PHYS 101

Ch. 8

Momentum, Impulse and Collisions

Chapter 8

Chapter Eight

Momentum, Impulse and Collisions

- ***Momentum and Impulse***
- ***Conservation of Momentum***
- ***Momentum Conservation and Collisions***
- ***Elastic Collisions***
- ***Center of Mass***

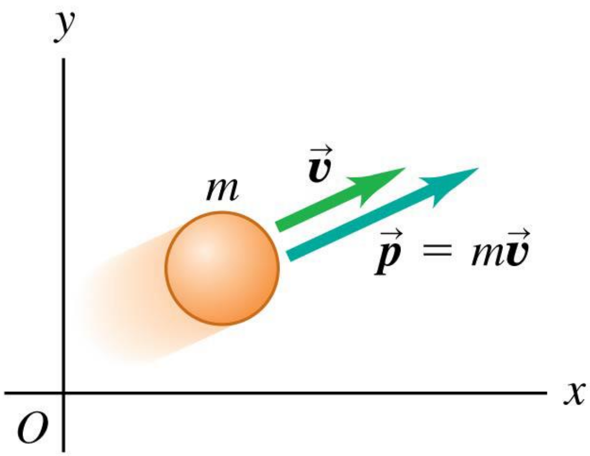
Momentum and Impulse

Newton's Second Law in Terms of Momentum

Momentum

The momentum of a particle is the product of its mass and its velocity and given by

$$\vec{p} = m\vec{v}$$



which m is the mass of the particle and $\vec{v}$ is its velocity.

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Momentum and Impulse

Newton's second law can be written in terms of momentum

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt}$$

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}.$$

The Impulse-Momentum Theorem

The change in momentum of a particle during a time interval is equal to the impulse of the net force acting on the particle during that interval

$$\Delta\vec{p} = \vec{J}$$ (linear momentum–impulse theorem).

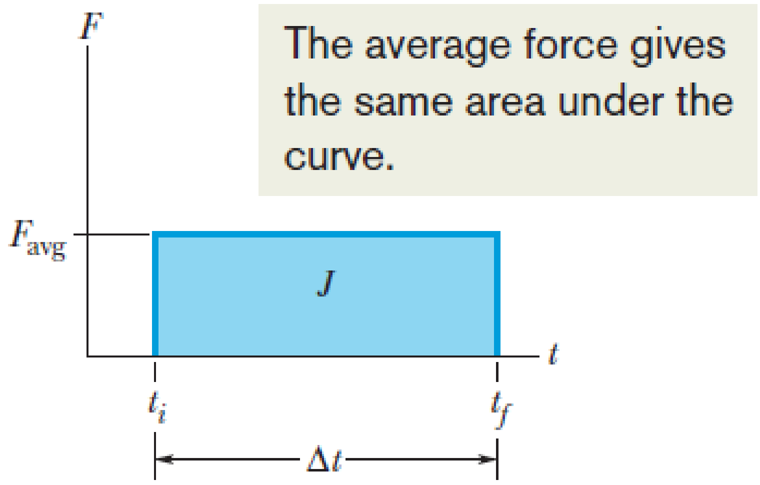
Momentum and Impulse

Impulse

- The impulse of a force is the product of the force and the time interval during which it acts.

- It can be given by

$$J = F_{avg}\,\Delta t.$$



Momentum and Impulse

Example 1:

A force was applied on an object of mass 50 kg which changed its speed from 13 m/s to 45 m/s. The momentum for each speed is:

Solution:

(D)

(A) 730 kg.m/s & 4450 kg.m/s
(B) 850 kg.m/s & 3250 kg.m/s
(C) 450 kg.m/s & 6550 kg.m/s
(D) 650 kg.m/s & 2250 kg.m/s

Momentum and Impulse

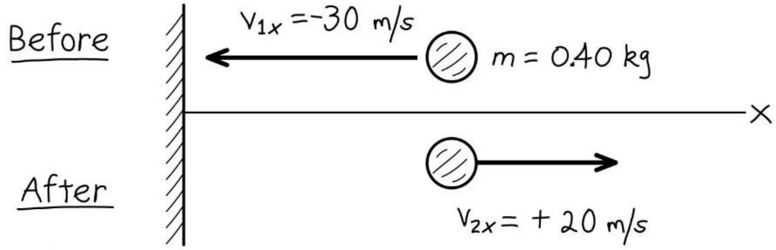
Example 2:

A 0.40 kg ball is initially moving to the left at 30 m/s. After hitting the wall, the ball is moving to the right at 20 m/s. The impulse of the net force on the ball during its collision with the wall is:

Solution:

(A)

(A) 20 kg.m/s to the right
(B) 20 kg.m/s to the left
(C) 4.0 kg.m/s to the right
(D) 4.0 kg.m/s to the left



Momentum and Impulse

Example 3:

During a collision with a wall, the velocity of a 0.200-kg ball changes from 20 m/s toward the wall to 12.0 m/s away from the wall. If the time when the ball was in contact with the wall is 60.0 ms, the magnitude of the average force applied to the ball is

Solution:

(B)

(A) 40.0 N
(B) 107 N
(C) 16.7 N
(D) 26.7 N

Momentum and Impulse

Example 4:

A time-varying horizontal force $F(t) = 4.5t^4 + 8.75t^2$ acts for 0.500 s on a 12.25-kg object. The impulse imparted to the object by this force is:

Solution:

(B)

(A) 8.75 N.s horizontally
(B) 13.9 N.s horizontally
(C) 18.2 N.s horizontally
(D) 23.4 N.s horizontally

Conservation of Momentum

CONSERVATION OF MOMENTUM

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.

$$\frac{d\vec{P}}{dt} = 0 \qquad \vec{P} = \text{constant}$$

If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.

$$\vec{P}_i = \vec{P}_f$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots$$

$$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots$$

Conservation of Momentum

Example 5:

On a smooth horizontal frictionless floor, an object slides into a spring which is attached to another stationary mass. Afterward, both objects are moving at the same speed. What is conserved during this interaction?

Solution:

(C)

(A) momentum only
(B) momentum and kinetic energy
(C) momentum and mechanical energy
(D) momentum and potential energy

Conservation of Momentum

Example 6:

A baseball is thrown vertically upward and feels no air resistance. As it is rising

Solution:

(B)

(A) momentum and mechanical energy are conserved.
(B) momentum not conserved, but mechanical energy conserved.
(C) momentum and kinetic energy are conserved.
(D) kinetic energy conserved, but momentum not conserved.

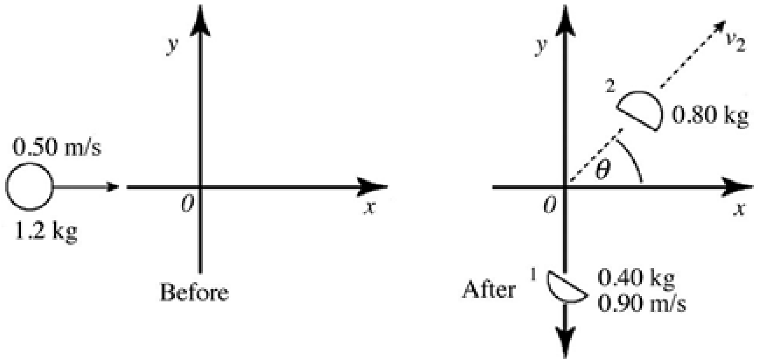
Conservation of Momentum

Example 7:

A 1.2-kg spring-activated toy bomb slides on a smooth surface along the x-axis with a speed of 0.50 m/s. At the origin 0, the bomb explodes into two fragments. Fragment 1 has a mass of 0.40 kg and a speed of 0.90 m/s along the negative y-axis. In the figure, the angle θ, made by the velocity vector of fragment 2 and the x-axis, is closest to

Solution:

(A)



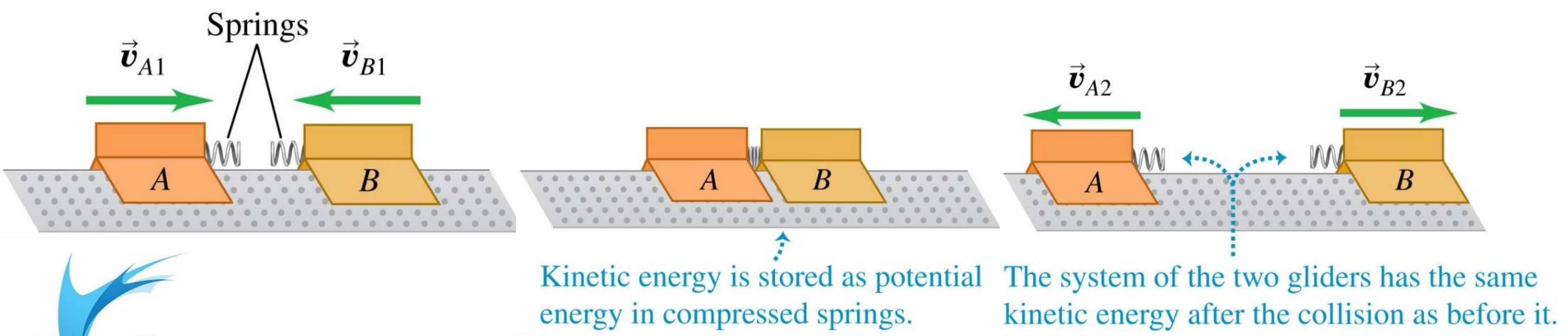
(A) 31°. (B) 37°.
(C) 53°. (D) 59°.

Momentum Cons. and Collisions

MOMENTUM CONSERVATION AND COLLISIONS

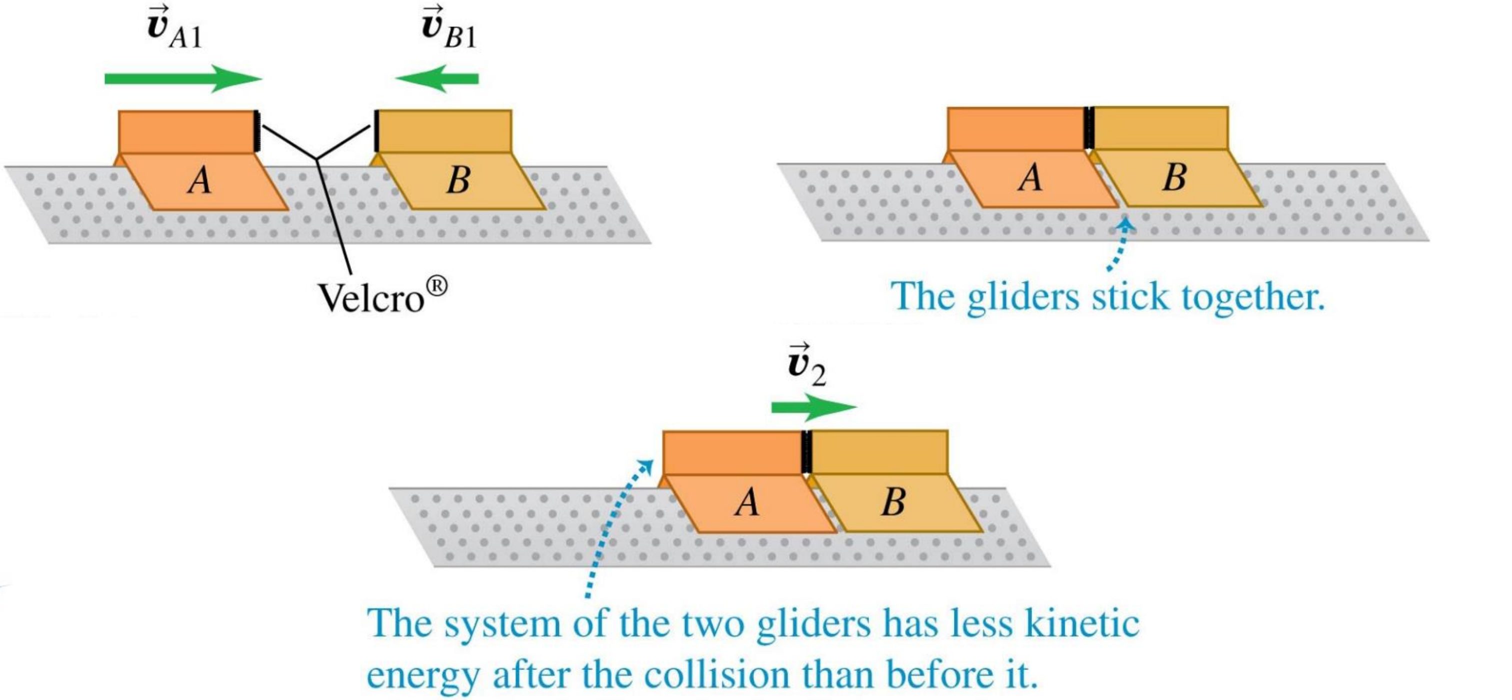
Elastic and Inelastic Collisions

In an elastic collision, the total kinetic energy of the system is the same after the collision as before.



Momentum Cons. and Collisions

In an inelastic collision, the total kinetic energy after the collision is less than before the collision.



Momentum Cons. and Collisions

- In an inelastic collision, the total kinetic energy after the collision is less than before the collision.
- A collision in which the bodies stick together is called a completely inelastic collision.
- In any collision in which the external forces can be neglected, the total momentum is conserved.
- In elastic collisions only, the total kinetic energy before equals the total kinetic energy after.

Momentum Cons. and Collisions

Completely Inelastic Collisions

Conservation of momentum gives the relationship

$$m_A\vec{v}_{A1} + m_B\vec{v}_{B1} = (m_A + m_B)\vec{v}_2$$

for $(v_{B1x} = 0)$ x-component

$$v_{2x} = \frac{m_A}{m_A + m_B}v_{A1x}$$

Momentum Cons. and Collisions

kinetic energies K_1 and K_2 before and after the collision,

$$K_1 = \tfrac{1}{2} m_A {v_{A1x}}^2$$

$$K_2 = \tfrac{1}{2}(m_A + m_B){v_{2x}}^2 = \tfrac{1}{2}(m_A + m_B)\left(\frac{m_A}{m_A + m_B}\right)^2 {v_{A1x}}^2$$

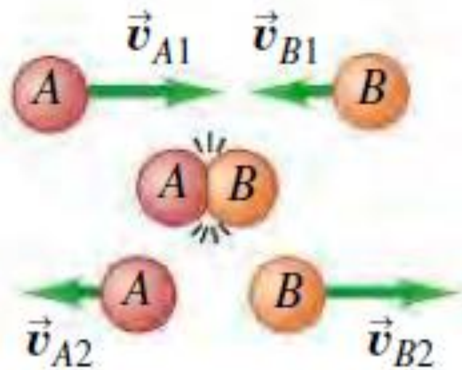
The ratio of final to initial kinetic energy is

$$\frac{K_2}{K_1} = \frac{m_A}{m_A + m_B} \qquad \text{(completely inelastic collision, } B \text{ initially at rest)}$$

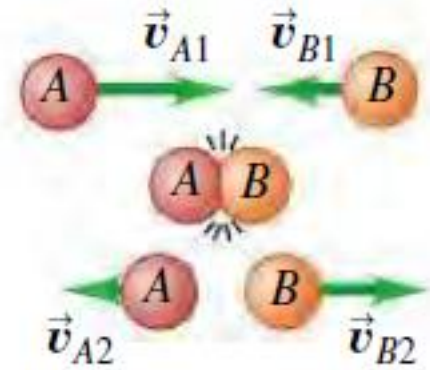
Momentum Cons. and Collisions

Classifying Collisions

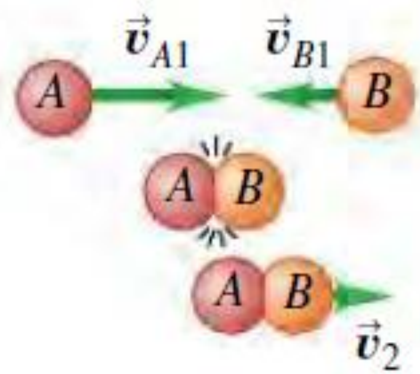
Elastic:
Kinetic energy conserved.



Inelastic:
Some kinetic energy lost.



Completely inelastic:
Bodies have same final velocity.



Example 8:

Two objects of the same mass move along the same line in opposite directions. The first mass is moving with speed v. The objects collide, stick together, and move with speed 0.100v in the direction of the velocity of the first mass before the collision. What was the speed of the second mass before the collision?

Solution:

(D)

(A) 1.20v
(B) 10.0v
(C) 0.900v
(D) 0.800v

Momentum Cons. and Collisions

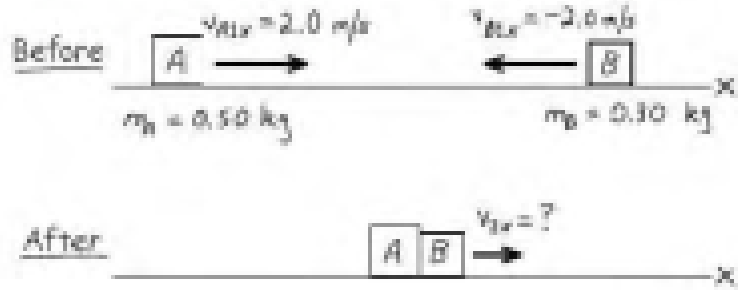
Example 9:

Two gliders with different masses move toward each other on a frictionless air track. The gliders are equipped so that they stick together when they collide. Find the common final x-velocity?

Solution:

(B)

(A) 0.2 m/s
(B) 0.5 m/s
(C) 0.9 m/s
(D) 1.2 m/s



Momentum Cons. and Collisions

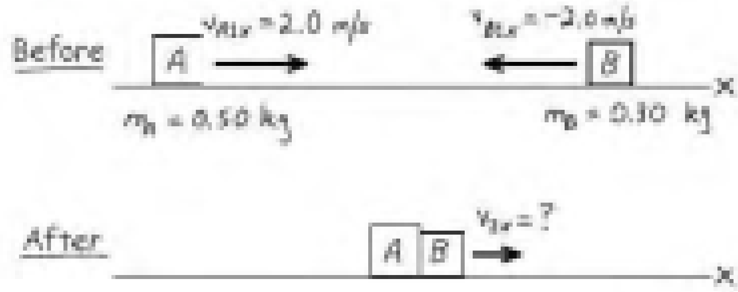
Example 10:

Referring to Example 9, compare the initial and final kinetic energies of the system.

Solution:

(D)

(A) 1/2
(B) 1/4
(C) 1/8
(D) 1/16



Elastic Collisions

ELASTIC COLLISIONS

Let's look at a *one-dimensional* elastic collision between two bodies *A* and *B*,

From conservation of kinetic energy we have

$$\frac{1}{2}m_A v_{A1x}{}^2 + \frac{1}{2}m_B v_{B1x}{}^2 = \frac{1}{2}m_A v_{A2x}{}^2 + \frac{1}{2}m_B v_{B2x}{}^2$$

and conservation of momentum gives

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

If the masses m_A and m_B and the initial velocities v_{A1x} and v_{B1x} are known, we can solve these two equations to find the two final velocities v_{A2x} and v_{B2x}.

Elastic Collisions

Elastic Collisions, One Body Initially at Rest

$$v_{B1x} = 0$$

$$\frac{1}{2}m_A v_{A1x}{}^2 = \frac{1}{2}m_A v_{A2x}{}^2 + \frac{1}{2}m_B v_{B2x}{}^2$$

$$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x}$$

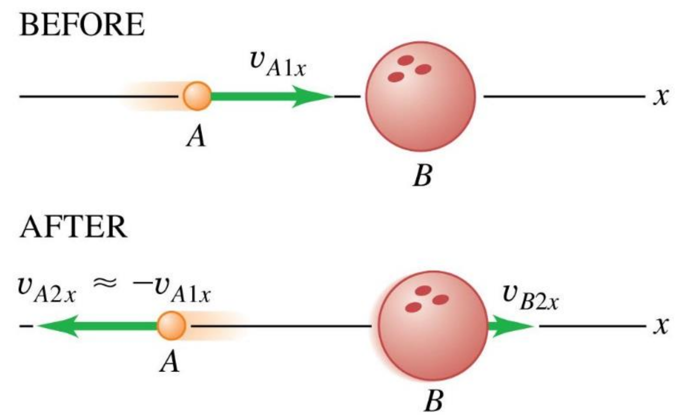
$$v_{B2x} = v_{A1x} + v_{A2x}$$

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x}$$

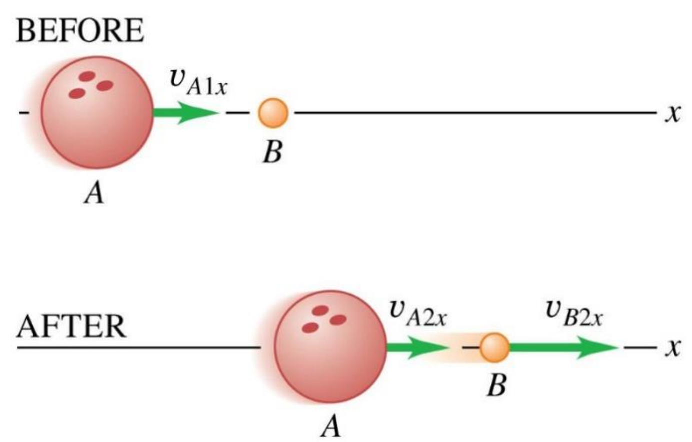
$$v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x}$$

Elastic Collisions

When B is much more massive than A, then A reverses its velocity direction, and B hardly moves.

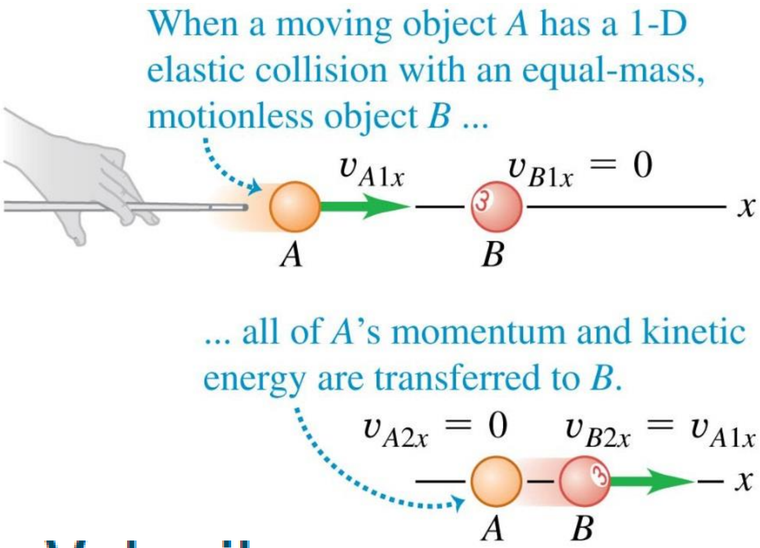


When B is much less massive than A, then A slows a little bit, while B picks up a velocity of about twice the original velocity of A.



Elastic Collisions

When A and B have similar masses, then A stops after the collision and B moves with the original speed of A.



Elastic Collisions and Relative Velocity

$$v_{A1x} = v_{B2x} - v_{A2x}$$

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$$

Elastic Collisions

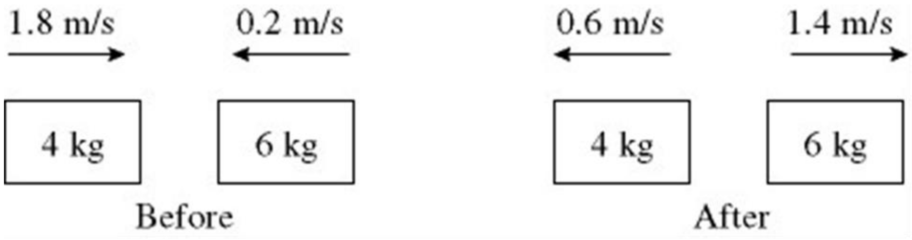
Example 11:

In the figure, determine the character of the collision. The masses of the blocks, and the velocities before and after are given. The collision is

Solution:

(A)

(A) perfectly elastic.
(B) partially inelastic.
(C) completely inelastic.
(D) not possible



Elastic Collisions

Example 12:

A 1.0 kg object travelling at 1.0 m/s collides head on with a 2.0 kg object initially at rest. Find the velocity of each object after impact if the collision is perfectly elastic.

Solution:

(C)

(A) v_{1f} = - 1/2 *m*/*s* and v_{2f} = 2/3 *m*/*s*.
(B) v_{1f} = - 1/4 *m*/*s* and v_{2f} = 2/3 *m*/*s*.
(C) v_{1f} = - 1/3 *m*/*s* and v_{2f} = 2/3 *m*/*s*.
(D) v_{1f} = - 1/3 *m*/*s* and v_{2f} = 2/5 *m*/*s*.

Centre of Mass

CENTER OF MASS

The center of mass of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.

center of mass (com)

Centre of Mass

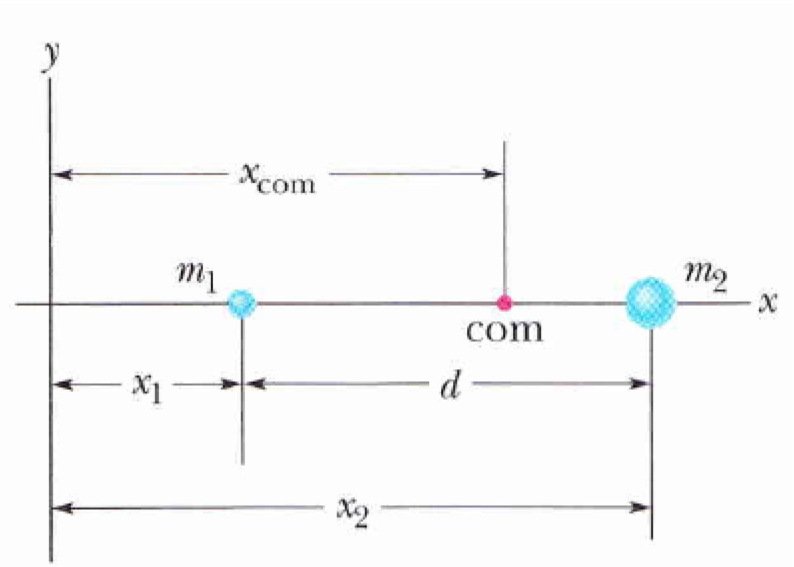
Systems of Particles

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{M},$$

$$M = m_1 + m_2$$

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{M}$$

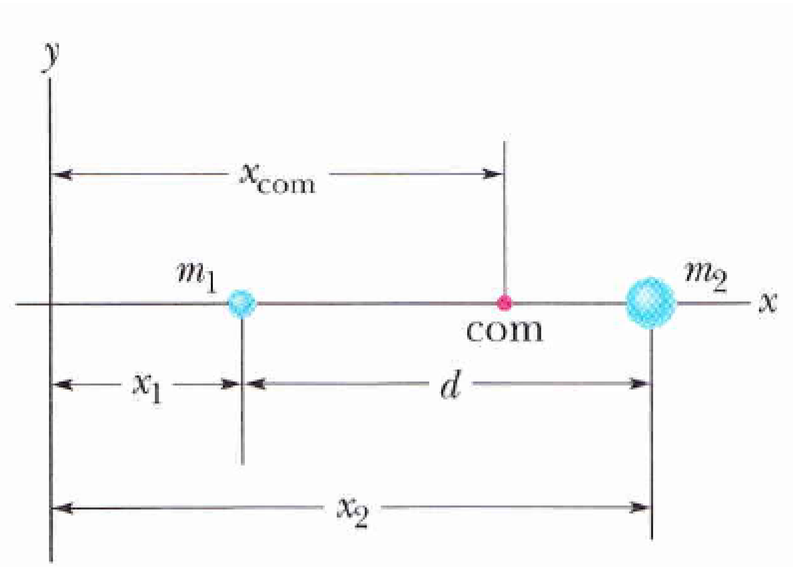
$$= \frac{1}{M}\sum_{i=1}^{n} m_i x_i .$$



$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \qquad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \qquad z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i .$$

Centre of Mass

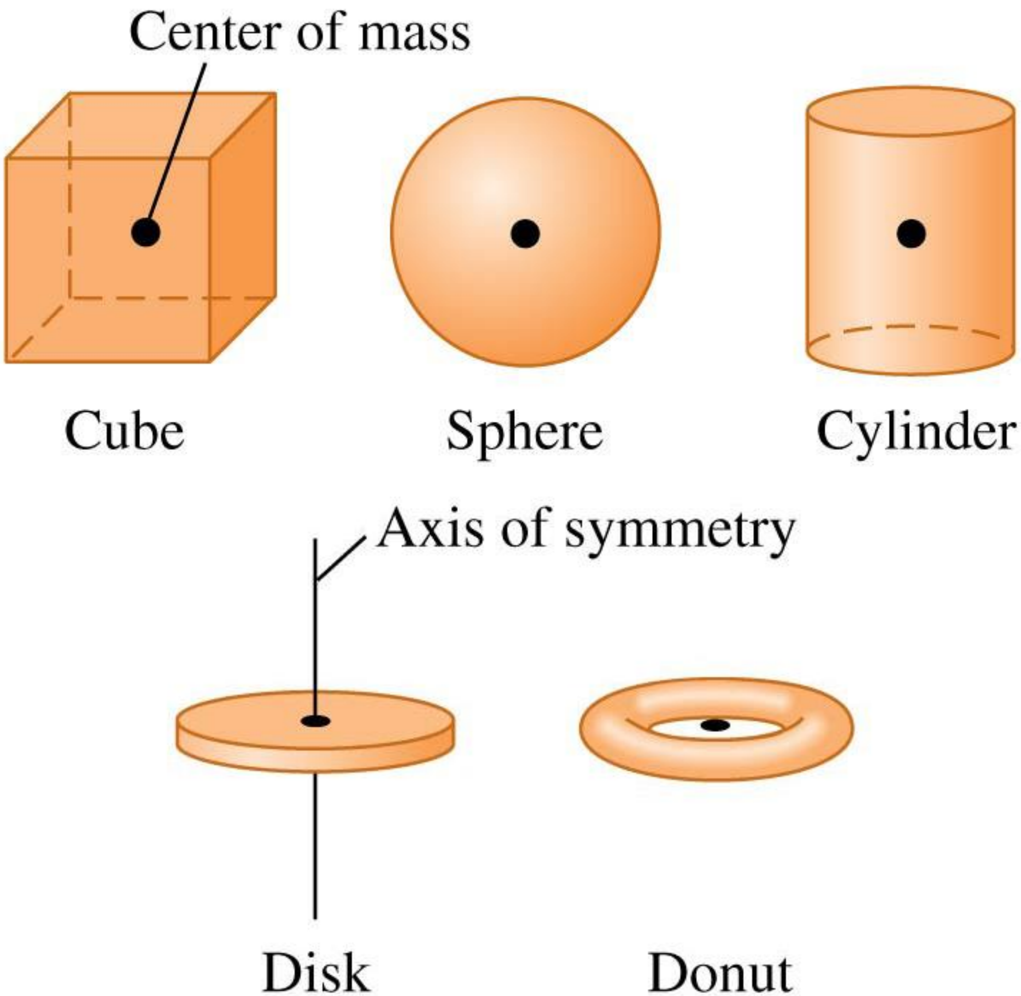
$$\vec{r}_i = x_i\hat{\mathrm{i}} + y_i\hat{\mathrm{j}} + z_i\hat{\mathrm{k}}.$$



$$\vec{r}_{com} = x_{com}\hat{\mathrm{i}} + y_{com}\hat{\mathrm{j}} + z_{com}\hat{\mathrm{k}}.$$

$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i,$$

Centre of Mass



Centre of Mass

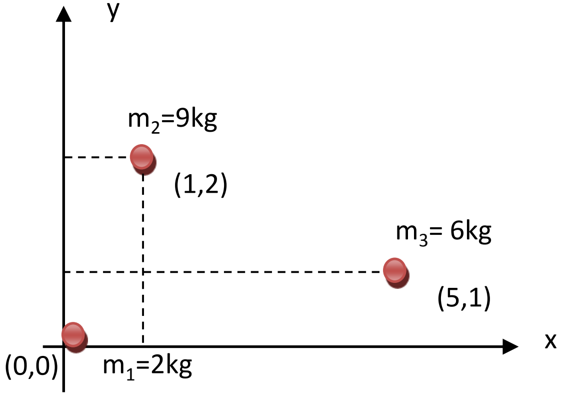
Example 13:

The center of mass of the objects shown in the Figure is:

Solution:

(B)

(A) (3.54, 6.54) m
(B) (2.29, 1.41) m
(C) (4.25, 3.45) m
(D) (5.65, 1.54) m



Centre of Mass

Motion of the Center of Mass

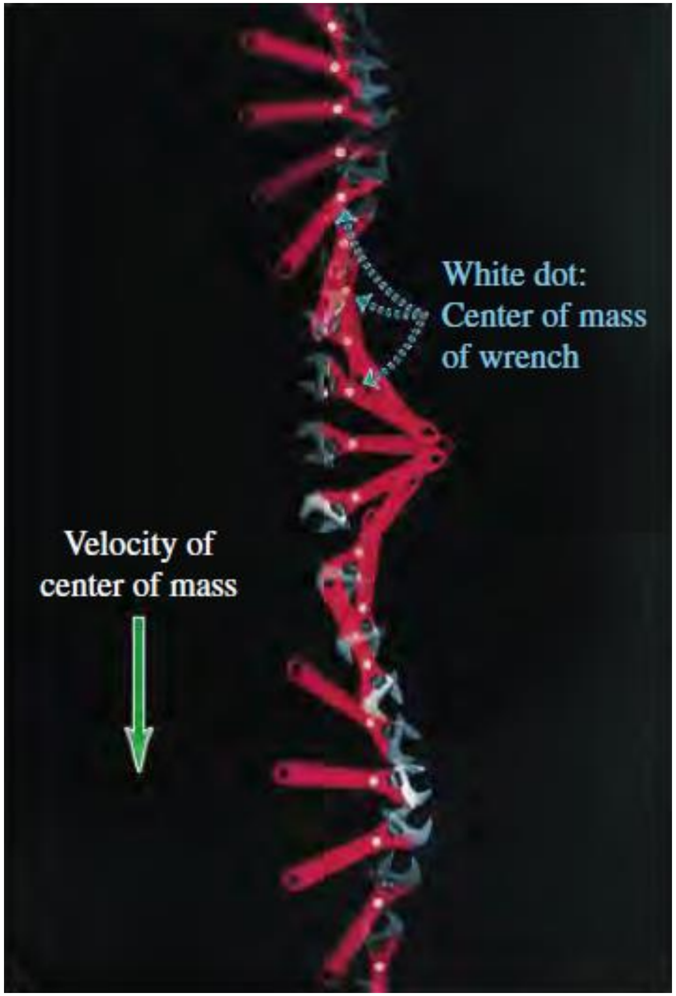
$$M\vec{r}_{com} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots + m_n\vec{r}_n,$$

Differentiating with respect to time gives

$$M\vec{v}_{com} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n.$$

Differentiating with respect to time gives

$$M\vec{a}_{com} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n.$$



Centre of Mass

External Forces and Center-of-Mass Motion

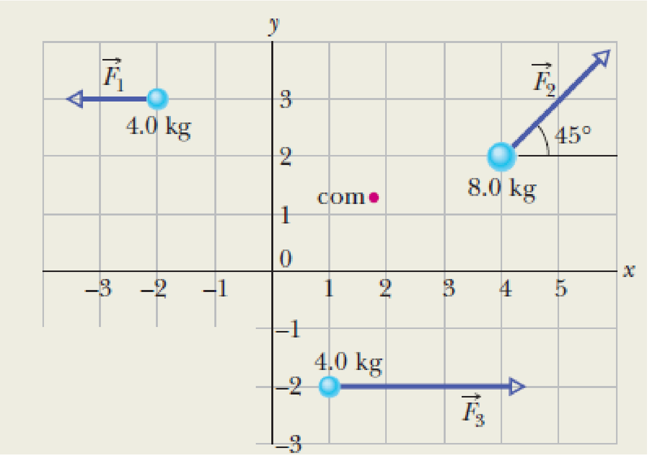
$$\vec{F}_{\text{net}} = M\vec{a}_{\text{com}} \quad \text{(system of particles).}$$

$$F_{\text{net},x} = Ma_{\text{com},x} \qquad F_{\text{net},y} = Ma_{\text{com},y} \qquad F_{\text{net},z} = Ma_{\text{com},z}.$$

Centre of Mass

Example 14:

Three particles as in Fig are initially at rest. experiences an external force. The directions are indicated, and the magnitudes are F1=6.0 N, F_2=12 N, and F_3=14 N. The acceleration of the center of mass of the system is:



Solution:

(B)

(A) 0.74 m/s^2
(B) 1.16 m/s^2
(C) 2.36 m/s^2
(D) 4.02 m/s^2