0.6 Operations on Functions

Example 1 : Let $F(x) = \sqrt[4]{x+1}$ and $G(x) = \sqrt{9-x^2}$, with respective natural domains [-1,∞) and $[-3,3]$.

Find formulas for: $F + G$, $F - G$, $F \cdot G$, F/G and F^5 , and give their natural domains

Solution

Operation	Domain
$(F+G)(x) = \sqrt[4]{x+1} + \sqrt{9-x^2}$	$[-1, 3]$
$(F-G)(x) = \sqrt[4]{x+1} - \sqrt{9-x^2}$	$[-1, 3]$
$(F \cdot G)(x) = \sqrt[4]{x+1} \cdot \sqrt{9-x^2}$	$[-1, 3]$
$(F/G)(x) = \sqrt[4]{x+1}/\sqrt{9-x^2}$	$[-1, 3)$
$(F^5)(x) = \left(\sqrt[4]{x+1}\right)^5 = (x+1)^{5/4}$	$[-1, \infty)$

- **Composition of Functions (تركيب الدوال) :**

$$(f \ o \ g)(x) = f(\ g(x)\ )$$

Example 2 : Let $f(x) = \dfrac{6x}{x^2-9}$ and $g(x) = \sqrt{3x}$, then find :

(a) $(f \ o \ g)(x)$ and give its domain.
(b) $(f \ o \ g)(12)$

Solution

(a) $(f \ o \ g)(x) = f\big(g(x)\big) = f\big(\sqrt{3x}\big) = \dfrac{6\sqrt{3x}}{(\sqrt{3x})^2-9} = \dfrac{6\sqrt{3x}}{3x-9}$

its domain = $R - \{3\}$

(b) $(f \ o \ g)(12) = f\big(g(12)\big) = f\big(\sqrt{36}\big) = f(6) = \dfrac{6(6)}{6^2-9} = \dfrac{36}{36-9} = \dfrac{36}{27} = \dfrac{4}{3}$

<u>Example 3 :</u> Write the function $p(x) = (x + 2)^5 as\ a\ composite\ function$

gof

<div align="center">**Solution**</div>

$$p(x) = gof = g\big(f(x)\big)$$
$$\therefore g(x) = x^5 \quad and \ f(x) = x + 2$$