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Ch. 3 - Part 1

- Measures of Central Tendency for Ungrouped data.
- Measures of Variation for ungrouped data.

STAT.110

جمال السعدي
رياضيات - إحصاء



Ch.3 Part 1

جمال السدي

مقاييس النزعة المركزية
Measures of central tendency

Mean, Median and Mode.

- statistic: is measures for the sample.
- parameter: is measures for the population.

Individual data:

$$\frac{\text{مجموع القيم}}{\text{عددها}} = \text{الوسط الحسابي} *$$

The mean (average)

Is the sum of values, divided by total number of values.

$$\bar{x} = \frac{\sum x}{n}$$

For the sample

$$\mu = \frac{\sum x}{N}$$

For the population

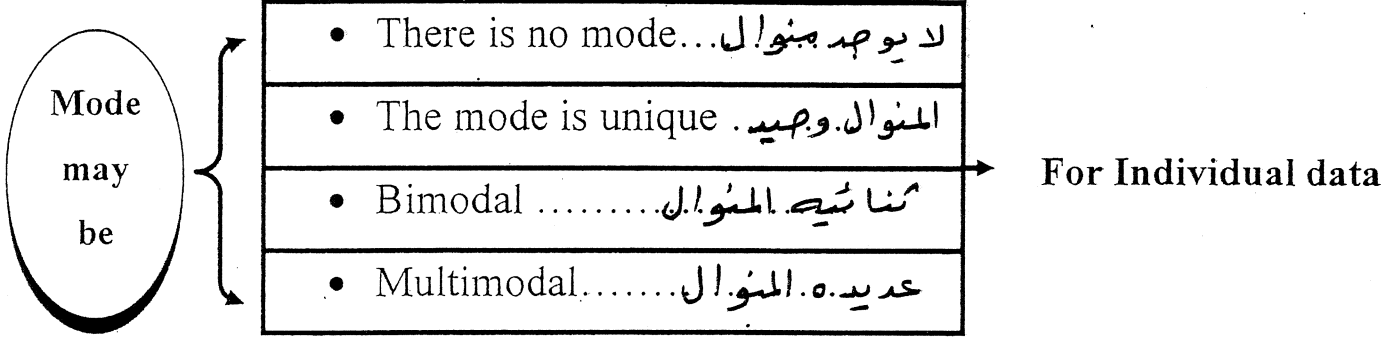
The median: (MD) $\left\{ \begin{array}{l} X_{\frac{n+1}{2}} \dots\dots\dots n \text{ is odd} \\ [X_{\frac{n}{2}} + (X_{\frac{n}{2} + 1})] \div 2 \dots\dots\dots n \text{ is even} \end{array} \right.$

- الوسيط: هو نقطة منتصف البيانات بعد ترتيبها تصاعدياً أو تنازلياً.
- Arrange the data from low to high and select the middle point.

The Mode: → (the most typical case)

❖ Is the value that occurs most often in the data.
المَنوال : هو القيمة الأكثر تكراراً من البيانات

❖ The modal class is the class with the largest frequency.
الفئة المَوالية : هي الفئة المناظرة لأعلى تكرار.



Note

- The mean is unique
- The median is unique

• **The Midrange:** تخمين تقريبي
is a rough estimate of the middle.

$$\text{Midrange} = \frac{H + L}{2}$$

• **The range** = Highest value - lowest value
→ المدى = H - L

• **The variance:** التباين
is the average of the squares of the distance each value is from the mean.

• **The standard deviation:** الانحراف المعياري
is the square root of the variance.

* الانحراف المعياري هو الجذر التربيعي للتباين.

Important Formulas

	Sample	Population
• Mean	$\bar{X} = \frac{\sum X}{n}$	$\mu = \frac{\sum X}{N}$
• Variance	$S^2 = \frac{\sum X^2}{n} - \bar{X}^2$ <p>OR $S^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1}$</p> <p style="text-align: center;">الأكثر استخداماً</p>	$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$
• Standard deviation	$S = \sqrt{S^2}$	$\sigma = \sqrt{\sigma^2}$
• Coefficient of variation	$C.Var = \frac{S}{\bar{X}} .100\%$	$C.Var = \frac{\sigma}{\mu} .100\%$

Note

Sample: \bar{X} , S^2 , S

Population: μ , σ^2 , σ

Note

Measures of variation:

Range, variance and standard deviation

Note

When: C. var for sample 1 > C.var for sample 2

Then: sample 1 is more than variable sample 2

Example:

Twelve major earthquakes had Richter magnitudes shown here

7.0 , 6.2 , 7.7 , 8.0 , 6.4 , 6.2

7.2 , 5.4 , 6.4 , 6.5 , 7.2 , 5.4

Find ① Mean

② Median

③ Mode

④ Midrange

⑤ Range

⑥ Variance

⑦ Stander deviation

⑧ Coefficient of variation

Solution

Arrange the data:

5.4 , 5.4 , 6.2 , 6.2 , 6.4 , 6.4 , 6.5 , 7 , 7.2 , 7.2 , 7.7 , 8

$$\textcircled{1} \text{ Mean : } \bar{X} = \frac{\sum x}{n} = \frac{79.6}{12} = 6.63$$

$$\textcircled{2} \text{ Median: } \bar{X} = \frac{X_6 + X_7}{2} = \frac{6.4 + 6.5}{2} = 6.45$$

$$\textcircled{3} \text{ Mode: } D = 5.4 , 6.2 , 6.4 , 7.2$$

$$\textcircled{4} \text{ Midrange } = \frac{H + L}{2} = \frac{8 + 5.4}{2} = 6.7$$

$$\textcircled{5} \text{ Range } = H - L = 8 - 5.4 = 2.6$$

$$\textcircled{6} \text{ Variance: } S^2 = \frac{\sum x^2}{n} - \bar{X}^2 = \frac{535.34}{12} - (6.63)^2 \cong 0.65$$

$$\textcircled{7} \text{ Stander deviation: } S = \sqrt{S^2} = \sqrt{0.65} \cong 0.81$$

⑧ Coefficient of variation

$$\begin{aligned} \text{C. var} &= \frac{S}{\bar{X}} \times 100 \% \\ &= \frac{0.81}{6.63} \times 100 \% \\ &= 12.22 \% \end{aligned}$$

• لاحظ الأسماء المميزة من كل جملة •

Exercises:

* حدد المقياس الأنسب لكل موقف

Describe which measure of Central tendency: Mean, Median, and Mode was probably used in each situation.

(a) One half of the factory workers make more than \$ 5.37 per hour, and one half make less than \$ 5.37 per hour.

▪ One half = Median (Median)

(b) The average number of children per family is 1.8.

▪ Average + عدد = Mean (Mean)

(c) Most people prefer red convertibles over any other color.

▪ Most = Mode (Mode)

(d) The average person cuts the lawn once a week.

▪ Average + once a week = Mode (Mode)
Twice

(e) the most common fear today is fear of speaking in public.

▪ Most = Mode (Mode)

(f) The average age of college professor is 42.3 years.

▪ Average + عدد = Mean (Mean)

- The weighed mean is used when the values in a data set are not equally represented.

Example:

In a survey of third – grade students, this distribution was obtained for the number of "best friends" each had.

Number of students	Number of best friends
8	1
6	2
5	3
3	0

Find the average number of best friends for the class use the weighted mean.

Solution

$$\bar{X}_w = \frac{\sum w \times x}{\sum w} = \frac{8 \times 1 + 6 \times 2 + 5 \times 3 + 3 \times 0}{8 + 6 + 5 + 3} = \frac{35}{22} = 1.6$$

Example:

- The average score on an English final exam was 85,
With a standard deviation of 5.
- The average score on a history final exam was 110.
With a standard deviation of 8 which class was more
variable?

Solution

We find coefficient of variation

- For English exam:

$$C.Var = \frac{S}{\bar{X}} \cdot 100\% = \frac{5}{85} \cdot 100\% = \underline{\underline{5.8\%}}$$

- For history exam:

$$C.Var = \frac{S}{\bar{X}} \cdot 100\% = \frac{8}{110} \cdot 100\% = \underline{\underline{7.2\%}}$$

∴ History exam is more variable.

Grouping data

Important formulas

* صيغ هامة : تُعطى من ورقة الأسئلة

$$\bullet \text{ Width} = \frac{\text{Rang}}{\text{number of class}}$$

$$\bullet \text{ Range} = \text{Highest value} - \text{lowest value}$$

$$= H - L$$

$$\bullet \text{ Mean } \bar{X} = \frac{\sum X.F}{\sum F}$$

$$\bullet \text{ Variance } S^2 = \frac{\sum F.X^2 - [(\sum F.X)^2 / n]}{n-1}$$

$$\bullet \text{ Standard deviation } S = \sqrt{s^2}$$

$$\bullet \text{ Coefficient of variation } C \text{ var} = \frac{S}{\bar{X}} . 100\%$$

- Z score or standard score: * الدرجة المعيارية (مقياس)
 Tells how many standard deviation of the data above or below the mean.

$$\text{For sample } z = \frac{X - \bar{X}}{S}$$

$$\text{For population } z = \frac{x - \mu}{\sigma}$$

Example:

This distribution represents the data for weights of fifth - grade boys.

Find: ① Mean

② standard deviation

Example		Solution		
Weight	Frequency f	X	X.f	X ² .f
52.5 - 55.5	9	54	486	26244
55.5 - 58.5	12	57	684	38988
58.5 - 61.5	17	60	1020	61200
61.5 - 64.5	22	63	1386	87318
64.5 - 67.5	15	66	990	65340
	$n = \sum f = 75$		$\sum X.f = 4566$	$\sum X^2.f = 279090$

① Mean: $\bar{X} = \frac{\sum X.f}{\sum f} = \frac{4566}{75} = \boxed{60.88}$

② Variance: $S^2 = \frac{\sum x^2f - \frac{(\sum X.f)^2}{n}}{n - 1} = \frac{279090 - \frac{(4566)^2}{75}}{75 - 1} = \boxed{15.02594595}$

Standard deviation: $S = \sqrt{S^2} = \boxed{3.876}$

Example:

A final exam has a mean of 84 and a standard deviation of 4.
Find: the corresponding Z score for each raw score.

- (a) 87 (b) 79 (c) 93

Solution

$$Z = \frac{X - \mu}{\sigma}$$

$$\mu = 84$$

$$\sigma = 4$$

(a) $Z = \frac{87 - 84}{4} = 0.75$

(b) $Z = \frac{79 - 84}{4} = -1.25$

(c) $Z = \frac{93 - 84}{4} = 2.25$

Example:

A student scores 60 on a math test that has mean of 54 a standard dev. Of 3 and she scores 80 on a history test.

With a mean 78 and a standard dev. Of 2

On which test did she perform better?

Solution

$$Z = \frac{X - \bar{X}}{S}$$

$$Z_1 = \frac{60 - 54}{3} = \underline{\underline{2}}$$

$$Z_2 = \frac{80 - 78}{2} = \underline{\underline{1}}$$

$$\longrightarrow Z_1 > Z_2$$

Then:

Score on math is better relative position.