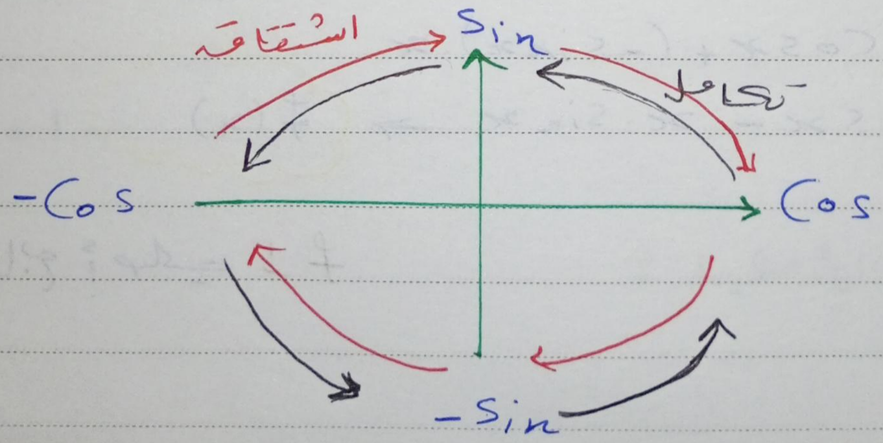
التكامل

مراجعة قواعد الاشتقاق

f	f'
a	0
ax	a
x^n	$n x^{n-1}$
$\sqrt{g}$	$\frac{g'}{2\sqrt{g}}$
$f \cdot g$	$f' g + g' f$
$\frac{f}{g}$	$\frac{f' g - g' f}{g^2}$
$\sin$	$\cos$
$\cos$	$-\sin$
$\tan x$	$1+\tan^2 x$ أو $\frac{1}{\cos^2 x}$



* تابع أصلي

اشتقاق

$F(x)$

$f(x)$

$f'(x)$

تكامل

* أمثلة

$F(x) = x^2$

$f(x) = 2x$

أثبت أن F تابع أصلي لـ f

$F'(x) = 2x \rightarrow f(x)$

F تابع أصلي لـ f

$F(x) = \tan x - x$

$f(x) = \tan^2 x$

$F'(x) = 1 + \tan^2 x - 1 \Rightarrow \tan^2 x \rightarrow f(x)$

F تابع أصلي لـ f

$F(x) = x \cdot \cos x$

$f(x) = \cos x - x \cdot \sin x$

$F'(x) = (1) \cdot \cos x + (-\sin x) \cdot x$

$F'(x) = \cos x - x \cdot \sin x \rightarrow f(x)$

F تابع أصلي لـ f

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$F(x) = (x + 1)^2$

$f(x) = \frac{2(x^4 - 1)}{x^3}$

$F'(x) = 2\left(x + \frac{1}{x}\right) \cdot \left(1 - \frac{1}{x^2}\right)$

$= 2\left[x - \cancel{\frac{1}{x}} + \cancel{\frac{1}{x}} - \frac{1}{x^3}\right]$

$= 2\left(\frac{x^4 - 1}{x^3}\right) \Longrightarrow f(x)$

F تابع أصلي لـ f

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$F(x) = \frac{-1}{x(x-1)}$

$f(x) = \frac{2x - 1}{x^2(x-1)^2}$

$F'(x) = \frac{2x - 1}{x^2(x-1)^2} \Longrightarrow f(x)$

F تابع أصلي لـ f

$F(x) = x \ln x - x$

$f(x) = \ln x$

$F'(x) = \ln x + \frac{1}{x} x - 1$

$= \ln x + 1 - 1$

$= \ln x$

F تابع أصلي لـ f

$f(x) = \frac{1}{x \ln x}$

$F(x) = \ln(\ln x)$

$F'(x) = \frac{\frac{1}{x}}{\ln x} \Rightarrow \frac{1}{x \ln x} \Rightarrow f(x)$

F تابع أصلي لـ f

$f(x) = \sqrt{e^x}$

$F(x) = 2\sqrt{e^x}$

$F'(x) = \frac{\cancel{2} \cdot e^x}{\cancel{2}\sqrt{e^x}} \Rightarrow \sqrt{e^x} \Rightarrow f(x)$

F تابع أصلي لـ f

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*ملاحظة:

$f(x) = e^g$

$f(x) = g' \cdot e^g \Rightarrow e^x = e^x$

تمرين رقم 2 ص 22

$f(x) = \frac{x^2+3x-1}{x-1}$ $\qquad$ $g(x) = \frac{x^2+7x-5}{x-1}$

هل F و G تابعان أصليان لـ f

$F'(x) = \frac{(2x+3)(x-1) - x^2 - 3x + 1}{(x-1)^2}$

$= \frac{2x^2 - 2x \cancel{+3x} - 3 - x^2 \cancel{-3x} + 1}{(x-1)^2}$

$F'(x) = \frac{x^2 - 2x - 2}{(x-1)^2}$

F تابع أصلي لـ f

$$G'(x) = \frac{x^2 - 2x - 2}{(x-1)^2}$$

F تابع أصلي لـ f

⇐ فإن F و G تابعان أصليان لـ f

$$\int a \, d(x) = ax + C$$

f	F
3	$3x$
5	$5x$
$\sqrt{5}$	$\sqrt{5}\,x$
$\frac{1}{2}$	$\frac{1}{2}x$
$\frac{1}{3}$	$\frac{1}{3}x$
-1	$-x$
$\ln 2$	$\ln 2x$

$$\int x^n \, d(x) = \frac{x^{n+1}}{n+1} + C$$

f	F
x^2	$\frac{x^3}{3}$
x	$\frac{x^2}{2}$
x^3	$\frac{x^4}{4}$
$\sqrt{x} = x^{\frac{1}{2}}$	$\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$

$x^{-\frac{1}{2}}$ | $\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}$

$\dfrac{1}{\sqrt{x}} = \dfrac{1}{x^{\frac{1}{2}}}$ | $\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}$

* مثال:

$f(x) = \sqrt{\sqrt{\sqrt{x}}}$ $\qquad \left(\left((x)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$F(x) = \dfrac{x^{\frac{1}{8}+1}}{\frac{1}{8}+1} = \dfrac{x^{\frac{9}{8}}}{\frac{9}{8}} + C$

$f(x) = \dfrac{1}{\sqrt{\sqrt{x}}}$ $\qquad \dfrac{1}{x^{\frac{1}{4}}}$

$F(x) = \dfrac{x^{-\frac{1}{4}+1}}{-\frac{1}{4}+1} = \dfrac{x^{\frac{3}{4}}}{\frac{3}{4}} + C$

تكامل مجموع = مجموع التكاملات

* $\int (F+g) \cdot d(x)$

$= \int F \cdot d(x) + \int g \cdot d(x)$

* أمثلة

1- $\int (x+2) \cdot d(x)$

$= \dfrac{x^2}{2} + 2x + C$

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$$2- \int (x^3 + x^2) \cdot d(x)$$
$$= \frac{x^4}{4} + \frac{x^3}{3} + C$$

$$\int g^n \cdot g' \cdot d(x) \Rightarrow \frac{g^{n+1}}{n+1} + C$$

$$1- \int (4x+1)^2 \cdot 4 \cdot d(x)$$
$$= \frac{(4x+1)^3}{3} + C$$

$$2- \int (x^2+1)^2 \cdot 2x \cdot d(x)$$
$$= \frac{(x^2+1)^3}{3} + C$$

$$3- \int (e^x+1)^2 \cdot e^x \cdot d(x)$$
$$= \frac{(e^x+1)^3}{3} + C$$

$$4- \int (\sqrt{x}+1)^3 \cdot \frac{1}{2\sqrt{x}} \cdot d(x)$$
$$= \frac{(\sqrt{x}+1)^4}{4} + C$$

5- $\int (4x+1)^2 \cdot d(x)$

$= \frac{1}{4} \int (4x+1)^2 \cdot 4 \; d(x)$

$= \frac{1}{4} \cdot \frac{(4x+1)^3}{3} + C$

6- $\int (x^4+1)^5 \cdot 4x^3 \cdot d(x)$

$= \frac{(x^4+1)^6}{6} + C$

7- $\int \frac{(x+1)}{\sqrt[3]{x^2+2x+1}} \cdot d(x)$

$\int \frac{(x+1)}{(x^2+2x+1)^{\frac{1}{3}}} \cdot d(x)$

$\frac{1}{2} \int 2(x+1)\,(x^2+2x+1)^{-\frac{1}{3}}$

$\frac{1}{2} \; \frac{(x^2+2x+1)^{\frac{2}{3}}}{\frac{2}{3}}$

$= \frac{3}{4} \; (x^2+2x+1)^{\frac{2}{3}}$

$= \frac{3}{4} \; \sqrt[3]{(x^2+2x+1)^2}$

8- $3\int (x^{\frac{1}{3}}+1)^2 \cdot \frac{1}{3} x^{-\frac{2}{3}} \, d(x)$

$= 3 \cdot \frac{(x^{\frac{1}{3}}+1)^3}{3} \Rightarrow (x^{\frac{1}{3}}+1)^3$

9- $\frac{1}{4}\int (x^4+1)^2 \cdot 4x^3 \, d(x)$

$= \frac{1}{4} \cdot \frac{(x^4+1)^3}{3} = \frac{(x^4+1)^3}{12}$

- $\int \cos x \, d(x) = \sin x$

- $\int \sin x \, d(x) = -\cos x$

- $\int (1+\tan^2 x) \, d(x) = \tan x$

أمثلة *

- $\int f' \cdot \cos(f) \cdot d(x) = \sin(f)$
- $\int 2\cos(2x+1) \cdot d(x) = \sin(2x+1)$
- $\frac{1}{2}\int 2\cos(2x+1) \cdot d(x) = \frac{1}{2} \sin(2x+1)$
- $\frac{1}{2}\int 2x \cdot \cos(x^2) \cdot d(x) = \frac{1}{2} \sin x^2$

$2\int \frac{1}{2\sqrt{x}} \cdot \cos(\sqrt{x}+1) \cdot d(x)$

$= 2\sin(\sqrt{x}+1)$

* قواعد :

- $\int \cos(f) \cdot d(x) = \frac{1}{f'} \sin(f)$
- $\int \sin(f) \cdot d(x) = -\frac{1}{f'} \cos(f)$
- $\int (1+\tan^2(f)) \cdot d(x) = \frac{1}{f'} \tan(f)$
- $\int \frac{1}{\cos^2(f)} \cdot d(x) = \frac{1}{f'} \tan(f)$
- $\int \frac{1}{\sin^2 x} \, d(x) = -\cot(x)$

* حل التمارين الآتية :

- $\int \cos(2x+1) \cdot d(x) = \frac{1}{2} \sin(2x+1)$
- $\int \sin(2x+1) \cdot d(x) = -\frac{1}{2} \cos(2x+1)$
- $\int (1+\tan^2(2x+1)) \, d(x) = \frac{1}{2} \tan(2x+1)$

$\int \frac{1}{\cos^2(2x+1)} \, d(x) = \frac{1}{2} \tan(2x+1)$

$\int \frac{1}{\sin^2(x)} \, d(x) = -\cot(x)$

* قاعدة:

$\int \frac{g'}{g} \, d(x) \Rightarrow \ln|g|$

$]0,+\infty[\rightarrow g$

$]-\infty,0[\rightarrow -g$

$\int \frac{1}{x+2} \, d(x) \; ; \;]0,+\infty[$

$\ln|x+2| = \ln(x+2)$

$; \;]-\infty,0[$

$\ln(-x-2)$

$\frac{1}{2}\int 2 \frac{(x+1)}{x^2+2x+3} \, d(x)$

$x^2+2x+3 \Rightarrow 2x+2$

$\frac{1}{2} \ln|x^2+2x+3|$

$$\int \frac{3}{x+2}\, d(x)$$

$$3\int \frac{1}{x+2}\, d(x)$$

$$3 \ln|x+2|$$

$$\int \frac{5}{2x+3}\, d(x)$$

$$\frac{5}{2}\int \frac{2\times 1}{2x+3}\, d(x)$$

$$\frac{5}{2}\int \frac{2}{2x+3}\, d(x)$$

$$= \frac{5}{2} \ln|2x+3|$$

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* قاعدة:

$$\int e^{g}\, d(x) = \frac{1}{g'}\, e^{g}$$

أمثلة:

$$\int e^{2x+1}\, d(x) = \frac{1}{2}\, e^{2x+1}$$

$$\int e^{x^2}\, d(x) = \frac{1}{2x}\, e^{x^2}$$

$$\int 2\, e^{2x+1}\, d(x) = 2 \cdot \frac{1}{2} e^{2x+1} \Rightarrow e^{2x+1}$$

* دساتیر

- $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$
- $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x)$
- $\sin x = 2 \cdot \sin\frac{x}{2} \cdot \cos\frac{x}{2}$
- $\sin^2 x = 1 - \cos^2 x$
- $\cos^2 x = 1 - \sin^2 x$
- $\sin^2 x + \cos^2 x = 1$

- $\cos(x) \cdot \cos(y) = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$
- $\sin(x) \cdot \sin(y) = -\frac{1}{2}[\cos(x+y) - \cos(x-y)]$
- $\cos(x) \cdot \sin(y) = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$
- $\sin(x) \cdot \cos(y) = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$

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* أمثلة

- $\int \cos^2(x) \cdot d(x)$

$$\int \left[\frac{1}{2} + \frac{1}{2}\cos(2x)\right] \cdot d(x)$$

$$= \frac{1}{2}x + \frac{1}{2} \cdot \frac{1}{2}\sin(2x)$$

$$= \frac{1}{2}x + \frac{1}{4}\sin(2x)$$

- $\int \cos^4(x) \cdot d(x)$

$$\int (\cos^2 x)^2 \cdot d(x)$$

$$\int \left[\frac{1}{2} + \frac{1}{2}\cos(2x)\right]^2 \cdot d(x)$$

$$\int \left[\frac{1}{4} + \frac{1}{2}\cos(2x) + \frac{1}{4}\cos^2(2x)\right] \cdot d(x)$$

$$\int \left[\frac{1}{4} + \frac{1}{2}\cos(2x) + \frac{1}{4}\left[\frac{1}{2} + \frac{1}{2}\cos(4x)\right]\right] \cdot d(x)$$

$$\left[\frac{1}{4}x + \frac{1}{4}\sin(2x) + \frac{1}{8}x + \frac{1}{32}\sin(4x)\right]$$

$$\Rightarrow \frac{3}{8}x + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)$$

$$\bullet \int \tan x \cdot d(x) \quad ; \quad \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[$$

$$- \int \frac{-\sin x}{\cos x} \, d(x)$$

$$\Rightarrow -\ln|\cos x| \Rightarrow -\ln(-\cos x)$$

$$\bullet \int \cot^2(x) \, d(x)$$

$$\int \frac{\cos^2 x}{\sin^2 x} \cdot d(x)$$

$$\int \frac{1-\sin^2 x}{\sin^2 x} \, d(x)$$

$$\int \left(\frac{1}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} \right) d(x)$$

$$\int \left(\frac{1}{\sin^2 x} - 1 \right) d(x)$$

$$= -\cot(x) - x$$

$$\bullet -\int -\cancel{\sin x} \cdot \cos^2 x \cdot d(x)$$

$$= -\frac{\cos^3 x}{3}$$

$$\bullet \int \frac{\sin x}{\cos^3 x} \cdot d(x)$$

$$= -\int -\sin x \cdot \cos^{-3} x \cdot d(x)$$

$$= -\frac{\cos^{-2} x}{-2} \Rightarrow \frac{\cos^{-2} x}{2}$$

$$\bullet \int \cos(3x) \cdot \cos(x) \cdot d(x)$$

$$= \int \frac{1}{2} \left[\cos(4x) + \cos(2x) \right] \cdot d(x)$$

$$= \int \frac{1}{2} \cos(4x) + \frac{1}{2} \cos(2x) \cdot d(x)$$

$$= \frac{1}{2} \cdot \frac{1}{4} \sin(4x) + \frac{1}{2} \cdot \frac{1}{2} \sin(2x)$$

$$= \frac{1}{8} \sin(4x) + \frac{1}{4} \sin(2x)$$

* التكامل المحدد

$$\int_a^b f(x) . d(x) = [F(x)]_a^b$$

$F(b) - F(a)$

$$\int_0^2 1 . d(x) = [x]_0^2$$

$= 2 - 0 = 2$

$$\int_{-1}^{2} (2x - 1) . d(x)$$

$$\left[2 . \frac{x^2}{2} - x\right]_{-1}^{2} \Rightarrow (4 - 2) - (1 + 1) \Rightarrow 2 - 2 = 0$$

$$\int_1^2 \frac{3}{x+1} . d(x)$$

$$3 \int_1^2 \frac{1}{x+1} . d(x)$$

$$3 [\ln|x+1|]_1^2$$

$$= 3 [\ln(3) - \ln(2)]$$

$$= 3 \ln(3) - 3 \ln(2)$$

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$$\bullet \int_{2}^{3} \frac{3}{x-1} \cdot d(x)$$

$$3 \int_{2}^{3} \frac{1}{x-1} \cdot d(x)$$

$$3 \left[Ln|x-1| \right]_{2}^{3} \Rightarrow 3 \left[Ln(2) - Ln(1) \right]$$

$$\Rightarrow 3 Ln(2)$$

* أمثلة

$$1- I = \int_{-1}^{2} (2x-2) \cdot d(x)$$

$$= \left[2\frac{x^2}{2} - 2x \right]_{-1}^{2} = (4-4) - (1-(2)(-1))$$

$$= -3$$

$$2- I = \frac{1}{2} \int_{0}^{1} 2(x-2)(\overset{2x-4}{x^2 - 4x+3}) \, d(x)$$

$$= \frac{1}{2} \left[\frac{(x^2-4x+3)^2}{2} \right]_{0}^{1}$$

$$= \frac{1}{2} \left[\frac{(1-4+3)^2}{2} - \frac{(3)^2}{2} \right]_{0}^{1}$$

$$= \frac{1}{2} \left(\frac{-9}{2} \right) = \frac{-9}{4}$$

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3- $I = \int_2^4 \frac{2}{x-1}$ • I

$= 2 \int_2^4 \frac{1}{x-1} \cdot d(x)$

$2 \left[Ln|x-1| \right]_2^4 \Rightarrow 2 \left[Ln(3) - Ln(1) \right]$

$= 2 Ln 3$

4- $I = \int_1^2 \frac{x^4 + x^3 + x^2 + x}{x^2} \cdot d(x)$

$= \int_1^2 \left[\frac{x^4}{x^2} + \frac{x^3}{x^2} + \frac{x^2}{x^2} + \frac{x}{x^2} \right] \cdot d(x)$

$= \int_1^2 \left[x^2 + x + 1 + \frac{1}{x} \right] \cdot d(x)$

$= \left[\frac{x^3}{3} + \frac{x^2}{2} + x + Ln|x| \right]_1^2$

$= \left[\frac{8}{3} + \frac{4}{2} + 2 + Ln(2) \right] - \left[\frac{1}{3} + \frac{1}{2} + 1 \right]$

$= \left(\frac{8}{3} + 2 + 2 + Ln(2) \right] - \left(\frac{11}{6} \right)$

$= \frac{20}{3} + Ln(2) - \frac{11}{6} \Rightarrow \frac{29}{6} + Ln(2)$

5- $I = \int_1^2 \frac{(x+1)^2}{x} \cdot d(x)$

$$= \int_1^2 \frac{x^2+2x+1}{x} \cdot d(x)$$

$$= \int_1^2 \frac{x^2}{x} + \frac{2x}{x} + \frac{1}{x} \cdot d(x)$$

$$= \int_1^2 x + 2 + \frac{1}{x} \cdot d(x)$$

$$= \left[\frac{x^2}{2} + 2x + \text{Ln}|x| \right]_1^2$$

$$= \left[2 + 4 + \text{Ln}(2) \right] - \left[\frac{1}{2} + 2 + \text{Ln}(1) \right]$$

$$= 6 + \text{Ln}(2) - \frac{5}{2} \Rightarrow \frac{7}{2} + \text{Ln}(2)$$

6- $I = \int_{0}^{\frac{\pi}{4}} \frac{\cos x - \sin x}{\cos + \sin x}$

$\left[\ln |\cos x + \sin x| \right]_{0}^{\frac{\pi}{4}}$

$= \left[\ln \left(\cos \frac{\pi}{4} + \sin \frac{\pi}{4} \right) - \left[\ln \left(\cos (0) + \sin (0) \right) \right] \right]$

$= \ln \sqrt{2} - 0 = \ln \sqrt{2}$

7- $I = \int_{0}^{1} \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

$= \left[\ln |e^{x} + e^{-x}| \right]_{0}^{1} = \ln (e^{1} + e^{-1}) - \ln (e^{0} + e^{0})$

$= \ln \left(e + \frac{1}{e} \right) - \ln (2) = \ln \left(\frac{e^{2}+1}{e} \right) - \ln (2)$

$= \ln \left(\frac{e^{2}+1}{2e} \right)$

8- $I = \int_{-2}^{-1} \frac{2x-1}{x-1} \cdot d(x)$

Note { درجة البسط = درجة المقام

درجة البسط > درجة المقام

⟸ قسمة إقليدية

$$\begin{array}{r|l} 2x-1 & x-1 \\ \cancel{-2x} \mp 2 & 2 \\ \hline 1 & \end{array}$$

$= \int_{-2}^{-1} \left(2 + \frac{1}{x-1} \right) d(x)$

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$$= \left[2x + Ln|x-1|\right]_{-2}^{-1} = \left[2x + Ln(-x+1)\right]_{-2}^{-1}$$

ضربنا بـ -
لأنو المجال سالب

$$= -2 + Ln(2) - \left[-4 + Ln(3)\right]_{-2}^{-1}$$

$$= 2 + Ln(2) - Ln(3)$$

$$= 2 + Ln\left(\frac{2}{3}\right)$$

9- $I = \displaystyle\int_{\frac{3\pi}{2}}^{2\pi} \sqrt{2 - 2\cos x}\; d(x)$

$$= \int_{\frac{3\pi}{2}}^{2\pi} \sqrt{2(1 - \cos(2x)}\; d(x)$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} \sqrt{2.(2\sin^2 x)}\; d(x)$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} 2.|\sin x|\; d(x)$$

$$= \int_{\frac{3\pi}{2}}^{2\pi} 2(-\sin)\; d(x)$$

$$= -2\int_{\frac{3\pi}{2}}^{2\pi} \sin x\; d(x)$$

$= +2 \left[\cos x\right]_{\frac{3\pi}{2}}^{2\pi}$ $= I$

$= 2\left[\cos(2\pi)\right] - \left[\cos\left(\frac{3\pi}{2}\right)\right]$

$= 2(1) - 0 \Rightarrow 2$

10 - $I = \int_{-3}^{-1} x|x+2| \, d(x)$

دراسة إشارة

$x+2 = 0 \Rightarrow x = -2$

x	$-\infty$	-2	$+\infty$
$x+2$	$-$	0	$+$

$I = \int_{-3}^{-2} x(-x-2)\, d(x) + \int_{-2}^{-1} x(x+2) \, d(x)$

$I = \int_{-3}^{-2} (-x^2-2x)\, d(x) + \int_{-2}^{-1} (x^2+2x) \, d(x)$

$I = \left[-\frac{x^3}{3} - x^2\right]_{-3}^{-2} + \left[\frac{x^3}{3} + x^2\right]_{-2}^{-1}$

$= \left[\frac{8}{3} - 4\right] - \left[\frac{27}{3} - 9\right] + \left[\frac{-1}{3} + 1\right] - \left[\frac{-8}{3} + 4\right]$

$= \frac{8}{3} - 4 + \frac{2}{3} + \frac{8}{3} - 4 \Rightarrow I = 6 - 8 = -2$

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11- $I = \int_1^3 2 - |2-x| \, d(x)$

$2 - x = 0 \Rightarrow x = 2$

x	$-\infty$		2		$+\infty$
$2-x$		+	0	−	

$I = \int_1^2 2 - (2-x) \, d(x) + \int_2^3 2 - (-2+x) \, d(x)$

$= \int_1^2 2 - 2 + x \, d(x) + \int_2^3 2 + 2 - x \, d(x)$

$= \int_1^2 x \, d(x) + \int_2^3 4 - x \, d(x)$

$= \left[\frac{x^2}{2}\right]_1^2 + \left[4x - \frac{x^2}{2}\right]_2^3$

$I = \left[2 - \frac{1}{2}\right] + \left[12 - \frac{9}{2}\right] - \left[8 - 2\right]$

$I = \frac{3}{2} + \frac{15}{2} - 6 = 9 - 6 = 3$

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12 $I = \int \frac{x+1}{x-2} \, d(x)$

$I = \int \left(1 + \frac{3}{x-2}\right) . d(x)$

$$\begin{array}{r|l} x-2 & \overline{\;x+1\;} \quad 1 \\ & \underline{-\cancel{x} \pm 2} \\ & \quad 3 \end{array}$$

$I = x + 3 . \ln|x-2|$

* تفريقة الكسور

درجة المقام أكبر من درجة البسط

$\boxed{\frac{\quad}{\quad}} = \frac{A}{(\quad)} + \frac{B}{(\quad)}$

* تمرين :

1 - $I = \int \frac{1}{x^2 - x - 2} \, d(x)$

$\frac{1}{x^2 - x - 2} = \frac{A}{(x-2)} + \frac{B}{(x+1)}$

$\frac{1}{\cancel{x^2 - x - 2}} = \frac{A(x+1) + B(x-2)}{\cancel{(x-2)(x+1)}}$

$\Rightarrow 1 = A(x+1) + B(x-2)$

$A = -1$ | $B = 2$

$1 = B(-3)$ | $1 = 3A$

$B = \frac{-1}{3}$ | $A = \frac{1}{3}$

$$\int \frac{\frac{1}{3}}{(x-2)} + \frac{\frac{-1}{3}}{(x+1)} \, d(x)$$

$$= \frac{1}{3} \int \frac{1}{x-2} \, d(x) - \frac{1}{3} \int \frac{1}{x+1} \, d(x)$$

$$= \frac{1}{3} \ln|x-2| - \frac{1}{3} \ln|x+1|$$

2- $I = \int_0^1 \frac{2x+1}{x^2+3x+2} \, d(x)$

$$\frac{2x+1}{x^2+3x+2} = \frac{A}{(x+2)} + \frac{B}{(x+1)}$$

$$\frac{2x+1}{x^2+3x+2} = \frac{A(x+1) + B(x+2)}{(x+2)(x+1)}$$

$$2x+1 = A(x+1) + B(x+2)$$

$x = -1$ | $x = -2$

$x = -1$ | $-3 = -A$

$B = -1$ | $A = 3$

$$\Rightarrow \int_0^1 \frac{3}{x+2} + \frac{-1}{x+1} \, d(x)$$

$$\int_0^1 \frac{3}{x+2}\, d(x) + \int_0^1 \frac{-1}{x+1}\, d(x)$$

$$3\int_0^1 \frac{1}{x+2}\, d(x) - \int_0^1 \frac{1}{x+1}\, d(x)$$

$$= 3\left[\ln|x+2|\right]_0^1 - \left[\ln|x+1|\right]_0^1$$

$$= 3\left[\ln 3 - \ln 2\right] - \left[\ln 2 - \cancel{\ln 1}^{0}\right]$$

$$= 3\left[\ln 3 - \ln 2 - \ln 2\right]$$

$$= 3\left[\ln \frac{3}{2} - \ln 2\right] \Rightarrow 3\left[\ln \frac{3}{4}\right]$$

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3. $I = \int_0^1 \frac{1}{x^2+x-6}\, d(x)$

$$\frac{1}{x^2+x-6} = \frac{A}{(x+3)} + \frac{B}{(x-2)}$$

$$\frac{1}{x^2+x-6} = \frac{A(x-2)+B(x+3)}{(x+3)(x-2)}$$

$$1 = A(x-2) + B(x+3)$$

$x = -3$ | $x = 2$

$1 = -5A \Rightarrow A = \frac{-1}{5}$ | $1 = 5 \cdot B \Rightarrow B = \frac{1}{5}$

$$\int_0^1 \frac{-\frac{1}{5}}{x+3} + \frac{\frac{1}{5}}{x-2} \; d(x)$$

$$\int_0^1 \frac{-\frac{1}{5}}{x+3} \, d(x) + \int_0^1 \frac{\frac{1}{5}}{x-2} \, d(x)$$

$$= -\frac{1}{5} \int_0^1 \frac{1}{x+3} \, d(x) + \frac{1}{5} \int_0^1 \frac{1}{x-2} \, d(x)$$

$$= -\frac{1}{5} \Big[\ln|x+3| \Big]_0^1 + \frac{1}{5} \Big[\ln|x-2| \Big]_0^1$$

$$= -\frac{1}{5} \Big[\ln(x+3) \Big]_0^1 + \frac{1}{5} \Big[\ln(-x+2) \Big]_0^1$$

$$= -\frac{1}{5} \Big[\ln 4 - \ln 3 \Big] + \frac{1}{5} \Big[\overset{0}{\cancel{\ln(1)}} - \ln 2 \Big]$$

$$= -\frac{1}{5} \ln\left(\frac{4}{3}\right) - \frac{1}{5} \ln 2$$

* التكامل بالتجزئة:

! قانونه:

$$\int u \cdot v' = [u \cdot v]_a^b - \int_a^b v \cdot u'$$

سؤال

هو عبارة عن جداء تابعين مختلفين.

- $\int \overset{u}{x^n} \cdot \overset{v'}{\sin x} \cdot d(x)$

- $\int \overset{u}{x^n} \cdot \overset{v'}{\cos x} \cdot d(x)$

- $\int \overset{v'}{x^n} \cdot \overset{u}{\ln x} \cdot d(x)$

$\int \overset{u}{x^n} \cdot \overset{v'}{e^x} \cdot d(x)$

$\int \overset{v'}{e^x} \cdot \underset{\sin x}{\overset{u}{\cos x}} \cdot d(x)$ (دوري يكرر نفسه)

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* ترتيب التوابع حسب الأقوى.

1. اللوغاريتمي
2. كثير حدود
3. المثلثي
4. الأسي

الأقوى هو: u الأضعف هو: v'

* مثال :

1- $I_1 = \int_0^{\pi} x \cdot \sin x \cdot d(x)$

$u = x \Rightarrow u' = 1$

$v' = \sin x \Rightarrow v = -\cos x$

$= \left[x \cdot (-\cos x)\right]_0^{\pi} + \int_0^{\pi} \cos x \cdot d(x)$

$= \left[x \cdot (-\cos x)\right]_0^{\pi} + \left[\sin x\right]_0^{\pi}$

$= \left[\pi(-\cos \pi) \cancel{-0}\right] + \left[\cancel{\sin \pi - \sin 0}\right]$

$= \boxed{\pi}$

2- $I_2 = \int_0^{\pi} x^2 \cos x \cdot d(x)$

$u = x^2 \Rightarrow u' = 2x$

$v' = \cos x \Rightarrow v = \sin x$

$= \left[x^2 \cdot \sin x\right]_0^{\pi} - \int_0^{\pi} 2x \cdot \sin x \cdot d(x)$

$= \left[x^2 \cdot \sin x\right]_0^{\pi} - 2 \cdot \int_0^{\pi} \underbrace{x \cdot \sin x}_{I_1} \cdot d(x)$

$u = x \Rightarrow u' = 1$

$v' = \sin x \Rightarrow v = -\cos x$

$\left[x \cdot (-\cos x)\right]_0^{\pi} + \int_0^{\pi} \cos x \cdot d(x)$