324 Stat
Lecture Notes

(4) Some Discrete Probability Distributions

(Book: Chapter 5 ,pg 143)

4.1 Discrete Uniform Distribution:

Discrete Uniform is not in the book, it should be studied from the notes

- If the random variable **X** assume the values with equal probabilities, then the discrete uniform distribution is given by:

$$P(X,K)=\frac{1}{K} \quad , \quad X=x_1,x_2,\ldots,x_K$$
$$= 0 \qquad elsewhere \qquad (1)$$

Theorem:

- The mean and variance of the discrete uniform distribution $P(X, K)$ are:

$$\mu = \frac{\sum_{i=1}^{K} X_i}{K} \quad , \quad \sigma^2 = \frac{\sum_{i=1}^{K} (X - \mu)^2}{K} \qquad (2)$$

EX (1):

- When a die is tossed once, each element of the sample space $S = \{1,2,3,4,5,6\}$ occurs with probability 1/6. Therefore we have a uniform distribution with:

$$P(X,6) = \frac{1}{6}, \quad X = 1,2,3,4,5,6$$

- Find:
- 1. $P(1 \le x < 4)$
- 2. $P(x < 3)$
- 3. $P(3 < x < 6)$

Solution:

- 1. $P(1 \le x < 4)$

$$= P(x = 1) + P(x = 2) + P(x = 3) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$$

- 2. $P(x < 3) = P(x = 1) + P(x = 2) = \frac{2}{6}$

- 3. $P(3 < x < 6) = P(x = 4) + P(x = 5) = \frac{2}{6}$

EX (2):

- For example (1): Find μ *and* σ^2

Solution:

$$\mu = \frac{\sum_{i=1}^{k} X_i}{k} = \frac{1+2+3+4+5+6}{6} = 3.5$$

$$\sigma^2 = \frac{\sum_{i=1}^{k} (X_i - \mu)^2}{k}$$

$$= \frac{(1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2}{6} = \frac{35}{12}$$

The Bernoulli Process:

- Bernoulli trials are an experiment with:

1) Only two possible outcomes.
2) Labeled as success (S), failure (F).
3) Probability of success $= p$, probability of failure $= q = 1 - p$ $(p + q = 1)$.

Binomial Distribution:

- * The **Binomial** trials must possess the following properties:

1) The experiment consists of **n** repeated trials.
2) Each trial results in an outcome that may be classified as a success or a failure.
3) The probability of success denoted by **p** remains constant from trial to trial.
4) The repeated trials are independent.
5) The parameters of binomial are n,p.

Binomial Distribution:

- The probability distribution of the binomial random variable **X**, the number of successes in **n** independent trials is:

$$b(x,n,p)=\binom{n}{x}p^{X}q^{n-X} \quad , \quad x=0,1,2,....,n \qquad (5)$$

Theorem:

- The mean and the variance of the **binomial** distribution b(x, n, p) are:

$$\mu = np \qquad and \qquad \sigma^2 = npq \qquad (6)$$

EX (4):

- According to a survey by the Administrative Management society, **1/3** of U.S. companies give employees four weeks of vacation after they have been with the company for **15** years. Find the probability that among **6** companies surveyed at random, the number that gives employees **4** weeks of vacation after **15** years of employment is:

a) anywhere from **2** to **5**;

b) fewer than **3**;

c) at most 1;

d) at least **5**;

e) greater than **2**;

f) calculate μ *and* σ^2

Solution:

$$n = 6, p = 1/3, q = 2/3$$

$$(a) P(2 \le x \le 5) = P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5)$$

$$= \binom{6}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4 + \binom{6}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3 + \binom{6}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + \binom{6}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^1$$

$$= \frac{240}{729} + \frac{160}{729} + \frac{60}{729} + \frac{12}{729} = \frac{472}{729} = 0.647$$

$$(b) P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2)$$

$$= \binom{6}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^6 + \binom{6}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^5 + \binom{6}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4$$

$$= \frac{64}{729} + \frac{192}{729} + \frac{240}{729} = \frac{496}{729} = 0.68$$

$$(c)\, P(x \le 1) = P(x = 0) + p(x = 1)$$

$$= \binom{6}{0}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^{6} + \binom{6}{1}\left(\frac{1}{3}\right)^{1}\left(\frac{2}{3}\right)^{5}$$

$$= \frac{64}{729} + \frac{192}{729} = \frac{256}{729} = 0.351$$

$$(d)\, P(x \ge 5) = P(x = 5) + P(x = 6)$$

$$= \binom{6}{5}\left(\frac{1}{3}\right)^{5}\left(\frac{2}{3}\right)^{1} + \binom{6}{6}\left(\frac{1}{3}\right)^{6}\left(\frac{2}{3}\right)^{0}$$

$$= \frac{12}{729} + \frac{1}{729} = \frac{13}{729} = 0.018$$

$$(e)\, P(x>2) = 1 - P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$$

$$= 1 - \left\{ \binom{6}{0} \left(\frac{1}{3}\right)^{0} \left(\frac{2}{3}\right)^{6} + \binom{6}{1} \left(\frac{1}{3}\right)^{1} \left(\frac{2}{3}\right)^{5} + \binom{6}{2} \left(\frac{1}{3}\right)^{2} \left(\frac{2}{3}\right)^{4} \right\}$$

$$= 1 - \left\{ \frac{64}{729} + \frac{192}{729} + \frac{240}{729} \right\} = 1 - \frac{496}{729} = \frac{233}{729} = 0.319$$

$$(f\ )\, \mu = np = (6)\left(\frac{1}{3}\right) = \frac{6}{3} = 2$$

$$\sigma^{2} = npq = (6)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{12}{9} = 1.333$$

EX (5):

- The probability that a person suffering from headache will obtain relief with a particular drug is **0.9**. **Three** randomly selected sufferers from headache are given the drug. Find the probability that the number obtaining relief will be:

a) exactly zero;
b) at most one;
c) more than one;
d) two or fewer;
e) Calculate μ and σ^2

See Ex 5.2 pg 146

Solution:

$$n = 3, p = 0.9, q = 0.1$$

$$(a)\, P(X = 0) = \binom{3}{0}(0.9)^0(0.1)^3 = 0.001$$

$$(b)\, P(x \le 1) = P(x = 0) + P(x = 1)$$

$$= \binom{3}{0}(0.9)^0(0.1)^3 + \binom{3}{1}(0.9)^1(0.1)^2$$

$$= 0.001 + 0.027 = 0.028$$

$$(c)\,P(X>1)=1-p(x\le 1)=1-[P(x=0)+P(x=1)]$$

$$=1-[\binom{3}{0}(0.9)^0(0.1)^3+\binom{3}{1}(0.9)^1(0.1)^2$$

$$=1-(0.001+0.027)=1-0.028=0.972$$

$$(d)\,P(x\le 2)=P(x=0)+P(x=1)+P(x=2)$$

$$=\binom{3}{0}(0.9)^0(0.1)^3+\binom{3}{1}(0.9)^1(0.1)^2+\binom{3}{2}(0.9)^2(0.1)^1$$

$$=0.001+0.027+0.243=0.271$$