

CHAPTER 4 : EXPONENTIAL & LOGARITHMIC FUNCTIONS

الدوال الأسية و اللوغاريتمية

SECTION (4 – 1) : EXPONENTIAL FUNCTIONS

Exercises 1 – 4 , Which of the following function is exponential ?

1. $f(x) = 2^x$

solution

f is exponential to the base $a = 2$

2. $f(x) = x^3$

solution:

ليست دالة اسية لكنها دالة كثيرة حدود

f is not exponential

3. $f(x) = \sqrt{x}$

solution:

f is not exponential

4. $f(x) = (\sqrt{7})^x$

solution

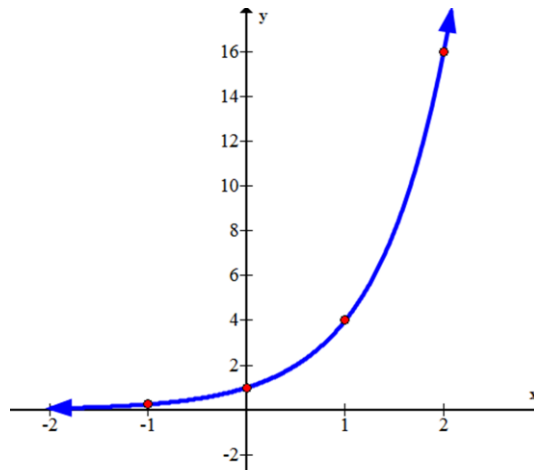
f is exponential to the base $a = \sqrt{7}$

Exercises 5 – 9 , Sketch the graph of each of the following exponential functions

5. $f(x) = 4^x$

solution:

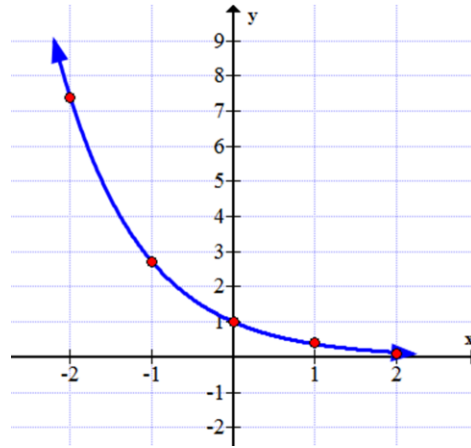
x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16



6. $f(x) = e^{-x}$

solution:

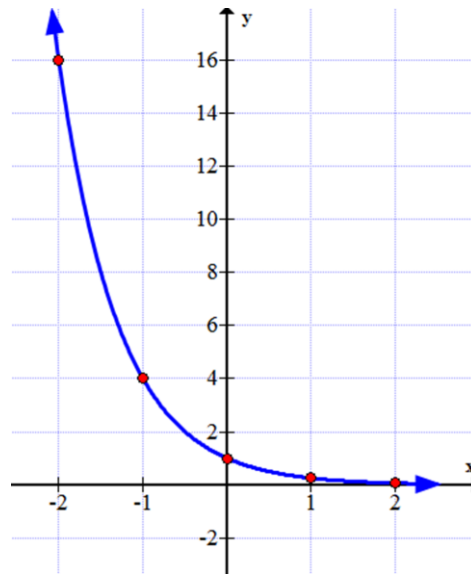
x	-2	-1	0	1	2
y	7.4	2.7	1	0.4	0.1



7. $f(x) = 4^{-x}$

solution:

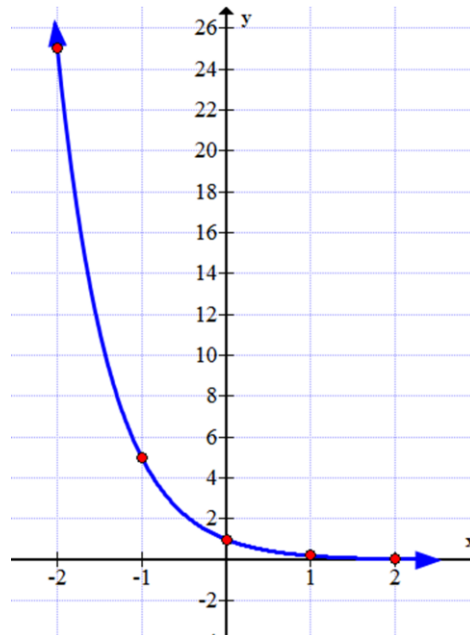
x	-2	-1	0	1	2
y	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$



8. $f(x) = 5^{-x}$

solution:

x	-2	-1	0	1	2
y	25	5	1	$\frac{1}{5}$	$\frac{1}{25}$



Exercise 10 – 25 , Sketch the graph of the each of the following function using translation method

13. $f(x) = 4^x - 1$

solution:

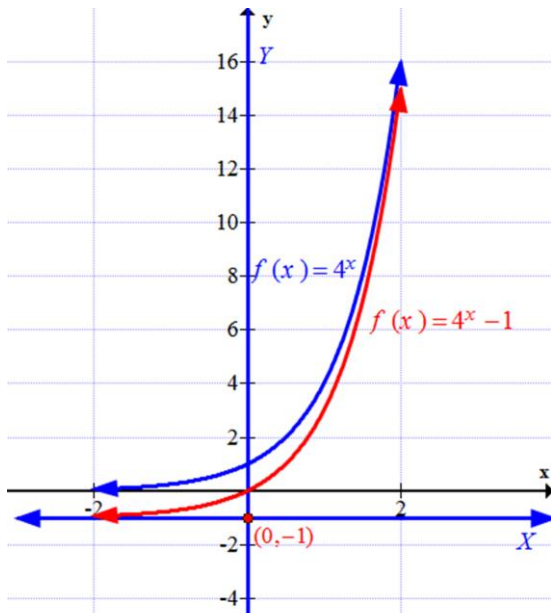
* First : We graph $f(x) = 4^x$

x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

* Second : $y = 4^x - 1$, $y + 1 = 4^x \Rightarrow$ we take $h = 0$, $k = -1$

Let $X = x$, $Y = y + 1 \Rightarrow Y = 4^X$

* The equation $Y = 4^X$ is exponential in XY – coordinate system
its origin is $(h, k) = (0, -1)$



خطوات الحل:

أولاً: نرسم الدالة الأم $y = 4^x$ (Exercise 5)

ثانياً: نضع المعادلة بالصيغة $y + 1 = 4^x$ لإيجاد قيمة h و k مركز الاحداثيات الجديدة $(0, -1)$

ثالثاً: نرسم المعادلة المطلوبة تطابق المعادلة $y = 4^x$ لكن بانسحاب لأسفل بمقدار 1

14. $f(x) = 4^{x+2} - 3$

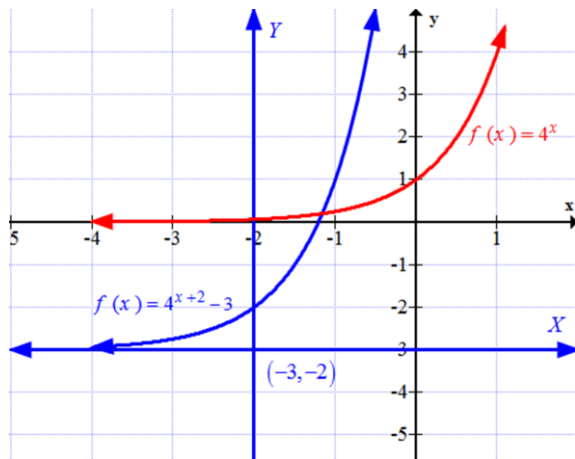
solution:

* First : We graph $f(x) = 4^x$

* Second : $y = 4^{x+2} - 3$, $y + 3 = 4^{x+2} \Rightarrow$ we take $h = -2$, $k = -3$

Let $X = x + 2$, $Y = y + 3 \Rightarrow Y = 4^X$

* The equation $Y = 4^X$ is exponential in XY - coordinate system , its origin is $(h, k) = (-2, -3)$



خطوات الحل:
 أولاً: نرسم الدالة الأم $y = 4^x$ (Exercise 5)
 ثانياً: نضع المعادلة بالصيغة $y + 3 = 4^{x+2}$ لإيجاد قيمة h و k مركز الإحداثيات الجديدة $(-2, -3)$
 ثالثاً: نرسم المعادلة المطلوبة تطابق المعادلة $y = 4^x$ لكن بانسحاب لأسفل بمقدار 3- وانسحاب لليساار بمقدار 2-

17. $f(x) = e^{x+2}$

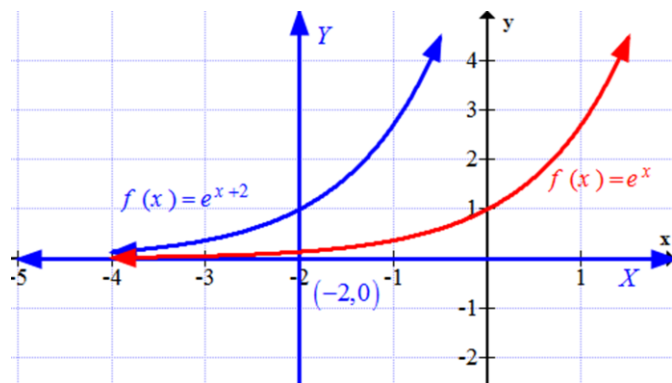
solution:

* First : We graph $f(x) = e^x$

* Second : $y = e^{x+2} \Rightarrow$ we take $h = -2$, $k = 0$

Let $X = x + 2$, $Y = y \Rightarrow Y = e^X$

* The equation $Y = e^X$ is exponential in XY - coordinate system , its origin is $(h, k) = (-2, 0)$



20. $f(x) = e^{x+2} - 3$

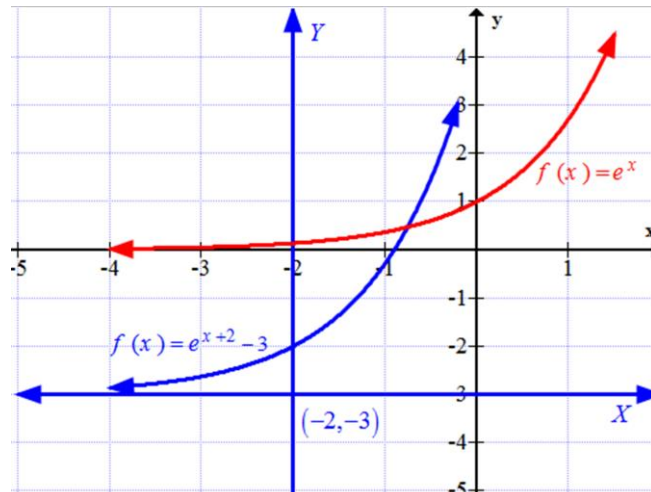
solution:

* First : We graph $f(x) = e^x$

* Second : $y = e^{x+2}$, $y + 3 = e^{x+2} \Rightarrow$ we take $h = -2$, $k = -3$

Let $X = x + 2$, $Y = y + 3 \Rightarrow Y = e^X$

* The equation $Y = e^X$ is exponential in XY - coordinate system , its origin is $(h, k) = (-2, -3)$



22. $f(x) = 5^x + 2$

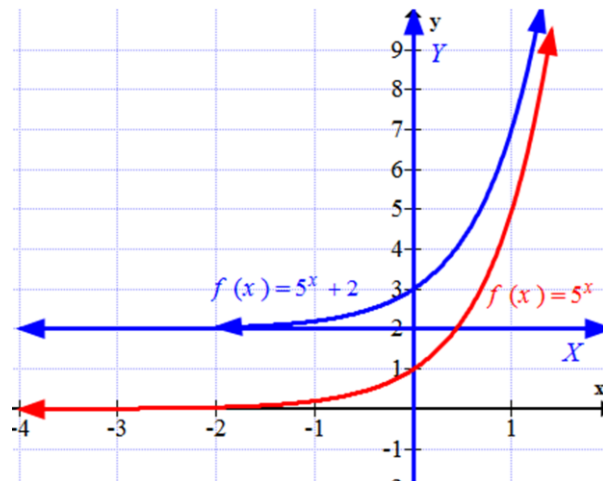
solution:

* First : We graph $f(x) = 5^x$

* Second : $y = 5^x + 2$, $y - 2 = 5^x \Rightarrow$ we take $h = 0$, $k = 2$

Let $X = x$, $Y = y - 2 \Rightarrow Y = 5^X$

* The equation $Y = 5^X$ is exponential in XY - coordinate system its origin is $(h, k) = (0, 2)$



25. $f(x) = 4^{-x+2} - 3$

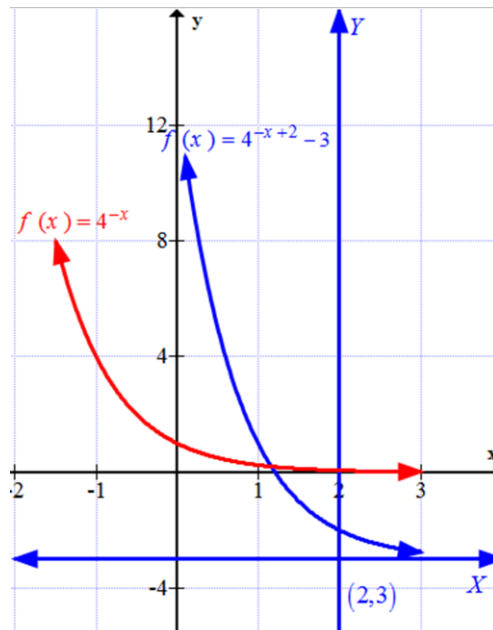
solution:

* First : We graph $f(x) = 4^{-x}$

* Second : $y = 4^{-x+2} - 3$, $y + 3 = 4^{-(x-2)} \Rightarrow$ we take $h = 2$, $k = 3$

Let $X = -(x - 2)$, $Y = y + 3 \Rightarrow Y = 4^{-X}$

* The equation $Y = 4^{-X}$ is exponential in XY - coordinate system , its origin is $(h, k) = (2, 3)$



Exercises 26 – 29 , Find the domain for the following functions

26. $f(x) = \frac{x + 2}{xe^x - e^x}$

مجال الدالة الكسرية : جميع الأعداد الحقيقية ما عدا أصفار المقام

solution:

$$xe^x - e^x = 0 \Rightarrow e^x(x - 1) = 0$$

$$x - 1 = 0 \quad \text{or} \quad e^x = 0 \text{ has no solution (because } e^x > 0 \text{)}$$

$$x = 1$$

The domain of f is $\mathbb{R} - \{0\}$

$$27. f(x) = \frac{5x+3}{(x^2-9)e^{2x+1}}$$

solution:

$$(x^2-9)e^{2x+1} = 0$$

$$x^2-9=0 \quad \text{or} \quad e^{2x+1}=0 \quad (\text{has no solution})$$

$$x = \pm 3$$

The domain of f is $\mathbb{R} - \{-3, 3\}$

$$28. f(x) = \frac{2x+3}{e^{-x+3}-e^{2x-1}}$$

solution:

$$e^{-x+3}-e^{2x-1} = 0$$

$$e^{-x+3} = e^{2x-1}$$

$$\text{we get} \quad -x+3 = 2x-1$$

$$-3x = -4$$

$$x = \frac{4}{3}$$

The domain of f is $\mathbb{R} - \left\{\frac{4}{3}\right\}$

$$\text{Ex : } f(x) = \frac{2x+3}{e^{-x+3}+3}$$

solution:

$$e^{-x+3}+3 = 0$$

$$e^{-x+3} = -3 \quad \text{has no solution (because exponential function } e^{-x+3} > 0 \text{)}$$

Domain of f is $(-\infty, \infty)$

الدالة الأسية دائما قيمتها أكبر من الصفر