Workshop Solutions to Sections 3.4 and 3.5

1) $\lim\limits_{x\to 3^+} \dfrac{2}{x-3} =$

Solution:

If $x \to 3^+$, then $x > 3 \implies x - 3 > 0$

$$\therefore \lim_{x\to 3^+} \frac{2}{x-3} = \infty$$

2) $\lim\limits_{x\to 3^-} \dfrac{2}{x-3} =$

Solution:

If $x \to 3^-$, then $x < 3 \implies x - 3 < 0$

$$\therefore \lim_{x\to 3^-} \frac{2}{x-3} = -\infty$$

3) $\lim\limits_{x\to 3^+} \dfrac{-2}{x-3} =$

Solution:

If $x \to 3^+$, then $x > 3 \implies x - 3 > 0$

$$\therefore \lim_{x\to 3^+} \frac{-2}{x-3} = -\infty$$

4) $\lim\limits_{x\to 3^-} \dfrac{-2}{x-3} =$

Solution:

If $x \to 3^-$, then $x < 3 \implies x - 3 < 0$

$$\therefore \lim_{x\to 3^-} \frac{2}{x-3} = \infty$$

5) $\lim\limits_{x\to -3^+} \dfrac{2}{x+3} =$

Solution:

If $x \to -3^+$, then $x > -3 \implies x + 3 > 0$

$$\therefore \lim_{x\to -3^+} \frac{2}{x+3} = \infty$$

6) $\lim\limits_{x\to -3^-} \dfrac{2}{x+3} =$

Solution:

If $x \to -3^-$, then $x < -3 \implies x + 3 < 0$

$$\therefore \lim_{x\to -3^-} \frac{2}{x+3} = -\infty$$

7) $\lim\limits_{x\to 2^+} \dfrac{3x-1}{x-2} =$

Solution:

If $x \to 2^+$, then $x > 2 \implies x - 2 > 0$ and $3x - 1 > 0$

$$\therefore \lim_{x\to 2^+} \frac{3x-1}{x-2} = \infty$$

8) $\lim\limits_{x\to 2^-} \dfrac{3x-1}{x-2} =$

Solution:

If $x \to 2^-$, then $x < 2 \implies x - 2 < 0$ and $3x - 1 > 0$

$$\therefore \lim_{x\to 2^-} \frac{3x-1}{x-2} = -\infty$$

9) $\lim\limits_{x\to -2^+} \dfrac{1-x}{(x+2)^2} =$

Solution:

If $x \to -2^+$, then $x > -2$

$\implies 1 - x > 0$ and $(x+2)^2 > 0$

$$\therefore \lim_{x\to -2^+} \frac{1-x}{(x+2)^2} = \infty$$

10) $\lim\limits_{x\to -2^-} \dfrac{1-x}{(x+2)^2} =$

Solution:

If $x \to -2^-$, then $x < -2$

$\implies 1 - x > 0$ and $(x+2)^2 > 0$

$$\therefore \lim_{x\to -2^+} \frac{1-x}{(x+2)^2} = \infty$$

11) $\lim\limits_{x\to -2^+} \dfrac{x-1}{(x+2)^2} =$

Solution:

If $x \to -2^+$, then $x > -2$

$\implies x - 1 < 0$ and $(x+2)^2 > 0$

$$\therefore \lim_{x\to -2^+} \frac{x-1}{(x+2)^2} = -\infty$$

12) $\lim\limits_{x\to -2^-} \dfrac{x-1}{(x+2)^2} =$

Solution:

If $x \to -2^-$, then $x < -2$

$\implies x - 1 < 0$ and $(x+2)^2 > 0$

$$\therefore \lim_{x\to -2^-} \frac{x-1}{(x+2)^2} = -\infty$$

13) $\lim\limits_{x\to 2^+} \dfrac{6x-1}{x^2-4} =$

Solution:

If $x \to 2^+$, then $x^2 > 4$

$\implies x^2 - 4 > 0$ and $6x - 1 > 0$

$$\therefore \lim_{x\to 2^+} \frac{6x-1}{x^2-4} = \infty$$

14) $\lim\limits_{x\to 2^-} \dfrac{6x-1}{x^2-4} =$

Solution:

If $x \to 2^-$, then $x^2 < 4$

$\implies x^2 - 4 < 0$ and $6x - 1 > 0$

$$\therefore \lim_{x\to 2^+} \frac{6x-1}{x^2-4} = -\infty$$

15) $\lim_{x\to -2^+} \frac{6x-1}{x^2-4} =$

Solution:

If $x \to -2^+$, then $x^2 < 4$

$$\Longrightarrow\ x^2-4<0 \text{ and } 6x-1<0$$

$$\therefore\ \lim_{x\to 2^+} \frac{6x-1}{x^2-4} = \infty$$

16) $\lim_{x\to -2^-} \frac{6x-1}{x^2-4} =$

Solution:

If $x \to -2^-$, then $x^2 > 4$

$$\Longrightarrow\ x^2-4>0 \text{ and } 6x-1<0$$

$$\therefore\ \lim_{x\to 2^+} \frac{6x-1}{x^2-4} = -\infty$$

17) $\lim_{x\to -2^-} \frac{6x-1}{x^2-x-6} =$

Solution:

$$f(x) = \frac{6x-1}{x^2-x-6} = \frac{6x-1}{(x-3)(x+2)}$$

If $x \to -2^-$, then $x < -2$

$$\Longrightarrow\ x-3<0,\ x+2<0 \text{ and } 6x-1<0$$

$$\therefore\ \lim_{x\to -2^-} \frac{6x-1}{x^2-x-6} = -\infty$$

18) $\lim_{x\to -2^+} \frac{6x-1}{x^2-x-6} =$

Solution:

$$f(x) = \frac{6x-1}{x^2-x-6} = \frac{6x-1}{(x-3)(x+2)}$$

If $x \to -2^+$, then $x > -2$

$$\Longrightarrow\ x-3<0,\ x+2>0 \text{ and } 6x-1<0$$

$$\therefore\ \lim_{x\to -2^+} \frac{6x-1}{x^2-x-6} = \infty$$

19) $\lim_{x\to 3^+} \frac{-1}{x^2-x-6} =$

Solution:

$$f(x) = \frac{-1}{x^2-x-6} = \frac{-1}{(x-3)(x+2)}$$

If $x \to 3^+$, then $x > 3$

$$\Longrightarrow\ x-3>0,\ x+2>0 \text{ and } -1<0$$

$$\therefore\ \lim_{x\to 3^+} \frac{-1}{x^2-x-6} = -\infty$$

20) $\lim_{x\to 3^-} \frac{-1}{x^2-x-6} =$

Solution:

$$f(x) = \frac{-1}{x^2-x-6} = \frac{-1}{(x-3)(x+2)}$$

If $x \to 3^-$, then $x < 3$

$$\Longrightarrow\ x-3<0,\ x+2>0 \text{ and } -1<0$$

$$\therefore\ \lim_{x\to 3^-} \frac{-1}{x^2-x-6} = \infty$$

21) $\lim_{x\to (\pi/2)^+} \tan x =$

Solution:

$$\lim_{x\to (\pi/2)^+} \tan x = -\infty$$

22) $\lim_{x\to (\pi/2)^-} \tan x =$

Solution:

$$\lim_{x\to (\pi/2)^-} \tan x = \infty$$

23) The vertical asymptote of $f(x) = \frac{1-x}{2x+1}$ is

Solution:

We see that the function $f(x)$ is not defined when $2x+1=0 \ \Longrightarrow\ x = -\frac{1}{2}$. Since

$$\lim_{x\to \left(-\frac{1}{2}\right)^+} \frac{1-x}{2x+1} = \infty$$

and

$$\lim_{x\to \left(-\frac{1}{2}\right)^-} \frac{1-x}{2x+1} = -\infty$$

then, $x = -\frac{1}{2}$ is a vertical asymptote.

24) The vertical asymptote of $f(x) = \frac{3-x}{x^2-4}$ is

Solution:

We see that the function $f(x)$ is not defined when $x^2-4=0 \ \Longrightarrow\ x = \pm 2$. Since

$$\lim_{x\to 2^+} \frac{3-x}{x^2-4} = \infty, \qquad \lim_{x\to 2^-} \frac{3-x}{x^2-4} = -\infty$$

and

$$\lim_{x\to -2^+} \frac{3-x}{x^2-4} = -\infty, \qquad \lim_{x\to -2^-} \frac{3-x}{x^2-4} = \infty$$

then, $x = \pm 2$ are vertical asymptotes.

25) The vertical asymptote of $f(x)=\frac{3-x}{x^2-x-6}$ is

Solution:

$$f(x)=\frac{3-x}{x^2-x-6}=\frac{3-x}{(x-3)(x+2)}=\frac{-(x-3)}{(x-3)(x+2)}=-\frac{1}{x+2}$$

We see that the function $f(x)$ is not defined when $x^2-x-6=0 \implies (x-3)(x+2)=0 \implies x=3$ or $x=-2$. Since

$$\lim_{x\to3}\frac{3-x}{x^2-x-6}=\lim_{x\to3}\frac{3-x}{(x-3)(x+2)}=\lim_{x\to3}\frac{-(x-3)}{(x-3)(x+2)}=\lim_{x\to3}\frac{-1}{x+2}=-\frac{1}{5}$$

then, $x=3$ is a removable discontinuity.

$$\lim_{x\to-2^+}\frac{3-x}{x^2-x-6}=\lim_{x\to-2^+}\frac{3-x}{(x-3)(x+2)}=\infty$$

and

$$\lim_{x\to-2^-}\frac{3-x}{x^2-x-6}=\lim_{x\to-2^-}\frac{3-x}{(x-3)(x+2)}=-\infty$$

then, $x=-2$ is a vertical asymptote only.

26) The vertical asymptote of $f(x)=\frac{7-x}{x^2-5x+6}$ is

Solution:

$$f(x)=\frac{7-x}{x^2-5x+6}=\frac{7-x}{(x-3)(x-2)}$$

We see that the function $f(x)$ is not defined when $x-3=0$ or $x-2=0 \implies x=3$ or $x=2$. Since

$$\lim_{x\to3^+}\frac{7-x}{x^2-5x+6}=\lim_{x\to3^+}\frac{7-x}{(x-3)(x-2)}=\infty$$

$$\lim_{x\to3^-}\frac{7-x}{x^2-5x+6}=\lim_{x\to3^-}\frac{7-x}{(x-3)(x-2)}=-\infty$$

and

$$\lim_{x\to2^+}\frac{7-x}{x^2-5x+6}=\lim_{x\to2^+}\frac{7-x}{(x-3)(x-2)}=-\infty$$

$$\lim_{x\to2^-}\frac{7-x}{x^2-5x+6}=\lim_{x\to2^-}\frac{7-x}{(x-3)(x-2)}=\infty$$

then, $x=3$ and $x=2$ are vertical asymptotes.

27) The vertical asymptote of $f(x)=\frac{x-7}{x^2+5x+6}$ is

Solution:

$$f(x)=\frac{x-7}{x^2+5x+6}=\frac{x-7}{(x+3)(x+2)}$$

We see that the function $f(x)$ is not defined when $x+3=0$ or $x+2=0 \implies x=-3$ or $x=-2$. Since

$$\lim_{x\to-3^+}\frac{x-7}{x^2+5x+6}=\lim_{x\to-3^+}\frac{x-7}{(x+3)(x+2)}=\infty$$

$$\lim_{x\to-3^-}\frac{x-7}{x^2+5x+6}=\lim_{x\to-3^-}\frac{x-7}{(x+3)(x+2)}=-\infty$$

and

$$\lim_{x\to-2^+}\frac{x-7}{x^2+5x+6}=\lim_{x\to-2^+}\frac{x-7}{(x+3)(x+2)}=-\infty$$

$$\lim_{x\to-2^-}\frac{x-7}{x^2+5x+6}=\lim_{x\to-2^-}\frac{x-7}{(x+3)(x+2)}=\infty$$

then, $x=-3$ and $x=-2$ are vertical asymptotes.

28) The vertical asymptote of $f(x)=\frac{x-7}{x^2+3x}$ is

Solution:

$$f(x)=\frac{x-7}{x^2+3x}=\frac{x-7}{x(x+3)}$$

We see that the function $f(x)$ is not defined when $x=0$ or $x+3=0 \implies x=0$ or $x=-3$. Since

$$\lim_{x\to-3^+}\frac{x-7}{x^2+3x}=\lim_{x\to-3^+}\frac{x-7}{x(x+3)}=\infty$$

$$\lim_{x\to-3^-}\frac{x-7}{x^2+3x}=\lim_{x\to-3^-}\frac{x-7}{x(x+3)}=-\infty$$

and

$$\lim_{x\to0^+}\frac{x-7}{x^2+3x}=\lim_{x\to0^+}\frac{x-7}{x(x+3)}=-\infty$$

$$\lim_{x\to0^-}\frac{x-7}{x^2+3x}=\lim_{x\to0^-}\frac{x-7}{x(x+3)}=\infty$$

then, $x=-3$ and $x=0$ are vertical asymptotes.

29) The vertical asymptote of $f(x)=\frac{x-7}{x^2-3x}$ is

Solution:

$$f(x)=\frac{x-7}{x^2-3x}=\frac{x-7}{x(x-3)}$$

We see that the function $f(x)$ is not defined when $x=0$ or $x-3=0 \implies x=0$ or $x=3$. Since

$$\lim_{x\to3^+}\frac{x-7}{x^2-3x}=\lim_{x\to3^+}\frac{x-7}{x(x-3)}=-\infty$$

$$\lim_{x\to3^-}\frac{x-7}{x^2-3x}=\lim_{x\to3^-}\frac{x-7}{x(x-3)}=\infty$$

and

$$\lim_{x\to0^+}\frac{x-7}{x^2-3x}=\lim_{x\to0^+}\frac{x-7}{x(x-3)}=\infty$$

$$\lim_{x\to0^-}\frac{x-7}{x^2-3x}=\lim_{x\to0^-}\frac{x-7}{x(x-3)}=-\infty$$

then, $x=3$ and $x=0$ are vertical asymptotes.

30) The vertical asymptotes of $f(x)=\frac{2x^2+1}{x^2-9}$ are

Solution:

$$f(x)=\frac{2x^2+1}{x^2-9}=\frac{2x^2+1}{(x+3)(x-3)}$$

We see that the function $f(x)$ is not defined when $x^2-9=0 \implies x=\pm3$. Since

$$\lim_{x\to3^+}\frac{2x^2+1}{x^2-9}=\lim_{x\to3^+}\frac{2x^2+1}{(x+3)(x-3)}=\infty$$

$$\lim_{x\to3^-}\frac{2x^2+1}{x^2-9}=\lim_{x\to3^-}\frac{2x^2+1}{(x+3)(x-3)}=-\infty$$

and

$$\lim_{x\to-3^+}\frac{2x^2+1}{x^2-9}=\lim_{x\to-3^+}\frac{2x^2+1}{(x+3)(x-3)}=-\infty$$

$$\lim_{x\to-3^-}\frac{2x^2+1}{x^2-9}=\lim_{x\to-3^-}\frac{2x^2+1}{(x+3)(x-3)}=\infty$$

then, $x=\pm3$ are vertical asymptotes.

31) The function $f(x)=\frac{x+1}{x^2-9}$ is continuous at $a=2$ because

$1-\quad f(2)=\frac{(2)+1}{(2)^2-9}=\frac{3}{-5}=-\frac{3}{5}$

$2-\quad \lim_{x\to 3^-}\frac{x+1}{x^2-9}=\lim_{x\to 2}\frac{(2)+1}{(2)^2-9}=\frac{3}{-5}=-\frac{3}{5}$

$3-\quad \lim_{x\to 2}\frac{x+1}{x^2-9}=f(2)$

OR

We know that $D_f=\mathbb{R}\setminus\{\pm 3\}$, so $\{2\}\in D_f$.

Note: Any function is continuous on its domain.

32) The function $f(x)=\frac{x+1}{x^2-9}$ is discontinuous at $a=\pm 3$ because we know that $D_f=\mathbb{R}\setminus\{\pm 3\}$, so $\{\pm 3\}\notin D_f$.

33) The function $f(x)=\frac{x+1}{x^2-9}$ is discontinuous at ± 3 because $\{\pm 3\}\notin D_f$.

34) The function $f(x)=\frac{x+1}{x^2-9}$ is continuous on its domain which is $D_f=\mathbb{R}\setminus\{\pm 3\}$.

35) The function $f(x)=\begin{cases}\frac{\sin 3x}{x}, & x\neq 0\\ 3, & x=0\end{cases}$ is continuous at $a=0$ because

1- $f(0)=3$

2- $\lim_{x\to 0}\frac{\sin 3x}{x}=3\lim_{x\to 0}\frac{\sin 3x}{3x}=3(1)=3$

3- $\lim_{x\to 0}f(x)=f(0)$

36) The function $f(x)=\begin{cases}\frac{\sin 3x}{x}, & x\neq 0\\ 5, & x=0\end{cases}$ is discontinuous at $a=0$ because

1- $f(0)=5$

2- $\lim_{x\to 0}\frac{\sin 3x}{x}=3\lim_{x\to 0}\frac{\sin 3x}{3x}=3(1)=3$

3- $\lim_{x\to 0}f(x)\neq f(0)$

37) The function $f(x)=\begin{cases}\frac{2x^2-3x+1}{x-1}, & x\neq 1\\ 7, & x=1\end{cases}$ is discontinuous at $a=1$ because

1- $f(1)=7$

2- $\lim_{x\to 1}\frac{2x^2-3x+1}{x-1}=\lim_{x\to 1}\frac{(2x-1)(x-1)}{x-1}=\lim_{x\to 1}(2x-1)=1$

3- $\lim_{x\to 1}f(x)\neq f(1)$

38) The function $f(x)=\begin{cases}\frac{2x^2-3x+1}{x-1}, & x\neq 1\\ 1, & x=1\end{cases}$ is continuous at $a=1$ because

1- $f(1)=1$

2- $\lim_{x\to 1}\frac{2x^2-3x+1}{x-1}=\lim_{x\to 1}\frac{(2x-1)(x-1)}{x-1}=\lim_{x\to 1}(2x-1)=1$

3- $\lim_{x\to 1}f(x)=f(1)$

39) The function $f(x)=\frac{x^2-x-2}{x-2}$ is discontinuous at $a=2$ because $\{2\}\notin D_f$.

40) The function $f(x)=\begin{cases}2x+3, & x>2\\ 3x+1, & x\leq 2\end{cases}$ is continuous at $a=2$ because

1- $f(2)=3(2)+1=7$

2- $\lim_{x\to 2^+}(2x+3)=2(2)+3=7$

$\lim_{x\to 2^-}(3x+1)=3(2)+1=7$

$\therefore\ \lim_{x\to 2}f(x)=7$

3- $\lim_{x\to 2}f(x)=f(2)$

41) The function $f(x)=\frac{x+3}{\sqrt{x^2-4}}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2-4>0\ \Rightarrow\ x^2>4\ \Rightarrow\ \sqrt{x^2}>\sqrt{4}$$
$$\Rightarrow\ |x|>2\ \Leftrightarrow\ x>2\ \text{ or }\ x<-2$$

Hence,

$D_f=(-\infty,-2)\cup(2,\infty)$.

42) The function $f(x)=\sqrt{x^2-4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2-4\geq 0\ \Rightarrow\ x^2\geq 4\ \Rightarrow\ \sqrt{x^2}\geq\sqrt{4}$$
$$\Rightarrow\ |x|\geq 2\ \Leftrightarrow\ x\geq 2\ \text{ or }\ x\leq -2$$

Hence,

$D_f=(-\infty,-2]\cup[2,\infty)$.

43) The function $f(x)=\sqrt{4-x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$4-x^2\geq 0\ \Rightarrow\ -x^2\geq -4\ \Rightarrow\ x^2\leq 4$$
$$\Rightarrow\sqrt{x^2}\leq\sqrt{4}\ \Rightarrow\ |x|\leq 2\ \Leftrightarrow\ -2\leq x\leq 2$$

Hence,

$D_f=[-2,2]$.

44) The function $f(x)=\frac{x+3}{\sqrt{4-x^2}}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$4-x^2>0\ \Rightarrow\ -x^2>-4\ \Rightarrow\ x^2<4$$
$$\Rightarrow\ \sqrt{x^2}<\sqrt{4}\ \Rightarrow\ |x|<2\ \Leftrightarrow\ -2<x<2$$

Hence,

$D_f=(-2,2)$.

45) The function $f(x)=\frac{x+1}{x^2-4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2-4\neq 0\ \Rightarrow\ x^2\neq 4\ \Rightarrow\ x\neq\pm 2$$

Hence,

$D_f=\mathbb{R}\setminus\{\pm 2\}$

$=(-\infty,-2)\cup(-2,2)\cup(2,\infty)=\{x\in\mathbb{R}: x\neq\pm 2\}$.

46) The function $f(x)=\log_2(x+2)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x+2>0 \implies x>-2$$

Hence,

$D_f=(-2,\infty)$.

47) The function $f(x)=\sqrt{x-1}+\sqrt{x+4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x-1\geq 0 \text{ and } x+4\geq 0 \implies x\geq 1 \ \cap \ x\geq -4$$

Hence,

$D_f=[1,\infty)$.

48) The function $f(x)=5^x$ is continuous on its domain .

Hence,

$D_f=\mathbb{R}=(-\infty,\infty)$.

49) The function $f(x)=e^x$ is continuous on its domain .

Hence,

$D_f=\mathbb{R}=(-\infty,\infty)$.

50) The function $f(x)=\sin^{-1}(3x-5)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$-1\leq 3x-5\leq 1 \iff 4\leq 3x\leq 6 \iff \frac{4}{3}\leq x\leq 2 .$$

Hence,

$D_f=\left[\frac{4}{3},2\right]$.

51) The function $f(x)=\cos^{-1}(3x+5)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$-1\leq 3x+5\leq 1 \iff -6\leq 3x\leq -4 \iff -2\leq x\leq -\frac{4}{3} .$$

Hence,

$D_f=\left[-2,-\frac{4}{3}\right]$.

52) The number c that makes $f(x)=\begin{cases} c+x\ , & x>2 \\ 2x-c, & x\leq 2\end{cases}$ is continuous at $x=2$ is

Solution:

$\lim_{x\to 2} f(x)$ exists if

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^-} f(x)$$
$$\lim_{x\to 2^+}(c+x)=\lim_{x\to 2^-}(2x-c)$$
$$c+2=4-c$$
$$c+c=4-2$$
$$2c=2$$
$$c=1$$

53) The number c that makes $f(x)=\begin{cases} cx^2-2x+1, & x\leq -1 \\ 3x+2 \quad , & x>-1\end{cases}$ is continuous at -1 is

Solution:

$\lim_{x\to -1} f(x)$ exists if

$$\lim_{x\to -1^+} f(x)=\lim_{x\to -1^-} f(x)$$
$$\lim_{x\to -1^+}(3x+2\ )=\lim_{x\to -1^-}(cx^2-2x+1)$$
$$3(-1)+2=c(-1)^2-2(-1)+1$$
$$-1=c+3$$
$$c=-1-3$$
$$c=-4$$

54) The number c that makes

$f(x)=\begin{cases} \frac{\sin cx}{x}+2x-1, & x<0 \\ 3x+4 \quad , & x\geq 0\end{cases}$ is continuous at 0 is

Solution:

$\lim_{x\to 0} f(x)$ exists if

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^-} f(x)$$
$$\lim_{x\to 0^+}(3x+4)=\lim_{x\to 0^-}\left(\frac{\sin cx}{x}+2x-1\right)$$
$$3(0)+4=c(1)+2(0)-1$$
$$4=c-1$$
$$c=4+1$$
$$c=5$$

55) The value c that makes $f(x)=\begin{cases} cx^2+2x, & x\leq 2 \\ x^3-cx\ , & x>2\end{cases}$ is continuous at 2 is

Solution:

$\lim_{x\to 2} f(x)$ exists if

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^-} f(x)$$
$$\lim_{x\to 2^+}(x^3-cx\ )=\lim_{x\to 2^-}(cx^2+2x)$$
$$(2)^3-c(2)=c(2)^2+2(2)$$
$$8-2c=4c+4$$
$$-2c-4c=4-8$$
$$-6c=-4$$
$$c=\frac{-4}{-6}$$
$$c=\frac{2}{3}$$

56) The number c that makes $f(x)=\begin{cases} c^2x^2-1, & x\leq 3 \\ x+5 \quad , & x>3\end{cases}$ is continuous at 3 is

Solution:

$\lim_{x\to 3} f(x)$ exists if

$$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^-} f(x)$$
$$\lim_{x\to 3^+}(x+5)=\lim_{x\to 3^-}(c^2x^2-1)$$
$$(3)+5=c^2(3)^2-1$$
$$8=9c^2-1$$
$$9c^2=8+1$$
$$c^2=1$$
$$c=\pm 1$$

57) The number c that makes $f(x)=\begin{cases} x-2\ , & x>5 \\ cx-3, & x\leq 5\end{cases}$ is continuous at 5 is

Solution:

$\lim_{x\to 5} f(x)$ exists if

$$\lim_{x\to 5^+} f(x)=\lim_{x\to 5^-} f(x)$$
$$\lim_{x\to 5^+}(x-2)=\lim_{x\to 5^-}(cx-3)$$
$$(5)-2=c(5)-3$$
$$3=5c-3$$
$$5c=3+3$$
$$5c=6$$
$$c=\frac{6}{5}$$

58) The number c that makes $f(x) = \begin{cases} x+3\,, & x > -1 \\ 2x - c, & x \le -1 \end{cases}$ is continuous at -1 is Solution: $\lim_{x \to -1} f(x)$ exists if $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$ $\lim_{x \to -1^+} (x+3) = \lim_{x \to -1^-} (2x - c)$ $(-1) + 3 = 2(-1) - c$ $2 = -2 - c$ $c = -2 - 2$ $c = -4$	