Group Number:

$$
244
$$

First Midterm, March 2013
I) Choose the correct answer:
(a) If $A, B, C$ are square matrices of the same size, then

$$
(A-B)(C-A)+(C-B)(A-C)+(C-A)^{2}
$$

equals

(b) If $A$ and $B$ are $3 \times 3$ invertible square matrices and

$$
\operatorname{det}\left[2 A^{-1}\right]=\operatorname{det}\left[A^{3}\left(B^{-1}\right)^{T}\right]=-4,
$$

then

| $\operatorname{det}(A)=4$ <br> $\operatorname{det}(B)=4$ | $\operatorname{det}(A)=-4$ <br> $\operatorname{det}(B)=4$ |
| :--- | :--- | | $\operatorname{det}(A)=2$ |
| :--- |
| $\operatorname{det}(B)=-2$ |$\quad$| $\operatorname{det}(A)=-2$ |
| :--- |
| $\operatorname{det}(B)=2$ |

(c) If $A^{3}-2 B^{T}=\left[\begin{array}{cc}18 & -2 \\ -6 & 1\end{array}\right]$ and $B=\left[\begin{array}{cc}-5 & 3 \\ 1 & 0\end{array}\right]$, then the matrix $A$ is

II) Decide if the following statements are true (T) or false (F). Justify your answer.
(a) If $A$ and $B$ are two matrices, such that $A \cdot B=O$, then either $A=O$ or $B=O$.

F
(b) If $A$ and $B$ are square matrices of the same size, such that $A+B$ is symmetric, then both $A$ and $B$ are symmetric.

F
(c) If $A$ is a $n \times n$ square matrix, $n>1$ and $k \in \mathbb{R}, k \neq 0, k \neq \pm 1$, then $\operatorname{det}[k A]=k \cdot \operatorname{det}[A]$.

III) Let $A=\left[\begin{array}{ccc}2 & 3 & -5 \\ 0 & 1 & -3 \\ 0 & 0 & 2\end{array}\right]$.
(a) Find $\operatorname{det}[A]$;
(b) Find adj $[A]$;
(c) Find $A^{-1}$;
(d) Solve the system $A \mathbf{x}=\mathbf{b}$, where $\mathbf{x}=\left[\begin{array}{l}x_{1} \\ x_{2} \\ x_{3}\end{array}\right]$ and $\mathbf{b}=\left[\begin{array}{c}-1 \\ 0 \\ 2\end{array}\right]$.
IV) Find $a$, such that the matrix $A=\left[\begin{array}{ccc}1 & a & a \\ a & 1 & a \\ a & a & 1\end{array}\right]$ is not invertible.

