Solution:</u>

$$\cos x = \frac{3}{5} = \frac{adj}{hyp}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$

$$\therefore \tan x = \frac{1}{\cot x} = \frac{opp}{adj} = \frac{4}{3}$$

74) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\sin x =$

Solution:

$$\cos x = \frac{3}{5} = \frac{adj}{hyp}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$

$$\therefore \sin x = \frac{opp}{hyp} = \frac{4}{5}$$

75) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$, then $\csc x =$

Solution:

$$\cos x = \frac{3}{5} = \frac{adj}{hyp}$$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$

$$\therefore \csc x = \frac{1}{\sin x} = \frac{hyp}{opp} = \frac{5}{4}$$

76) $\sin\left(\frac{5\pi}{6}\right) =$

Solution:

$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

So, we deduce now that $\sin\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$\sin\left(\frac{5\pi}{6}\right) = \sin(150^\circ) = \sin(180^\circ - 30^\circ) = \sin(30^\circ) =$

$\sin\pi 6=12$

77) $\cos\left(\frac{5\pi}{6}\right) =$

Solution:

$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

So, we deduce now that $\cos\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$$\cos\left(\frac{5\pi}{6}\right) = \cos(150^\circ) = \cos(180^\circ - 30^\circ)$$

$$= -\cos(30^\circ) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

78) $\tan\left(\frac{5\pi}{6}\right) =$

Solution:

$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

So, we deduce now that $\tan\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$$\tan\left(\frac{5\pi}{6}\right) = \tan(150^\circ) = \tan(180^\circ - 30^\circ)$$

$$= -\tan(30^\circ) = -\tan\left(\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

79) $\cot\left(\frac{5\pi}{6}\right) =$

Solution:

$\frac{5\pi}{6}$ rad $= \frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$

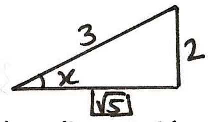
So, we deduce now that $\cot\left(\frac{5\pi}{6}\right)$ is in the second quarter.

$$\cot\left(\frac{5\pi}{6}\right) = \cot(150^\circ) = \cot(180^\circ - 30^\circ)$$

$$= -\cot(30^\circ) = -\cot\left(\frac{\pi}{6}\right) = -\sqrt{3}$$

80) If $\sin x = \frac{2}{3}$ and $0 < x < \frac{\pi}{2}$, then $\sec x =$

Solution:



$$\sin x = \frac{2}{3} = \frac{opp}{hyp}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$

$$\therefore \sec x = \frac{1}{\cos x} = \frac{hyp}{adj} = \frac{3}{\sqrt{5}}$$

81) If $\sin x = \frac{2}{3}$ and $0 < x < \frac{\pi}{2}$, then $\csc x =$

Solution:

$$\sin x = \frac{2}{3} = \frac{opp}{hyp}$$

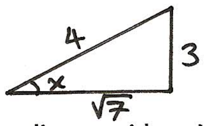
Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - 2^2} = \sqrt{9-4} = \sqrt{5}$

$$\therefore \csc x = \frac{1}{\sin x} = \frac{hyp}{opp} = \frac{3}{2}$$

82) If $\sin x = \frac{3}{4}$ and $0 < x < \frac{\pi}{2}$, then $\cos x =$

Solution:



$$\sin x = \frac{3}{4} = \frac{opp}{hyp}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{4^2 - 3^2} = \sqrt{16-9} = \sqrt{7}$

$$\therefore \cos x = \frac{adj}{hyp} = \frac{\sqrt{7}}{4}$$

83) If $\sin x = \frac{3}{4}$ and $0 < x < \frac{\pi}{2}$, then $\cot x =$

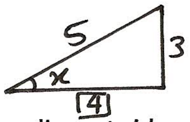
Solution:

$$\sin x = \frac{3}{4} = \frac{opp}{hyp}$$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{4^2 - 3^2} = \sqrt{16-9} = \sqrt{7}$

$$\therefore \cot x = \frac{1}{\tan x} = \frac{adj}{opp} = \frac{\sqrt{7}}{3}$$

<table>
<tr>
<td>
84) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\cos x =$
<u>Solution:</u>
$\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$


Now, we should find the length of the adjacent side using the Pythagorean Theorem, so
$|\text{adjacent}| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$
$\therefore\ \cos x = \frac{adj}{hyp} = \frac{4}{5}$
</td>
<td>
85) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\sec x =$
<u>Solution:</u>
$\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$
Now, we should find the length of the adjacent side using the Pythagorean Theorem, so
$|\text{adjacent}| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$
$\therefore\ \sec x = \frac{1}{\cos x} = \frac{hyp}{adj} = \frac{5}{4}$
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86) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\cot x =$
<u>Solution:</u>
$\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$
Now, we should find the length of the adjacent side using the Pythagorean Theorem, so
$|\text{adjacent}| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$
$\therefore\ \cot x = \frac{1}{\tan x} = \frac{adj}{opp} = -\frac{4}{3}$
</td>
<td>
87) If $\csc x = -\frac{5}{3}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\tan x =$
<u>Solution:</u>
$\csc x = \frac{5}{3} = \frac{1}{\sin x} = \frac{hyp}{opp}$
Now, we should find the length of the adjacent side using the Pythagorean Theorem, so
$|\text{adjacent}| = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$
$\therefore\ \tan x = \frac{1}{\cot x} = \frac{opp}{adj} = -\frac{3}{4}$
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<tr>
<td>88) If $f(x) = \sin x$, then $D_f = \mathbb{R}$</td>
<td>89) If $f(x) = \cos x$, then $D_f = \mathbb{R}$</td>
</tr>
<tr>
<td>88) If $f(x) = \sin x$, then $R_f = [-1,1]$</td>
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</table>

Workshop Solutions to Section 2.5 (1.5)

How to find the domain and range of the exponential function $f(x) = a^x$?

1- If $f(x) = c.a^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (\pm k, \infty)$$

2- If $f(x) = -c.a^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (-\infty, \pm k)$$

3- If $f(x) = c.e^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (\pm k, \infty)$$

4- If $f(x) = -c.e^{\pm x} \pm k$ where c and k are positive constants, then

$$D_f = \mathbb{R} \quad \text{and} \quad R_f = (-\infty, \pm k)$$

1) Find the domain of the function $f(x) = 4^x$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	2) Find the range of the function $f(x) = 4^x$. Solution: From Step (1) above, we deduce that $R_f = (0, \infty)$
3) Find the domain of the function $f(x) = 4^x - 3$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	4) Find the range of the function $f(x) = 4^x - 3$. Solution: From Step (1) above, we deduce that $R_f = (-3, \infty)$
5) Find the domain of the function $f(x) = 5 - 3^x$. Solution: From Step (2) above, we deduce that $D_f = \mathbb{R}$	6) Find the range of the function $f(x) = 5 - 3^x$. Solution: From Step (2) above, we deduce that $R_f = (-\infty, 5)$
7) Find the domain of the function $f(x) = 3^{-x} + 1$. Solution: From Step (1) above, we deduce that $D_f = \mathbb{R}$	8) Find the range of the function $f(x) = 3^{-x} + 1$. Solution: From Step (1) above, we deduce that $R_f = (1, \infty)$
9) Find the domain of the function $f(x) = e^x$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	10) Find the range of the function $f(x) = e^x$. Solution: From Step (3) above, we deduce that $R_f = (0, \infty)$
11) Find the domain of the function $f(x) = e^x - 3$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	12) Find the range of the function $f(x) = e^x - 3$. Solution: From Step (3) above, we deduce that $R_f = (-3, \infty)$
13) Find the domain of the function $f(x) = e^x + 1$. Solution: From Step (3) above, we deduce that $D_f = \mathbb{R}$	14) Find the domain of the function $f(x) = \frac{1}{1-e^x}$. Solution: $f(x)$ is defined when $1 - e^x \neq 0$ $\Leftrightarrow \; e^x \neq 1 \; \Leftrightarrow \; \ln e^x \neq \ln 1$ $\Leftrightarrow \; x \neq 0$ $\therefore \; D_f = \mathbb{R} \setminus \{0\}$

15) Find the domain of the function $f(x) = \frac{1}{1+e^x}$. <u>Solution:</u> $f(x)$ is defined when $1 + e^x \neq 0$. But there is no value of x makes $1 + e^x = 0$. Therefore, $D_f = \mathbb{R}$	16) Find the domain of the function $f(x) = \sqrt{1 + 3^x}$. <u>Solution:</u> $f(x)$ is defined when $1 + 3^x \geq 0$. But $1 + 3^x > 0$ always. Therefore, $D_f = \mathbb{R}$
17) If $4^{(x+1)} = 8$, then $x =$ <u>Solution:</u> $4^{(x+1)} = 8$ $(2^2)^{(x+1)} = 2^3$ $2^{2(x+1)} = 2^3$ $2(x+1) = 3$ $2x + 2 = 3$ $2x = 3 - 2 = 1$ $\therefore \ x = \frac{1}{2}$	18) If $4^{(x-1)} = 8$, then $x =$ <u>Solution:</u> $4^{(x-1)} = 8$ $(2^2)^{(x-1)} = 2^3$ $2^{2(x-1)} = 2^3$ $2(x-1) = 3$ $2x - 2 = 3$ $2x = 3 + 2 = 5$ $\therefore \ x = \frac{5}{2}$
19) If $9^{(x+1)} = 27$, then $x =$ <u>Solution:</u> $9^{(x+1)} = 27$ $(3^2)^{(x+1)} = 3^3$ $3^{2(x+1)} = 3^3$ $2(x+1) = 3$ $2x + 2 = 3$ $2x = 3 - 2 = 1$ $\therefore \ x = \frac{1}{2}$	20) If $9^{(x-1)} = 27$, then $x =$ <u>Solution:</u> $9^{(x-1)} = 27$ $(3^2)^{(x-1)} = 3^3$ $3^{2(x-1)} = 3^3$ $2(x-1) = 3$ $2x - 2 = 3$ $2x = 3 + 2 = 5$ $\therefore \ x = \frac{5}{2}$
21) If $5^{2(x-1)} = 125$, then $x =$ <u>Solution:</u> $5^{2(x-1)} = 125$ $5^{2(x-1)} = 5^3$ $2(x-1) = 3$ $2x - 2 = 3$ $2x = 3 + 2 = 5$ $\therefore \ x = \frac{5}{2}$	22) If $5^{2(x+1)} = 125$, then $x =$ <u>Solution:</u> $5^{2(x+1)} = 125$ $5^{2(x+1)} = 5^3$ $2(x+1) = 3$ $2x + 2 = 3$ $2x = 3 - 2 = 1$ $\therefore \ x = \frac{1}{2}$

Workshop Solutions to Section 2.6 (1.6)

1) The inverse of the function
$f = \{(0,3), (-2,1), (3,4), (5,-2), (1,7)\}$ is
$f^{-1} = \{(3,0), (1,-2), (4,3), (-2,5), (7,1)\}$

2) Find the inverse of the function $f(x) = 2x + 3$.

Solution:

Let $y = 2x + 3$

$2x = y - 3$

$x = \frac{y-3}{2}$

Now, change x with y $(x \Leftrightarrow y)$

$y = \frac{x-3}{2}$

$\therefore\ f^{-1}(x) = \frac{x-3}{2}$

3) Find the inverse of the function $f(x) = 3 - 2x$.

Solution:

Let $y = 3 - 2x$

$2x = 3 - y$

$x = \frac{3-y}{2}$

Now, change x with y $(x \Leftrightarrow y)$

$y = \frac{3-x}{2}$

$\therefore\ f^{-1}(x) = \frac{3-x}{2}$

4) Find the inverse of the function $f(x) = 3 - \frac{x}{2}$.

Solution:

Let $y = 3 - \frac{x}{2}$

$2y = 6 - x$

$x = 6 - 2y$

Now, change x with y $(x \Leftrightarrow y)$

$y = 6 - 2x$

$\therefore\ f^{-1}(x) = 6 - 2x$

5) Find the inverse of the function $f(x) = \sqrt{2x-3}$.

Solution:

Let $y = \sqrt{2x-3}$ by squaring both sides

$y^2 = 2x - 3$

$2x = y^2 + 3$

$x = \frac{y^2+3}{2}$

Now, change x with y $(x \Leftrightarrow y)$

$y = \frac{x^2+3}{2}$

$\therefore\ f^{-1}(x) = \frac{x^2+3}{2}$

6) Find the inverse of the function $f(x) = \sqrt[3]{3-2x}$.

Solution:

Let $y = \sqrt[3]{3-2x}$ by cubing both sides

$y^3 = 3 - 2x$

$2x = 3 - y^3$

$x = \frac{3-y^3}{2}$

Now, change x with y $(x \Leftrightarrow y)$

$y = \frac{3-x^3}{2}$

$\therefore\ f^{-1}(x) = \frac{3-x^3}{2}$

7) Find the inverse of the function
$f(x) = (2x+3)^2, x \in [0, \infty)$.

Solution:

Let $y = (2x+3)^2$

Take the square root for both sides

$\sqrt{y} = 2x + 3$

$2x = \sqrt{y} - 3$

$x = \frac{\sqrt{y}-3}{2}$

Now, change x with y $(x \Leftrightarrow y)$

$y = \frac{\sqrt{x}-3}{2}$

$\therefore\ f^{-1}(x) = \frac{\sqrt{x}-3}{2}$

8) Find the inverse of the function $f(x) = -(x-3)^3$.

Solution:

Let $y = -(x-3)^3$

$-y = (x-3)^3$

Take the cubic root for both sides

$\sqrt[3]{-y} = x - 3$

$x = \sqrt[3]{-y} + 3$

Now, change x with y $(x \Leftrightarrow y)$

$y = \sqrt[3]{-x} + 3$

$\therefore\ f^{-1}(x) = \sqrt[3]{-x} + 3$

9) Find the inverse of the function $f(x) = \frac{x}{x-3}$.

Solution:

Let $y = \frac{x}{x-3}$

$y(x-3) = x$

$xy - 3y = x$

$xy - x = 3y$

$x(y-1) = 3y$

$x = \frac{3y}{y-1}$

Now, change x with y $(x \Leftrightarrow y)$

$y = \frac{3x}{x-1}$

$\therefore\ f^{-1}(x) = \frac{3x}{x-1}$

10) Find the inverse of the function $f(x) = \frac{x-3}{x}$.

Solution:

Let $y = \frac{x-3}{x}$

$xy = x - 3$

$xy - x = -3$

$x(y-1) = -3$

$x = \frac{-3}{y-1} = -\frac{3}{y-1} = \frac{3}{-(y-1)} = \frac{3}{1-y}$

Now, change x with y $(x \Leftrightarrow y)$

$y = \frac{3}{1-x}$

$\therefore\ f^{-1}(x) = \frac{3}{1-x}$

11) Find the inverse of the function $f(x) = \frac{x+2}{x-3}$.

Solution:

Let $y = \frac{x+2}{x-3}$

$$y(x-3) = x+2$$
$$xy - 3y = x+2$$
$$xy - x = 3y+2$$
$$x(y-1) = 3y+2$$
$$x = \frac{3y+2}{y-1}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \frac{3x+2}{x-1}$$

$$\therefore\ f^{-1}(x) = \frac{3x+2}{x-1}$$

12) Find the inverse of the function $f(x) = \sqrt{x} + 5$.

Solution:

Let $y = \sqrt{x} + 5$

$\sqrt{x} = y - 5$ by squaring both sides

$$x = (y-5)^2$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = (x-5)^2$$

$$\therefore\ f^{-1}(x) = (x-5)^2$$

13) Find the inverse of the function $f(x) = \sqrt[3]{x^5}$.

Solution:

Let $y = \sqrt[3]{x^5}$

$$y = x^{\frac{5}{3}}$$

$$y^{\frac{3}{5}} = (x^{\frac{5}{3}})^{\frac{3}{5}}$$

$$x = \sqrt[5]{y^3}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \sqrt[5]{x^3}$$

$$\therefore\ f^{-1}(x) = \sqrt[5]{x^3}$$

14) Find the inverse of the function $f(x) = 2x^3 - 5$.

Solution:

Let $y = 2x^3 - 5$

$$2x^3 = y + 5$$

$x^3 = \frac{y+5}{2}$ take the cubic root for both sides

$$x = \sqrt[3]{\frac{y+5}{2}}$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = \sqrt[3]{\frac{x+5}{2}}$$

$$\therefore\ f^{-1}(x) = \sqrt[3]{\frac{x+5}{2}}$$

15) Find the inverse of the function $f(x) = \sqrt[3]{\frac{x+2}{5}}$.

Solution:

Let $y = \sqrt[3]{\frac{x+2}{5}}$ by cubing both sides

$$y^3 = \frac{x+2}{5}$$
$$5y^3 = x+2$$
$$x = 5y^3 - 2$$

Now, change x with y $(x \Leftrightarrow y)$

$$y = 5x^3 - 2$$

$$\therefore\ f^{-1}(x) = 5x^3 - 2$$

16) Evaluate

$$2^{log_2(5x+3)}.$$

Solution:

$$2^{log_2(5x+3)} = 5x + 3$$

17) Evaluate

$$log_2 2^{(5x+3)}.$$

Solution:

$$log_2 2^{(5x+3)} = 5x + 3$$

18) $log_2 64 - log_2 32 + log_2 2 = log_2 \frac{64\times 2}{32}$
$$= log_2 4 = log_2 2^2$$
$$= 2 log_2 2$$
$$= 2 \times 1 = 2$$

OR

$$log_2 64 - log_2 32 + log_2 2 = log_2 2^6 - log_2 2^5 + log_2 2$$
$$= 6 - 5 + 1 = 2$$

19) $log_3 27 - log_3 81 + 5\, log_3 3 = log_3 \frac{27\times 3^5}{81}$
$$= log_3 81 = log_3 3^4$$
$$= 4 log_3 3$$
$$= 4 \times 1 = 4$$

OR

$$log_3 27 - log_3 81 + 5\, log_3 = log_3 3^3 - log_3 3^4 + 5 \times 1$$
$$= 3 - 4 + 5 = 4$$

20) $log_3 54 - log_3 2 = log_3 \frac{54}{2}$
$$= log_3 27 = log_3 3^3 = 3$$

21) If $log_2(6+2x) = 1$, then $x =$

Solution:

$$log_2(6+2x) = 1$$
$$2^{log_2(6+2x)} = 2^1$$
$$6 + 2x = 2$$
$$2x = 2 - 6 = -4$$
$$x = -2$$

22) If $ln(x+3) = 5$, then $x =$

Solution:

$$ln(x+3) = 5$$
$$e^{ln(x+3)} = e^5$$
$$x + 3 = e^5$$
$$x = e^5 - 3$$

23) If $ln(x) = 5$, then $x =$

Solution:

$$ln(x) = 5$$
$$e^{ln(x)} = e^5$$
$$x = e^5$$

24) If $e^{(2x-3)} = 5$, then $x =$

Solution:

$$e^{(2x-3)} = 5$$
$$lne^{(2x-3)} = ln5$$
$$2x - 3 = ln5$$
$$2x = ln5 + 3$$
$$x = \frac{ln5+3}{2}$$

25) $log_3 2 = \frac{ln2}{ln3}$

26) $\log\ 25 + \log\ 4 = \log(25 \times 4)$
$$= \log 100 = \log 10^2$$
$$= 2$$

27) $log_3 18 - log_3 6 = log_3 \frac{18}{6}$
$$= log_3 3$$
$$= 1$$

28) $log_2 6 - log_2 15 + log_2 20 = log_2 \frac{6\times 20}{15}$
$$= log_2 8 = log_2 2^3$$
$$= 3$$

29) $e^{3ln2} = e^{ln2^3} = 2^3 = 8$

30) If $3^{2-x} = 6$, then $x =$

Solution:

$$3^{2-x} = 6$$
$$log_3 3^{2-x} = log_3 6$$
$$2 - x = log_3 6$$
$$x = 2 - log_3 6 = 2 - log_3(3 \times 2)$$
$$= 2 - (log_3 3 + log_3 2) = 2 - (1 + log_3 2)$$
$$= 2 - 1 - log_3 2$$
$$= 1 - log_3 2$$

31) Find the inverse of the function $f(x) = 5 + lnx$.

Solution:

Let $y = 5 + lnx$
$$lnx = y - 5$$
$$e^{lnx} = e^{y-5}$$
$$x = e^{y-5}$$

Now, change x with y $(x \Leftrightarrow y)$
$$y = e^{x-5}$$
$$\therefore\ f^{-1}(x) = e^{x-5}$$

32) Find the domain of the function
$$f(x) = \sin^{-1}(3x + 5).$$

Solution:

We know that the domain of $\sin^{-1}(x)$ is $[-1,1]$. So,
$$-1 \le 3x + 5 \le 1$$
$$-6 \le 3x \le -4$$
$$-2 \le x \le -\frac{4}{3}$$
$$\therefore\ D_f = \left[-2, -\frac{4}{3}\right]$$

33) Find the domain of the function
$$f(x) = \cos^{-1}(3x - 5).$$

Solution:

We know that the domain of $\cos^{-1}(x)$ is $[-1,1]$. So,
$$-1 \le 3x - 5 \le 1$$
$$4 \le 3x \le 6$$
$$\frac{4}{3} \le x \le 2$$
$$\therefore\ D_f = \left[\frac{4}{3}, 2\right]$$

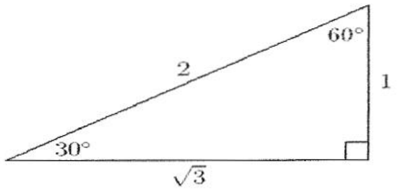
34) Find the domain of the function
$$f(x) = 2\sin^{-1}(x) + 1.$$

Solution:

We know that the domain of $\sin^{-1}(x)$ is $[-1,1]$. So,
$$\therefore\ D_f = [-1,1]$$

Before proceeding to the questions 35-55, we should be aware of the following well-known right triangles:

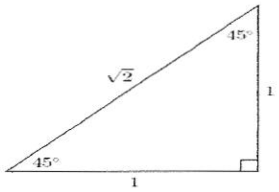
$30^o - 60^o$ Right Triangle



We know that $30^o = \frac{\pi}{6}$ and $60^o = \frac{\pi}{3}$, so

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \qquad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$
$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \qquad \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$
$$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \qquad \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$
$$\cot\left(\frac{\pi}{6}\right) = \sqrt{3} \qquad \cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$
$$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}} \qquad \sec\left(\frac{\pi}{3}\right) = 2$$
$$\csc\left(\frac{\pi}{6}\right) = 2 \qquad \csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$$

$30^o - 60^o$ Right Triangle



We know that $45^o = \frac{\pi}{4}$, so

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$
$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$
$$\tan\left(\frac{\pi}{4}\right) = 1$$
$$\cot\left(\frac{\pi}{4}\right) = 1$$
$$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$
$$\csc\left(\frac{\pi}{4}\right) = \sqrt{2}$$

35) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

Solution:

Let $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\sin\theta = \frac{\sqrt{3}}{2}$

Use the $30^o - 60^o$ right triangle to find θ. Thus,

$$\theta = \frac{\pi}{3}$$

36) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

Solution:

Let $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\sin\theta = \frac{\sqrt{3}}{2}$

Use the $30^o - 60^o$ right triangle to find θ. Thus,

$$\theta = \frac{\pi}{3}$$

37) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) =$

Solution:

Let $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\tan\theta = \frac{1}{\sqrt{3}}$

Use the $30^o - 60^o$ right triangle to find θ. Thus,

$$\theta = \frac{\pi}{6}$$

38) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) =$

Solution:

Let $\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\sin\theta = \frac{1}{\sqrt{2}}$

Use the $45^o - 45^o$ right triangle to find θ. Thus,

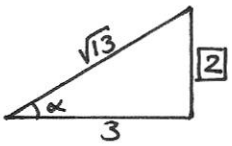
$$\theta = \frac{\pi}{4}$$

39) If $\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$, then $\tan\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$

$\cos\alpha = \frac{3}{\sqrt{13}} = \frac{adj}{hyp}$



Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{\left(\sqrt{13}\right)^2 - 3^2} = \sqrt{13-9} = \sqrt{4} = 2$$

$$\therefore \tan\alpha = \frac{opp}{adj} = \frac{2}{3}$$

40) If $\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$, then $\csc\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$

$\cos\alpha = \frac{3}{\sqrt{13}} = \frac{adj}{hyp}$

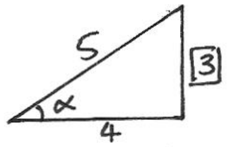
Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$$|\text{opposite}| = \sqrt{\left(\sqrt{13}\right)^2 - 3^2} = \sqrt{13-9} = \sqrt{4} = 2$$

$$\therefore \csc\alpha = \frac{1}{\sin\alpha} = \frac{hyp}{opp} = \frac{\sqrt{13}}{2}$$

41) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\csc\alpha =$

Solution:



$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \csc\alpha = \frac{1}{\sin\alpha} = \frac{hyp}{opp} = \frac{5}{3}$$

42) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\cot\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \cot\alpha = \frac{1}{\tan\alpha} = \frac{adj}{opp} = \frac{4}{3}$$

43) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\tan\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \tan\alpha = \frac{1}{\cot\alpha} = \frac{opp}{adj} = \frac{3}{4}$$

44) If $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$, then $\sin\alpha =$

Solution:

$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \sin\alpha = \frac{opp}{hyp} = \frac{3}{5}$$

45) $\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right) =$

Solution:

Let $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right) = \sin(\alpha) = \frac{opp}{hyp} = \frac{3}{5}$$

46) $\tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right) =$

Solution:

Let $\alpha = \cos^{-1}\left(\frac{4}{5}\right)$

$\cos\alpha = \frac{4}{5} = \frac{adj}{hyp}$

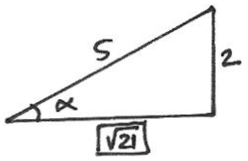
Now, we should find the length of the opposite side using the Pythagorean Theorem, so

$|\text{opposite}| = \sqrt{5^2 - 4^2} = \sqrt{25-16} = \sqrt{9} = 3$

$$\therefore \tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right) = \tan(\alpha) = \frac{opp}{adj} = \frac{3}{4}$$

47) $\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) =$

Solution:



Let $\alpha = \sin^{-1}\left(\frac{2}{5}\right)$

$\sin\alpha = \frac{2}{5} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{5^2 - 2^2} = \sqrt{25-4} = \sqrt{21}$

$$\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) = \sin(2\alpha)$$

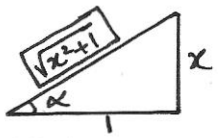
Now, use the identity $\sin(2x) = 2\sin x . \cos x$. Thus,

$$\sin\left(2\sin^{-1}\left(\frac{2}{5}\right)\right) = \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$

$$= 2 \times \frac{2}{5} \times \frac{\sqrt{21}}{5} = \frac{4\sqrt{21}}{25}$$

48) $\cos(\tan^{-1} x) =$

Solution:



Let $\alpha = \tan^{-1} x$

$\tan\alpha = x = \frac{opp}{adj}$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2+1}$

$$\cos(\tan^{-1} x) = \cos(\alpha) = \frac{adj}{hyp} = \frac{1}{\sqrt{x^2+1}}$$

49) $\sin(\tan^{-1} x) =$

Solution:

Let $\alpha = \tan^{-1} x$

$\tan\alpha = x = \frac{opp}{adj}$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2+1}$

$$\sin(\tan^{-1} x) = \sin(\alpha) = \frac{opp}{hyp} = \frac{x}{\sqrt{x^2+1}}$$

50) $\csc(\tan^{-1} x) =$

Solution:

Let $\alpha = \tan^{-1} x$

$\tan\alpha = x = \frac{opp}{adj}$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2+1}$

$$\csc(\tan^{-1} x) = \csc(\alpha) = \frac{1}{\sin\alpha} = \frac{hyp}{opp} = \frac{\sqrt{x^2+1}}{x}$$

51) $\sec(\tan^{-1} x) =$

Solution:

Let $\alpha = \tan^{-1} x$

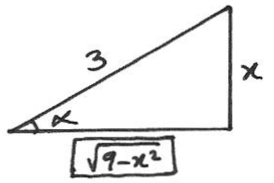
$\tan \alpha = x = \frac{opp}{adj}$

Now, we should find the length of the hypotenuse side using the Pythagorean Theorem, so

$|\text{hypotenuse}| = \sqrt{x^2 + 1^2} = \sqrt{x^2 + 1}$

$$\sec(\tan^{-1} x) = \sec(\alpha) = \frac{1}{\cos \alpha} = \frac{hyp}{adj} = \frac{\sqrt{x^2+1}}{1} = \sqrt{x^2+1}$$

52) $\sec\left(\sin^{-1}\frac{x}{3}\right) =$



Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$

$\sin \alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$

$$\sec\left(\sin^{-1}\frac{x}{3}\right) = \sec(\alpha) = \frac{1}{\cos \alpha} = \frac{hyp}{adj} = \frac{3}{\sqrt{9-x^2}}$$

53) $\cot\left(\sin^{-1}\frac{x}{3}\right) =$

Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$

$\sin \alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$

$$\cot\left(\sin^{-1}\frac{x}{3}\right) = \cot(\alpha) = \frac{1}{\tan \alpha} = \frac{adj}{opp} = \frac{\sqrt{9-x^2}}{x}$$

54) $\tan\left(\sin^{-1}\frac{x}{3}\right) =$

Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$

$\sin \alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$

$$\tan\left(\sin^{-1}\frac{x}{3}\right) = \tan(\alpha) = \frac{1}{\cot \alpha} = \frac{opp}{adj} = \frac{x}{\sqrt{9-x^2}}$$

55) $\cos\left(\sin^{-1}\frac{x}{3}\right) =$

Solution:

Let $\alpha = \sin^{-1}\frac{x}{3}$

$\sin \alpha = \frac{x}{3} = \frac{opp}{hyp}$

Now, we should find the length of the adjacent side using the Pythagorean Theorem, so

$|\text{adjacent}| = \sqrt{3^2 - x^2} = \sqrt{9 - x^2}$

$$\cos\left(\sin^{-1}\frac{x}{3}\right) = \cos(\alpha) = \frac{adj}{hyp} = \frac{\sqrt{9-x^2}}{3}$$

A) $f(x) = \dfrac{1}{x^3 - 8}$ is a Rational function

$$x^3 - 8 = 0$$

$$x^3 = 8$$

$$\sqrt[\cancel{3}]{x^{\cancel{3}}} = \sqrt[3]{8}$$

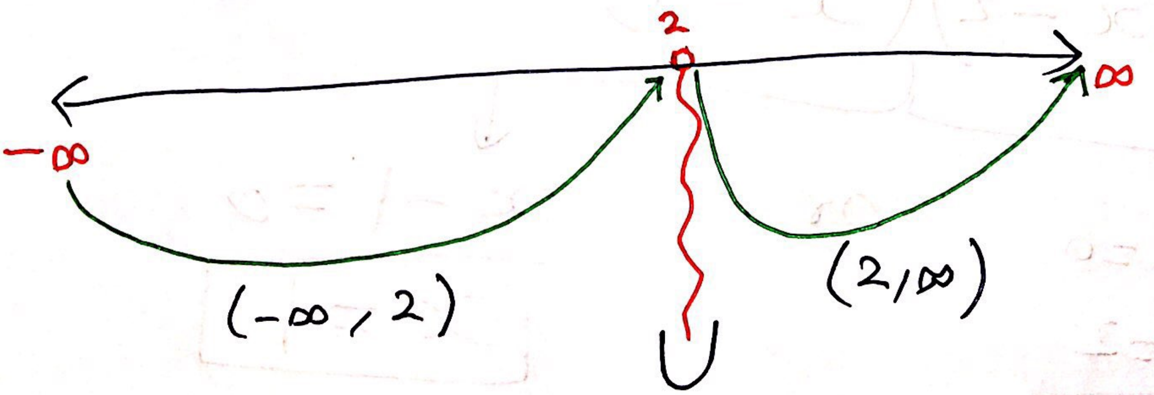
$$x = \sqrt[\cancel{3}]{2^{\cancel{3}}}$$

$$x = 2$$

$\therefore D_{f(x)} = \mathbb{R} - \{2\}$

or $= \{x \mid x \neq 2\}$

or $= (-\infty, 2) \cup (2, \infty)$



B) $f(x) = \dfrac{x}{3x^2 - 5x + 2}$ is a Rational function

$$3x^2 - 5x + 2 = 0$$

$$\begin{array}{cc|c} 3x & -2 & -2x \\ 1x & -1 & -3x \\ \hline & & -5x \end{array}$$

ضرب

ضرب

جمع

$$\therefore (3x-2)(x-1) = 0$$

$3x - 2 = 0$

$3x = 2$

$\boxed{x = 2/3}$

or

$x - 1 = 0$

$\boxed{x = 1}$

$$\Longrightarrow D_{f(x)} = \mathbb{R} - \{2/3, 1\}$$

c) $f(x) = \dfrac{x-3}{2x^2+5x-12}$ is a Rational function

$$2x^2 + 5x - 12 = 0$$

$$\begin{array}{cccc|l} 2 & x & - & 3 & -3x \\ & & \text{ضرب} & & \\ 1 & x & + & 4 & +8x \\ \hline & & & & +5x \end{array}$$

ج $\Rightarrow (2x-3)(x+4) = 0$

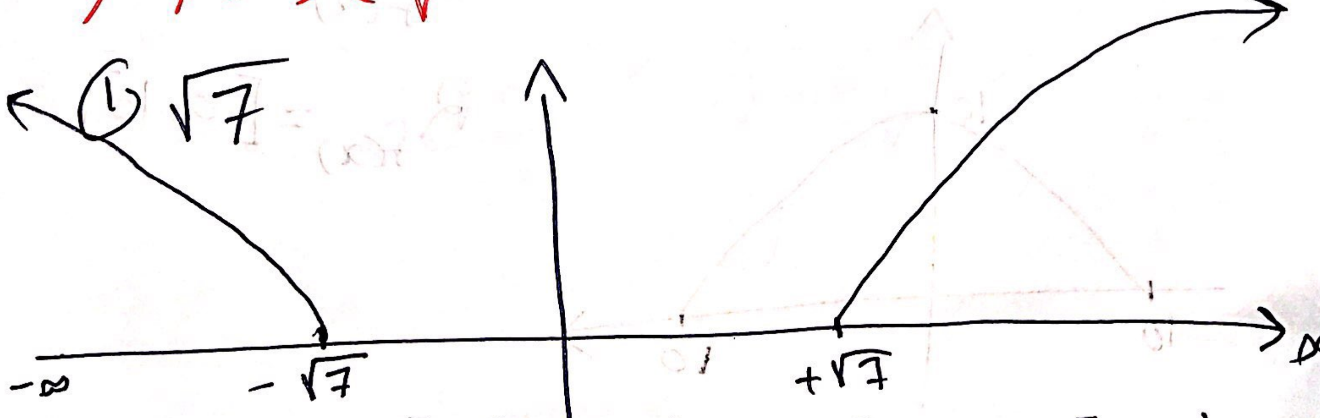
$2x-3=0$ or $x+4=0$

$2x = 3$

$x = 3/2$, $x = -4$

$\Rightarrow D_{f(x)} = \mathbb{R} - \{-4, \frac{3}{2}\}$

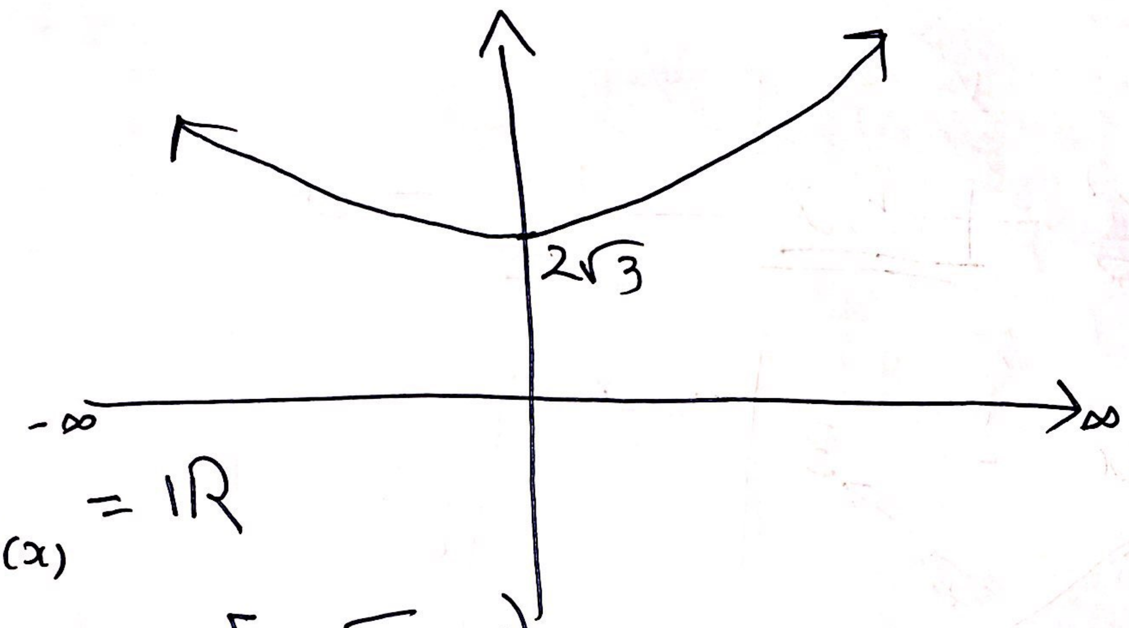
F) $f(x) = \sqrt{x^2 - 7}$



$D_{f(x)} = (-\infty, -\sqrt{7}] \cup [\sqrt{7}, \infty)$; Rang $= [0, \infty)$

D) $f(x) = \sqrt{x^2+12}$

① $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$



$D_{f(x)} = \mathbb{R}$

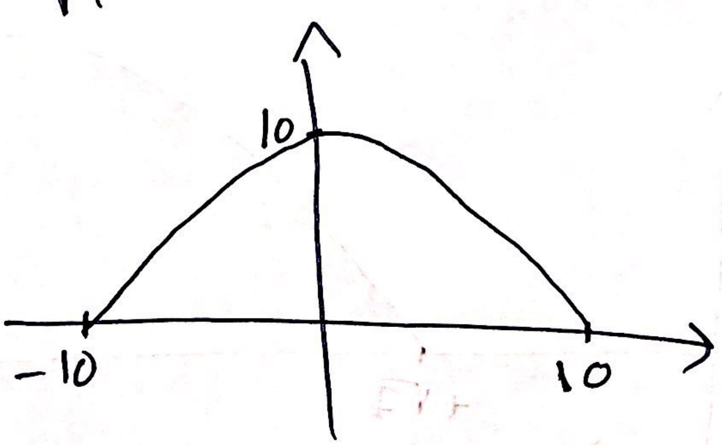
$R_{f(x)} = [2\sqrt{3}, \infty)$

E) $f(x) = \sqrt{100 - x^2}$

① $\sqrt{100} = 10$

* $D_{f(x)} = [-10, 10]$

$R_{f(x)} = [0, 10]$



1) $210°$

$$210° = \frac{210 \times \pi}{180}$$

$$= \frac{21\pi}{18}$$

$$= \frac{21 \div 3}{18 \div 3}\pi$$

$$= \frac{7}{6}\pi \quad \text{or} \quad \frac{7\pi}{6}$$

or

$$210° = \frac{210 \times \pi}{180} = \frac{21\pi}{18} = \frac{(3)(7)\pi}{(2)(3)(3)} = \frac{\cancel{(3)}(7)\pi}{(2)(3)\cancel{(3)}}$$

$$= \frac{7\pi}{6}$$

$$\begin{array}{r|l} 21 & 3 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

2) 300°

$$300^\circ = \frac{300\pi}{180}$$

$$= \frac{30\pi}{18}$$

$$= \frac{30 \div 2}{18 \div 2}\pi$$

$$= \frac{15}{9}\pi = \frac{15 \div 3}{9 \div 3}\pi$$

$$= \frac{5}{3}\pi \text{ or } \frac{5\pi}{3}$$

Or $300^\circ = \dfrac{300\pi}{180} = \dfrac{30\pi}{18} = \dfrac{\cancel{(2)}(3)\cancel{(3)}... }{}$

$$300^\circ = \frac{300\pi}{180} = \frac{30\pi}{18} = \frac{\cancel{(2)}\cancel{(3)}(5)\pi}{\cancel{(2)}(3)\cancel{(3)}}$$

$$= \frac{5\pi}{3}$$

30	2
15	3
5	5
1	

18	2
9	3
3	3
1	

③ 9°

$$9^\circ = \frac{9\pi}{180} = \frac{9 \div 3}{180 \div 3}\pi = \frac{3}{60}\pi$$

$$= \frac{3 \div 3}{60 \div 3}\pi$$

$$= \frac{1}{20}\pi$$

$$\text{or } = \frac{\pi}{20}$$

or

$$9^\circ = \frac{9\pi}{180} = \frac{\cancel{(3)}\cancel{(3)}\pi}{(2)(2)\cancel{(3)}\cancel{(3)}(5)}$$

$$= \frac{\pi}{(2)(2)(5)}$$

$$= \frac{\pi}{20}$$

9	3
3	3
1	

180	2
90	2
45	3
15	3
5	5
1	

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(4)

-315°

$$-315^\circ = -\frac{315\pi}{180} = -\frac{315 \div 5}{180 \div 5}\pi$$

$$= -\frac{63}{36}\pi$$

$$= -\frac{63 \div 3}{36 \div 3}\pi$$

$$= -\frac{21}{12}\pi$$

$$= -\frac{21 \div 3}{12 \div 3}\pi$$

$$= -\frac{7}{4}\pi \text{ or } -\frac{7\pi}{4}$$

$$\begin{array}{r} 63 \\ 5\,\overline{)315} \\ -30 \\ \hline 15 \\ 15 \\ \hline 00 \end{array} \qquad \begin{array}{r} 36 \\ 5\,\overline{)180} \\ 15 \\ \hline 30 \\ 30 \\ \hline 00 \end{array} \qquad \begin{array}{r} 21 \\ 3\,\overline{)63} \\ -6 \\ \hline 03 \\ -3 \\ \hline 00 \end{array} \qquad \begin{array}{r} 12 \\ 3\,\overline{)36} \\ -3 \\ \hline 06 \\ -6 \\ \hline 00 \end{array}$$

(5) $900^\circ = \frac{900\pi}{180}$

$$= \frac{90\pi}{18}$$

$$= \frac{90 \div 2}{18 \div 2}\pi$$

$$= \frac{45 \div 3}{9 \div 3}\pi$$

$$= \frac{15 \div 3}{3 \div 3}\pi$$

$$= \frac{5\pi}{1} = 5\pi$$

(6) $36^\circ = \frac{36\pi}{180} = \frac{36 \div 2}{180 \div 2}\pi$

$$= \frac{18 \div 2}{90 \div 2}\pi$$

$$= \frac{\cancel{18}\,9 \div 3}{45 \div 3}\pi = \frac{3 \div 3}{15 \div 3}\pi$$

$$= \frac{1}{5}\pi = \frac{\pi}{5}$$

(1) $4\pi = 4 \times 180° = 720°$

(3)
$$\begin{array}{r} 180 \\ \times \quad 4 \\ \hline 720 \end{array}$$

$\frac{7\pi}{2} = 7 \times 90° = 630°$

$\frac{8\pi}{3} = 8 \times 60 = 480°$

$\frac{11\pi}{4} = 11 \times 45 = 495°$

$$\begin{array}{r} 45 \\ 11 \\ \hline 45 \\ +\,45 \\ \hline 495 \end{array}$$

$\frac{5\pi}{6} = 5 \times 30 = 150°$

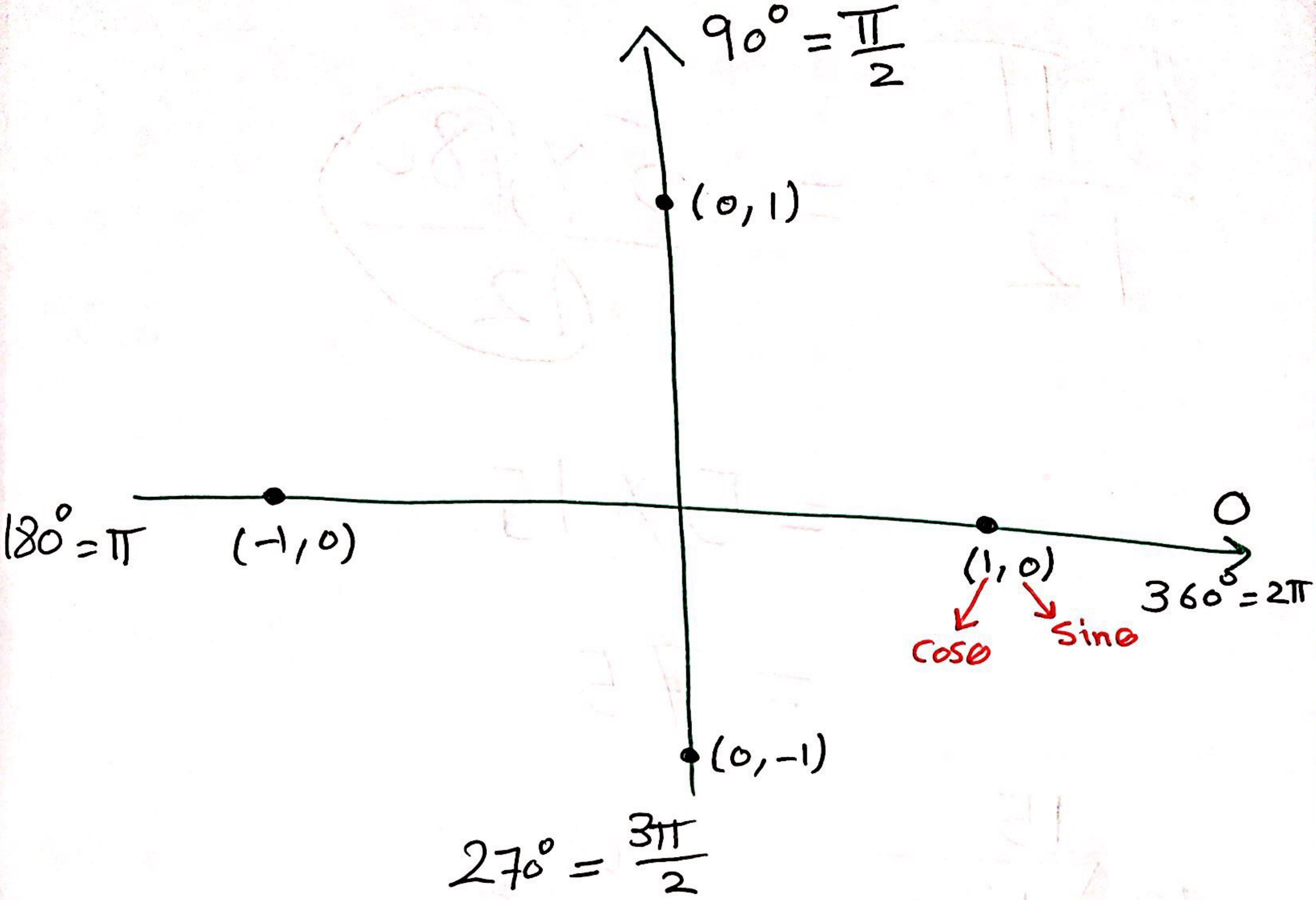
$$\frac{5\pi}{12} = \frac{5 \times 180}{12}$$

$$= 5 \times 15$$

$$= 75$$

$$\begin{array}{r} 15 \\ 12\overline{)180} \\ -\ 12 \\ \hline 60 \\ -\ 60 \\ \hline 00 \end{array}$$

$$\begin{array}{r} ^{2}15 \\ \times\ \ 5 \\ \hline 75 \end{array}$$



$\sin(2\pi) = 0$

$\sin(0) = 0$

$\cos(90^\circ) = 0$

$\cos\left(\frac{3\pi}{2}\right) = 0$

$\sin(270^\circ) = -1$

$\sin\left(\frac{\pi}{2}\right) = 1$

$\cos(\pi) = -1$

$\cos(360) = 1$

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$$30^\circ = \frac{\pi}{6} \qquad 45^\circ = \frac{\pi}{4} \qquad 60^\circ = \frac{\pi}{3}$$

$$\sin\theta \qquad \frac{1}{2} \qquad \frac{\sqrt{2}}{2} \qquad \frac{\sqrt{3}}{2}$$

$$\cos\theta \qquad \frac{\sqrt{3}}{2} \qquad \frac{\sqrt{2}}{2} \qquad \frac{1}{2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{\cot\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

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